Create Better Tests, Lessons, and Mathematical Thinkers with Rigorous Rubrics

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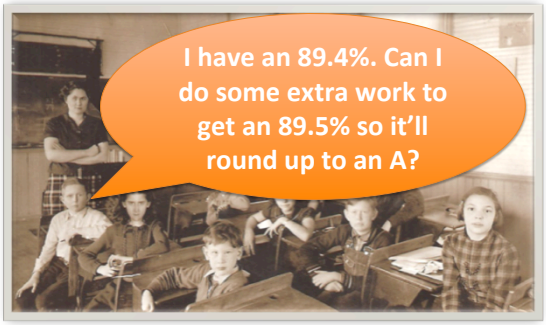
NCTM Regional 2015

Problems to Solve



Problems to Solve

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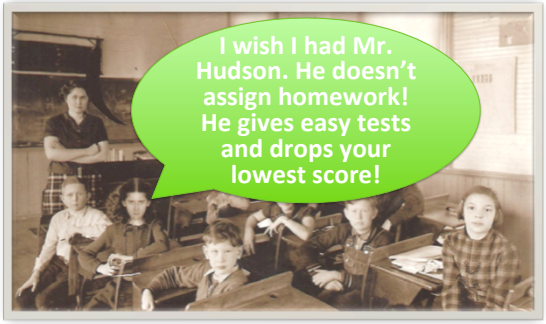

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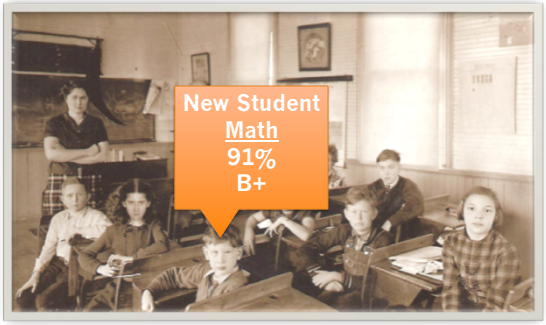
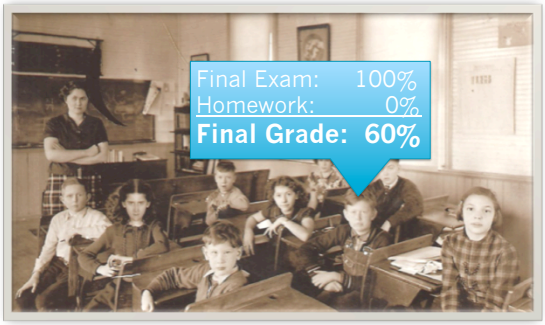


Problems to Solve



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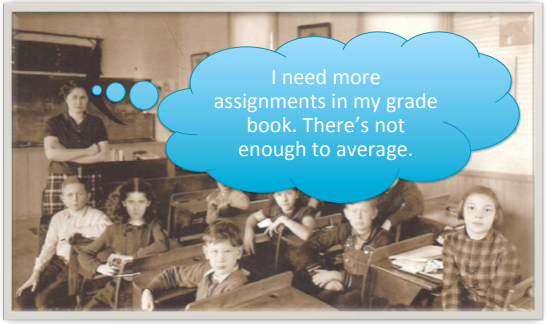


NY Times, 10/5/2014

The **final exam** for Math 96 ["developmental math"] would make up **35 percent** of the total grade, and as the day of the test approached, Mr. de Jesus knew that with the **demerits** he would face **for his poor attendance and his unfinished homework**, there was little chance he would pass. On the morning of the exam, he didn't show up, and **he failed the class for the third time**. As it happened, more than **40 percent of the students** in the class **also failed**.

Community College Students Face a Very Long Road to Graduation
by Ginia Bellafonte

Common Failure by Design

- Over-weighted final exam.
- Likely use of arithmetic mean.
- Penalties unrelated to content achievement
  - Attendance is not a proxy for understanding
  - A zero means no evidence of learning was collected
- 1 student failing 3 times
- 40% of students failing in a single term

We have the
WRONG GOALS
for students in Math

What are the RIGHT Goals?

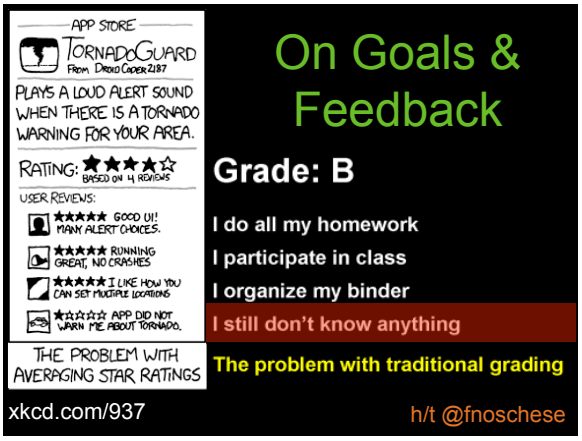
- What is mathematics?
- What are a mathematician's habits of mind and dispositions?
- What is a school math program "in business" to accomplish?

We give
POOR FEEDBACK
to students in Math

What is QUALITY Feedback?

What percentage did your doctor give you at your most recent routine check-up?

Wrong Goals
\+ Poor Feedback
Low Achievement



Key Questions

1. What are rigorous learning goals?
2. How is competence defined and measured?
3. How should student progress & learning be reported?
4. What manageable alternatives are there to traditional averages, percentages, & overly weighted finals?
5. How can we help all students meet higher standards?
6. What should assessments look like when students have technology readily available?
7. How can we prevent "point grubbing?"

Share which one resonates most for you.

Session as Advertised

Students often care more about points and less about understanding mathematics because **percent-based grading systems distract from key outcomes**. Learn how **teachers collaborated** to turn standards into Novice-Expert rubrics that improved tests, lessons and student performance. Hear how **rubrics transform curriculum, grading & rigor at any grade level.**

Algebra 1 End of Course Exam

	Below Basic	Basic	Proficient	Advanced
Algebra 1A	7 1%	94 18%	303 58%	118 23%
A	0	0	57	49
B	1	20	116	40
C	1	31	83	20
D	3	32	33	8
F	2	11	14	1

	Below Basic	Basic	Proficient	Advanced
Algebra 1B	68 18%	220 59%	84 23%	2 0.5%
A	2	18	23	1
B	6	61	28	0
C	16	60	22	1
D	23	63	6	0
F	21	18	5	0

Algebra 1 Program

Parkway School District St. Louis, MO	• K-12 with over 17,000 students • 4 Traditional HS, 1 Alternative HS • 60 HS Math Teachers
District Algebra 1 Curriculum Team	• Common Assessments & Grading Practices • Asst Supt, Principals, Dept Chairs in Support • Summer Workshops, Release Days
Confluence	• Mission "ALL" Students • Eliminated B Track • Teacher Evaluation Rubrics

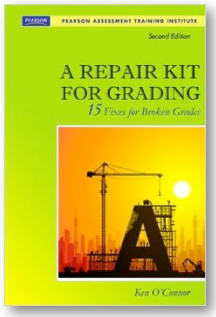
% Proficient & Advanced Algebra 1 End-of-Course Exam (MO)

	2009	2010	2011	2012
Central HS	50%	62%	66%	**81%**
North HS	43%	52%	52%	**68%**
South HS	47%	52%	67%	**78%**
West HS	72%	72%	77%	**86%**
District	51%	58%	66%	**78%**
Total Students	1001	960	947	813

A Repair Kit for Grading:

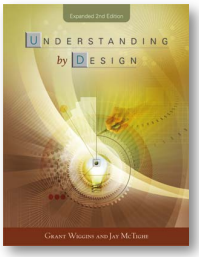
15 Fixes for Broken Grades

by Ken O'Connor



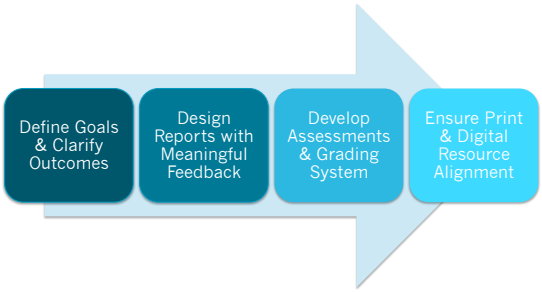
Plan Backwards

1. Identify desired results
2. Determine acceptable evidence
3. Plan learning experiences and instruction

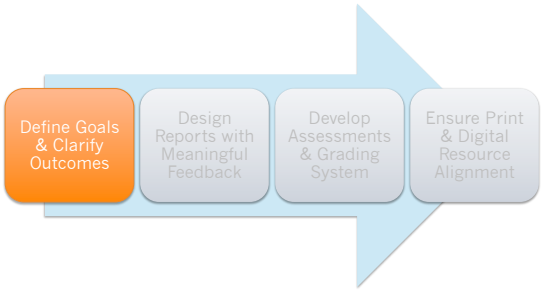


Understanding by Design, Wiggins & McTighe, ©2005

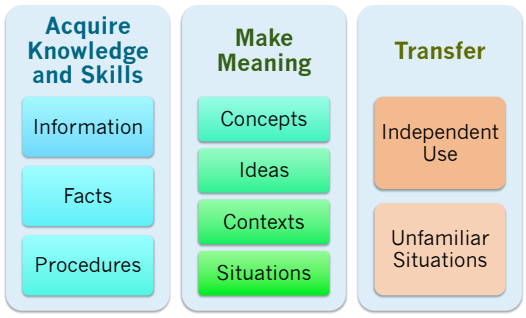
Math Program Design



Math Program Design



Learning Outcomes A-M-T



© Authentic Education

Pop Quiz

For a bicycle race, Donald's time was:

3 hours, 4 minutes, and 11 seconds.

Keina's time was:

2 hours, 58 minutes, and 39 seconds.

How long was Keina finished before Donald crossed the finish line?

one strategy

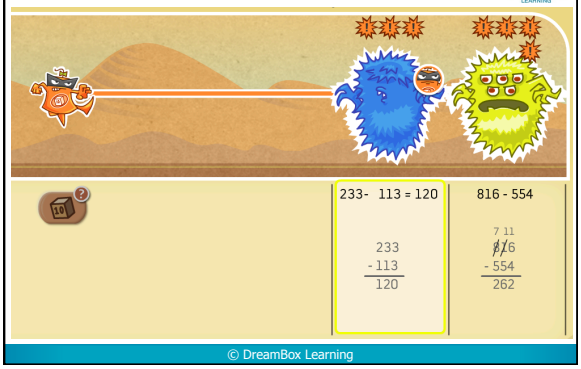
Hours	Minutes	Seconds
2	58	39
0	5	32

304 – 298 = ?

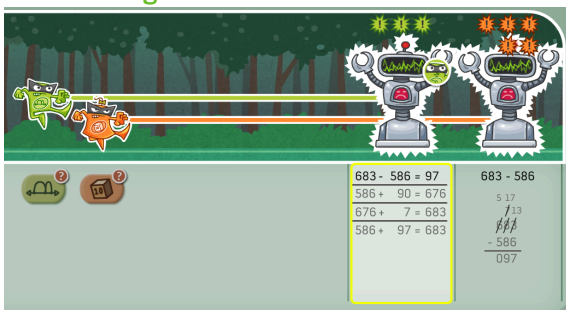
Invented Strategy?

$$\begin{array}{r} 304 \\ -\ 298 \\ \hline 006 \end{array}$$

Two Monsters (Problems). One Strategy.



Two Identical Robots (Problems). Two Strategies.



Do You Want Students to…

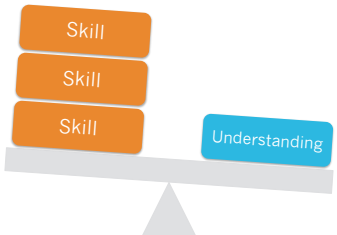
Acquisition	Meaning & Transfer
compute mean, median, and mode?	know which measure is best for the situation?
know 8 + 7?	not grab a pencil or calculator to solve 3,998 + 4,247?
know their 12 x 12 "facts?"	not grab a pencil or calculator to solve 13 x 12?

Do your students know you want all of these outcomes?

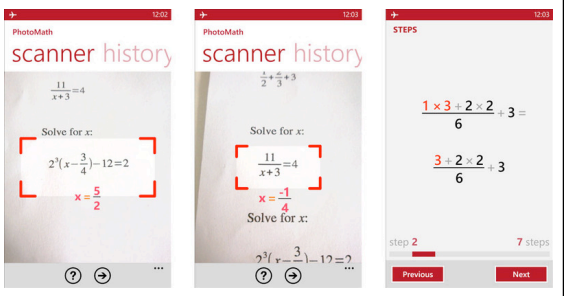
Acquisition

Meaning & Transfer

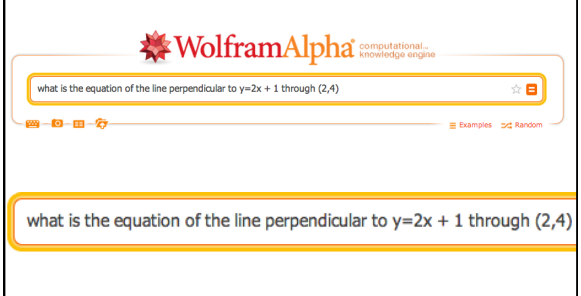
Skills Understanding



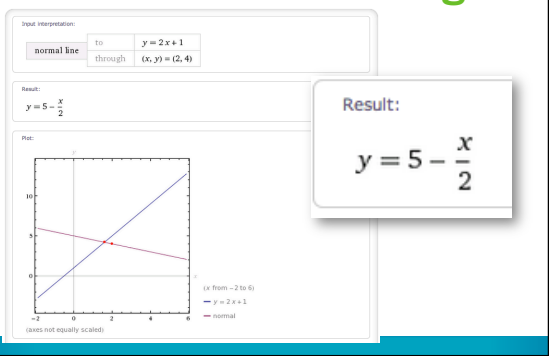
PhotoMath



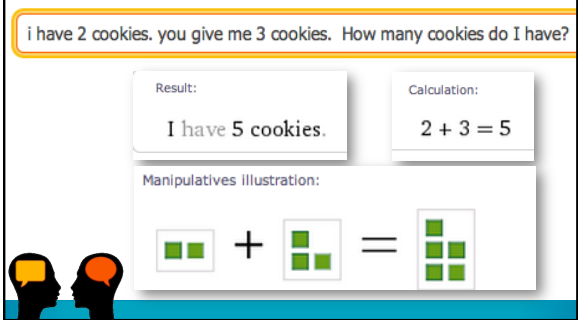
Algebra 1



Skill or Understanding?



What Grade does Wolfram|Alpha get?



Better Goals for Students

David Bressoud, (cont'd)

"There are **important mathematical ideas** behind these methods, and showing one knows how to solve these problems is **one way** of exhibiting working knowledge of these ideas."

http://www.maa.org/external_archive/columns/launchings/launchings_08_09.html

Better Goals for Students

David Bressoud, (cont'd)

"If a student feels that she or he has **learned nothing** that cannot be pulled directly from Wolfram|Alpha, then the course really has been **a waste of time**."

http://www.maa.org/external_archive/columns/launchings/launchings_08_09.html

What were the Goals?

From a 5th grade teacher in NY:

"I had a lot of good people teaching me math when I was a student – earnest and funny and caring. But the math they taught me wasn't good math. Every class was the same for eight years:

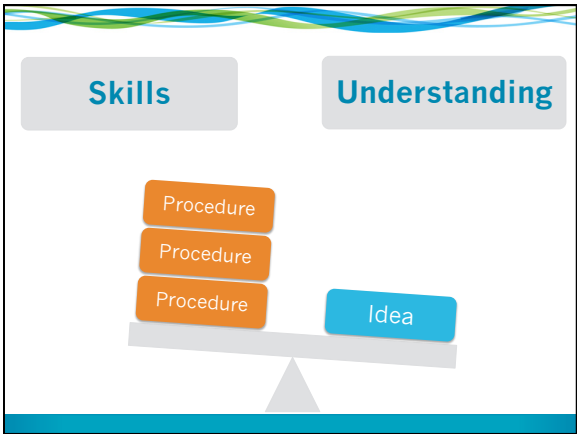
'Get out your homework, go over the homework, here's the new set of exercises, here's how to do them. Now get started. I'll be around.'"

Teaching What Matters Most by Strong, Silver, and Perini, © 2001, p. 55

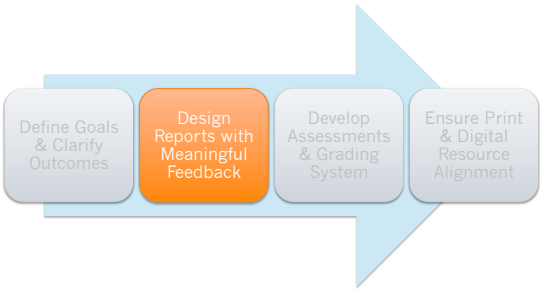
What were the Goals?

"They were so concerned with making sure we knew how to do every single procedure we never learned how to **think mathematically**. I did well in math but I never understood what I was doing. **I remember hundreds of procedures but not one single mathematical idea**."

p. 55, Teaching What Matters Most, Strong, Silver, & Perini, ©2001

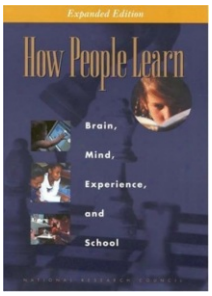


Math Program Design



How People Learn

1. Learning: From Speculation to Science
2. **How Experts Differ from Novices**
3. Learning and Transfer
4. How Children Learn
5. Mind and Brain



Measuring Anything

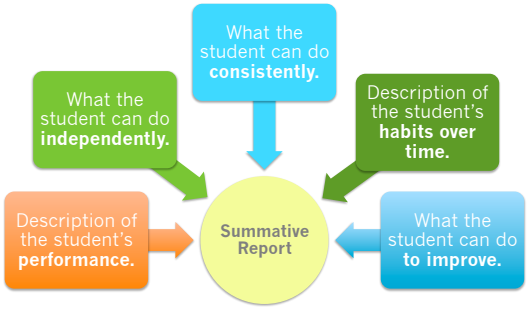
1. If it matters at all, it is detectable or observable
2. If it is detectable, it can be detected as an amount (or **a range of possible amounts**)
3. If it can be detected as a range of possible amounts, it is measurable

How to Measure Anything, D.W. Hubbard

Range of Understanding

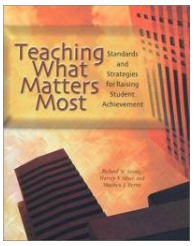
If you want it as an outcome for your students, you can build a Novice-Expert rubric for it

Rubrics Should Communicate…



Rubric for Thought

	Expert	Practitioner	Apprentice	Novice
Inquiry				
Knowledge Acquisition				
Problem Solving				
Communication				
Reflection				



Teaching What Matters Most by Strong, Silver, and Perini, © 2001, p. 58

Process: Inquiry

Expert	Proficient	Developing	Novice
Can formulate questions and seek answers independently; generates, tests, and refines hypotheses according to well-formulated criteria; uses evidence powerfully and persuasively; foresees and responds to counterarguments.	Looks for and uses questions to guide investigation; uses criteria to generate hypotheses; uses evidence effectively, but may fail to fully address counterarguments	Can use pre-drafted questions to direct investigation but needs help formulating her own; may have trouble telling quality hypotheses apart from guesses; substantiates some claims; pays little attention to counterarguments	Fails to look for questions to guide investigation; generates hypotheses haphazardly; fails to use evidence to substantiate claims

Teaching What Matters Most by Strong, Silver, and Perini, © 2001, p. 58

Process: Problem Solving

Expert	Practitioner	Developing	Novice
Is constantly looking for and posing relevant questions; experiments with a variety of solutions and perspectives	Restates problems; understands there is more than one way to attack a problem; surveys own understanding to determine progress toward solution	Accepts problems on their own terms (e.g., rarely restates them to make them more meaningful); often generates only one or two obvious solutions	Avoids difficult problems; looks for convenient solutions; rarely questions ideas

Teaching What Matters Most by Strong, Silver, and Perini, © 2001, p. 58

Concept & Skill: Central Tendency

Expert	Proficient	Developing	Novice
Apply new and unfamiliar statistical measures to make predictions and draw conclusions.	**Justify the most appropriate statistical measures** of center to make predictions and draw conclusions.	**Apply** mean, median, mode, and range to solve problems and make predictions. **(MO Alg 1 D2A)**	**Compute** mean, median, mode, and range given a data set.

Knowledge: Function Properties

Expert	Proficient	Developing	Novice
Given tables, graphs, or equations of **unfamiliar non-linear functions**, determine and define properties of those functions.	Given a table, graph, or equation, **classify** a function as **linear, quadratic, or exponential** and justify your answer.	Given a table, graph, or equation, **classify** a function as **linear or non-linear** and justify your answer. **(MO Alg 1 A1D)**	Given a table or graph, classify a relationship as a **function or non-function** and justify your answer.

Skill: Factoring Polynomials

Expert	Proficient	Developing	Novice
Factor polynomials with **more than three terms, more than one variable, or a degree higher than two**.	**Completely factor** any given **quadratic** expression. **MO Alg 1 A2B**	Factor trinomials with a **leading coefficient of one**.	Factor out a **Greatest Common Factor** (GCF) from any polynomial.

High School Algebra 1 Rubric, First Semester, 2011-12, Parkway Schools (rev. 10/11)

Standard	Expert	Proficient	Developing	Novice
1. Represent Relationships (Creating Mathematical Models)	Translate quickly and fluently between contexts, tables, graphs, and equations. Select the best representation of a problem in context based on audience and purpose.	1.1 Meaningfully and mathematically represent a contextual situation in multiple ways. Represent a problem in context with a data table, graph, and equation (linear, quadratic, and exponential).	Translate an equation into a graph.	Translate data in a table into a graph. Translate an equation or a graph into a data table.
	Compare all real numbers and place them on a number line.	1.2 Represent, compare, and order rational and irrational numbers, including approximate locations on a number line. **N1A***	Place numbers on a number line and write inequalities if they are all in the same format (i.e., decimals, fractions), if they have the same denominator or the same number of decimal places.	Place numbers on a number line and write inequalities if the numbers are all in the same format (either whole numbers, decimals to two places, or simple fractions).
2. Identify Relationships (Classifying Mathematical Models)	Given tables, graphs, or equations of unfamiliar non-linear functions, determine and define properties of those functions.	2.1 Given a table, graph, or equation, classify a function as linear, quadratic, or exponential and justify your answer.	Given a table, graph, or equation, classify a function as linear or non-linear and justify your answer. **A1D**	Given a table or graph, classify a relationship as a function or non-function and justify your answer.
	Determine several models (including unfamiliar, non-linear functions) that might represent a given situation. Of those options, justify the model that best represents the situation.	2.2 Determine the type(s) of functions (linear, quadratic, or exponential) that might model a given situation. Of those options, justify the type of function that best models the situation. **A3A**	Explain the similarities and differences of tables, graphs, or equations of linear, quadratic, and exponential relationships. **A1C**	Explain the similarities and differences in the tables or graphs of linear and non-linear functions.
3. Analyze Relationships (Making Predictions with Mathematical Models)	Justify the relevant domain and range of any relationship from context.	3.1 Justify the relevant domain and range of a linear, quadratic, or exponential relationship from context.	Determine the domain and range of a relationship from an equation or graph.	Determine the domain and range of relationships given a table.
	Generate an equation that might model a given situation that appears to be linear and use it to make predictions about future data.	3.2 Consider multiple equations that might model a situation. Select and justify the best model for predicting the relationship.	Make and justify predictions about a relationship when given a table.	Make and justify predictions about a relationship when given a graph, including scatter plots. **D3A**
	Apply new and unfamiliar statistical measures to make predictions and draw conclusions.	3.3 Justify the most appropriate statistical measures of center to make predictions and draw conclusions.	Apply mean, median, mode, and range to solve problems and make predictions. **D2A**	Find the mean, median, mode, and range of a set of numbers.

* The codes in bold, such as **N1A**, are the Missouri state Course Level Expectations (CLE) for Algebra 1

page 1

Standard	Expert	Proficient	Developing	Novice
4. Operate on Polynomials	Apply additional operations on polynomials, including division, negative exponents, fractional exponents, or powers raised to a power.	4.1 Perform exponent operations, addition, subtraction, and multiplication of polynomials. **A2B**	Multiply a monomial by any polynomial and a binomial by a binomial.	Add and subtract polynomials.
	Factor polynomials with more than three terms, more than one variable, or a degree higher than two.	4.2 Completely factor any given quadratic expression. **A2B**	Factor trinomials with a leading coefficient of one.	Factor out a Greatest Common Factor (GCF) from any polynomial.
5. Interpret and Create Graphs	Identify and interpret the intercepts, rate of change, and maximum and minimum of any function when given a table or a graph.	5.1 Identify and interpret the intercepts, rate of change, and maximum and minimum of any linear, quadratic, and exponential functions when given a table or a graph. **A4A**	Identify and interpret the rate of change of linear and exponential functions when given a table.	Identify the intercepts of any function when given a graph as well as maximums and minimums when applicable.
	Sketch a quick graph of any polynomial function when given its equation.	5.2 Sketch quick graphs of linear, quadratic, and exponential functions from equations in various forms.	Find intercepts given the equations of linear, exponential, and quadratic functions.	Find the maximum or minimum of a quadratic function given its equation.

Algebra 1 Rubric, First Semester, Parkway Schools, 2011-12 (rev. 10/11) page 2

Imagine receiving this level of detail when a new student transfers into your class.

	Proficient	Developing	Novice
	1.1 Meaningfully and mathematically represent a contextual situation in multiple ways. Represent a problem in context with a data table, graph, and equation (linear, quadratic, and exponential).	Translate an equation into a graph.	Translate data in a table into a graph. Translate an equation or a graph into a data table.
	1.2 Represent, compare, and order rational and irrational numbers, including approximate locations on a number line. **N1A***	Place numbers on a number line and write inequalities if they are all in the same format (i.e., decimals, fractions), if they have the same denominator or the same number of decimal places.	Place numbers on a number line and write inequalities if the numbers are all in the same format (either whole numbers, decimals to two places, or simple fractions).
2. Identify Relationships (Classifying Mathematical Models)	2.1 Given a table, graph, or equation, classify a function as linear, quadratic, or exponential and justify your answer.	Given a table, graph, or equation, classify a function as linear or non-linear and justify your answer. **A1D**	Given a table or graph, classify a relationship as a function or non-function and justify your answer.
	2.2 Determine the type(s) of functions (linear, quadratic, or exponential) that might model a given situation. Of those options, justify the type of function that best models the situation. **A3A**	Explain the similarities and differences of tables, graphs, or equations of linear, quadratic, and exponential relationships. **A1C**	Explain the similarities and differences in the tables or graphs of linear and non-linear functions.
3. Analyze Relationships (Making Predictions with Mathematical Models)	3.1 Justify the relevant domain and range of a linear, quadratic, or exponential relationship from context.	Determine the domain and range of a relationship from an equation or graph.	Determine the domain and range of relationships given a table.
	3.2 Consider multiple equations that might model a situation. Select and justify the best model for predicting the relationship.	Make and justify predictions about a relationship when given a table.	Make and justify predictions about a relationship when given a graph, including scatter plots. **D3A**
	3.3 Justify the most appropriate statistical measures of center to make predictions and draw conclusions.	Apply mean, median, mode, and range to solve problems and make predictions. **D2A**	Find the mean, median, mode, and range of a set of numbers.

Key Points

- Avoid negatives: "~~The student CAN'T…~~"
- Share with students & parents up front
- Expected to be Novice at the start?
  - Yes for content (i.e., parabolas, algorithms)
  - No for process (i.e., inquiry, problem solving, Common Core SMPs)
- These are not meant to create "ability groups." Engage rich tasks together.

Key Point

- If it's an outcome we want – or promise – we **MUST** assess it and report progress to students and parents
- If it's not assessed and reported, we'll never know if students have demonstrated it. (So why even bother to pretend we do it?)

The Mission of the Parkway School District is to ensure

all students are capable, curious, and confident learners who **understand and respond** to the **challenges** of an **ever-changing world**.

Curious Learners

Expert	Practitioner/ Developing	Novice
I continually ask insightful questions both inside and outside of class that extend the conversation and learning into new areas.	I ask some questions before and during class, but often only when prompted to, and only in relation to the current conversation and lesson.	I do not ask questions either before or after being presented with information in class. I ask unrelated questions or just ask for facts.
I don't care if I'm wrong, fail, or make a mistake. These experiences only improve my understanding.	When I'm presented with a challenge, I usually keep at it until I solve it. I'm not complacent with just simple answers.	If I'm not successful, I stop trying. I only ever think about problems someone else tells me about.
When I am presented with new information, I always ask questions to determine its value and credibility.	I often trust what I hear or read, but if something sounds really weird, I ask questions to learn more.	I immediately accept what is presented. I want an easy answer or method, so I can mindlessly use it forever.

Evidence of Curiosity: Assess Student Questions

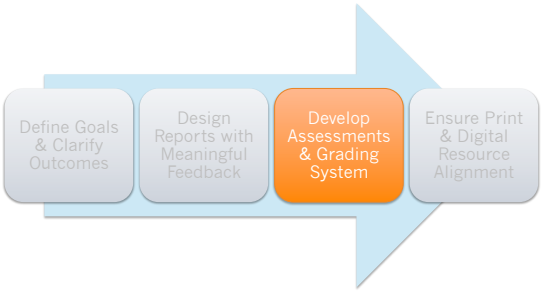
East High School has been recording the number of 12th graders who drop out of school before earning a diploma. The principal of East High School has asked you to help her reduce the number of students who drop out of school. She gives you this data table:

Dropping Out at East High School in 12th Grade from 2001-2011

School Year	2001-02	02-03	03-04	04-05	05-06	06-07	07-08	08-09	09-10	10-11
Number of Seniors who Dropped Out	21	24	25	48	24	27	25	28	32	30

1. Write two questions you would ask the principal at East High School about these dropout data.

Math Program Design



Assessment Design

- Rubrics = design schematics for tests.
  - If we couldn't write an assessment item aligned with the rubric, we re-worked the rubric until it described measurable student performance.
- Four test sections: a few problems aligned with each rubric category.
- Students couldn't only complete the Proficient and Expert items on a test.

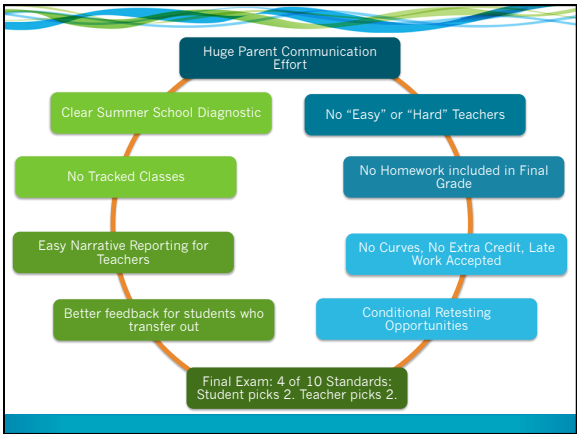
A, B, C … That's All

The semester grade will not be calculated using percentages, but rather by using the chart below. Please note that students not earning a C or higher will be required to re-take the first semester of Algebra 1.

Ratings	Number of Ratings (out of 11) used to Determine the Semester Grade			
	A	B	C	Retake 1st Semester
Expert	At least 5	--	--	--
Proficient	--	--	--	--
Developing	0	2 or fewer	4 or fewer	More than 4
Novice	0	0	0	Any

To help understand this assessment system, here are a few examples:

- 3 Expert ratings, 4 Proficient, and 4 Developing - Semester Grade is a C
- 2 Expert rating, 7 Proficient, and 2 Developing - Semester Grade is a B
- 7 Expert ratings, 2 Proficient, and 2 Developing - Semester Grade is a B
- 8 Expert ratings and 3 Proficient ratings - Semester Grade is an A
- 6 Proficient ratings and 5 Developing ratings - Retake 1st Semester
- 7 Proficient ratings, 3 Developing, and 1 Novice - Retake 1st Semester



Better Help Sessions

Juniors in Honors Pre-Calculus:

- "I have an 89.4%. Can I do some extra work to get an 89.5% so it'll round up to an A. My GPA could use a 4.0 instead of a 3.0."
- "I need help on Section 3.2."

Freshmen in Algebra 1:

- "I know I can compute mean, median, and mode, but I really don't know which one is best. Can you help me understand how to pick?"

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When you change the assessment system, you change the conversation with students.

3rd Grade Alternate Approach

On a 15-item test

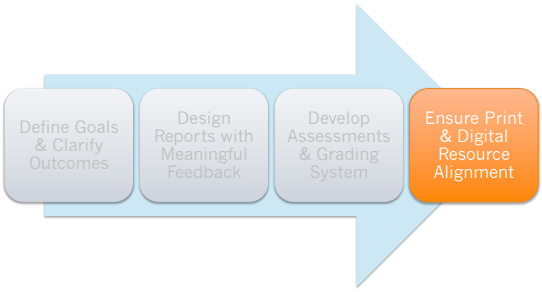
	Novice	Developing	Proficient	Expert	Total
Number of Items *Completely* Correct					
Number of Total Items	5	5	3	2	15

First Trimester # Correct Items 30 Total Test Items 4 Expert 6 Proficient 10 Developing 10 Novice		Combined Number of Proficient AND Expert Items Completely Correct (out of 10)					
		10	9	7-8	5-6	3-4	< 3
Combined Number of Novice AND Developing Items Completely Correct (out of 20)	18-20	A	A	B	B	C	N
	16-17	A	B	B	C	C	N
	14-15	B	B	C	C	N	N
	12-13	B	C	C	N	N	N
	10-11	C	C	N	N	N	N
	< 10	N	N	N	N	N	N

First Trimester # Correct Items 30 Total Test Items 4 Expert 6 Proficient 10 Developing 10 Novice		Combined Number of Proficient AND Expert Items Completely Correct (out of 10)					
		10	9	7-8	5-6	3-4	< 3
Combined Number of Novice AND Developing Items Completely Correct (out of 20)	18-20	28-30	29	25-28	23-26	21-24	N
	16-17	26-27	25-26	23-25	21-23	19-21	N
	14-15	24-25	23-24	21-23	19-21	N	N
	12-13	22-23	21-22	19-21	N	N	N
	10-11	20-21	19-20	N	N	N	N
	< 10	N	N	N	N	N	N

First Trimester % Correct Items 30 Total Test Items 4 Expert 6 Proficient 10 Developing 10 Novice		Combined Number of Proficient AND Expert Items Completely Correct (out of 10)					
		10	9	7-8	5-6	3-4	< 3
Combined Number of Novice AND Developing Items Completely Correct (out of 20)	18-20	93-100%	90-97%	83-93%	77-87%	70-80%	< 77%
	16-17	87-90%	83-87%	77-83%	70-77%	63-70%	< 67%
	14-15	80-83%	77-80%	70-77%	63-70%	< 63%	< 60%
	12-13	73-77%	70-73%	63-70%	< 63%	< 57%	N
	10-11	67-70%	63-67%	57-63%	< 57%	N	N
	< 10	< 67%	< 63%	< 60%	N	N	N

Math Program Design



What is Implied?

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IF…

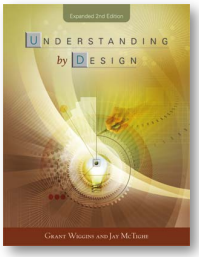
Students need to independently justify the most appropriate statistical measures of center to make predictions and draw conclusions.

THEN…

How do lessons and assignments need to be designed?

Plan Backwards

1. Identify desired results
2. Determine acceptable evidence
3. Plan **learning experiences** and **instruction**



Understanding by Design, Wiggins & McTighe, ©2005

Draft CC SMP 7 Rubric

Look for and make use of structure.

Expert	Proficient	Developing	Novice
I look for patterns and structures outside of class and make connections that are not immediately obvious. I use and analyze the value of these structures.	I look closely to discern useful patterns and structures in class. I step back to get an overview and shift perspective to gain more insight. I make use of structures I find to solve problems.	I look haphazardly for patterns when asked, but I'm not sure how to look or to know when I've found something useful. I am able to use simple structures.	I look for simple, obvious patterns. I am able to use simple structures with someone's help.

Who Did the Looking?

From a 5th grade teacher in NY:

"I had a lot of good people teaching me math when I was a student – earnest and funny and caring. But the math they taught me wasn't good math. Every class was the same for eight years:

'Get out your homework, go over the homework, here's the new set of exercises, here's how to do them. Now get started. I'll be around.'"

Teaching What Matters Most by Strong, Silver, and Perini, © 2001, p. 55

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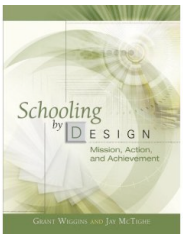
Learning Principle

- "An understanding is a **learner realization** about the power of an idea."
- "Understandings cannot be given."
- "Understandings have to be **engineered so that learners see for themselves** the power of an idea for making sense of things."



p. 113, Schooling by Design, Wiggins & McTighe, ©2007

Learning is not accomplished by putting thoughts into a mind, but rather by empowering a mind to generate thoughts.

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Thank you

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High School Algebra 1 Rubric, First Semester, 2011-12, Parkway Schools (rev. 10/11)

Standard	Expert	Proficient	Developing	Novice
1. Represent Relationships (Creating Mathematical Models)	Translate quickly and fluently between contexts, tables, graphs, and equations. Select the best representation of a problem in context based on audience and purpose.	1.1 Meaningfully and mathematically represent a contextual situation in multiple ways. Represent a problem in context with a data table, graph, and equation (linear, quadratic, and exponential).	Translate an equation into a graph.	Translate data in a table into a graph. Translate an equation or a graph into a data table.
	Compare all real numbers and place them on a number line.	1.2 Represent, compare, and order rational and irrational numbers, including approximate locations on a number line. **N1A***	Place numbers on a number line and write inequalities if they are all in the same format (i.e., decimals, fractions), if they have the same denominator or the same number of decimal places.	Place numbers on a number line and write inequalities if the numbers are all in the same format (either whole numbers, decimals to two places, or simple fractions).
2. Identify Relationships (Classifying Mathematical Models)	Given tables, graphs, or equations of unfamiliar non-linear functions, determine and define properties of those functions.	2.1 Given a table, graph, or equation, classify a function as linear, quadratic, or exponential and justify your answer.	Given a table, graph, or equation, classify a function as linear or non-linear and justify your answer. **A1D**	Given a table or graph, classify a relationship as a function or non-function and justify your answer.
	Determine several models (including unfamiliar, non-linear functions) that might represent a given situation. Of those options, justify the model that best represents the situation.	2.2 Determine the type(s) of functions (linear, quadratic, or exponential) that might model a given situation. Of those options, justify the type of function that best models the situation. **A3A**	Explain the similarities and differences of tables, graphs, or equations of linear, quadratic, and exponential relationships. **A1C**	Explain the similarities and differences in the tables or graphs of linear and non-linear functions.
3. Analyze Relationships (Making Predictions with Mathematical Models)	Justify the relevant domain and range of any relationship from context.	3.1 Justify the relevant domain and range of a linear, quadratic, or exponential relationship from context.	Determine the domain and range of a relationship from an equation or graph.	Determine the domain and range of relationships given a table.
	Generate an equation that might model a given situation that appears to be linear and use it to make predictions about future data.	3.2 Consider multiple equations that might model a situation. Select and justify the best model for predicting the relationship.	Make and justify predictions about a relationship when given a table.	Make and justify predictions about a relationship when given a graph, including scatter plots. **D3A**
	Apply new and unfamiliar statistical measures to make predictions and draw conclusions.	3.3 Justify the most appropriate statistical measures of center to make predictions and draw conclusions.	Apply mean, median, mode, and range to solve problems and make predictions. **D2A**	Find the mean, median, mode, and range of a set of numbers.

* The codes in bold, such as **N1A**, are the Missouri state Course Level Expectations (CLE) for Algebra 1

Standard	Expert	Proficient	Developing	Novice
4. Operate on Polynomials	Apply additional operations on polynomials, including division, negative exponents, fractional exponents, or powers raised to a power.	4.1 Perform exponent operations, addition, subtraction, and multiplication on polynomials. **A2B**	Multiply a monomial by any polynomial and a binomial by a binomial.	Add and subtract polynomials.
	Factor polynomials with more than three terms, more than one variable, or a degree higher than two.	4.2 Completely factor any given quadratic expression. **A2B**	Factor trinomials with a leading coefficient of one.	Factor out a Greatest Common Factor (GCF) from any polynomial.
5. Interpret and Create Graphs	Identify and interpret the intercepts, rate of change, and maximum and minimum of any function when given a table or a graph.	5.1 Identify and interpret the intercepts, rate of change, and maximum and minimum of any linear, quadratic, and exponential functions when given a table or a graph. **A4A**	Identify and interpret the rate of change of linear and exponential functions when given a table.	Identify the intercepts of any function when given a graph as well as maximums and minimums when applicable.
	Sketch a quick graph of any polynomial function when given its equation.	5.2 Sketch quick graphs of linear, quadratic, and exponential functions from equations in various forms.	Find intercepts given the equations of linear, exponential, and quadratic functions.	Find the maximum or minimum of a quadratic function given its equation.

Algebra 1 – Standard 3.3 Assessment **Name _____**
Measures of Central Tendency

Novice: (Find the mean, median and mode of a set of numbers)

Weights of professional wrestlers (in pounds):

178, 235, 300, 254, 277, 346, 402, 178, 234, 256, 345, 199, 256, 267

1. Find the median. What does this tell you?

2. Find the mode. What does this tell you?

3. Find the mean. What does this tell you?

4. Find the range. What does this tell you?

Developing: (Find the mean, median, range, and mode and use them to solve problems)

Depth of Jelly Fish in the Ocean (in feet):

103, 145, 209, 209, 389, 210, 266, 305, 207 276, 1045, 209, 215, 546, 278, 209, 196, 209

5. Find the mean, median, range and mode.

6. Who would use this information and why?

7. Which statistical measure would be most beneficial and why?

Proficient: (Apply statistical measures of center to solve problems)

8. In Math class, you current test scores are 78%, 98%, 82%, 82%, 100%, 75%, and 74%. What grade should your teacher give you and why?

9. Your parents said that they would buy you a car if you are able to get a 90% in the class. There is one more test left. Will you get your car? (Prove your answer)

10. The mean cost of baseball tickets is $15. The total cost is $360. How many friends can you take?

Expert: (Apply the most appropriate statistical measures of center to solve problems, explain why others are inappropriate)

Shoe Sizes Males and Females

	Mean	Median	Mode	Range
Male	9.5	10	8.5	6
Female	8	8	7	7

11. You are the owner of "Cool Shoes" in the mall. You have collecting the following data and hope to make some decisions on what size shoes to order for the next season. You are planning to order 200 pairs of shoes (100 women and 100 men). How many of each size of shoes would you order and why?

    Women's order form:

    Men's order form:

Store	Customers	Sales
1	907	11.20
2	926	11.05
3	506	6.84
4	741	9.21
5	789	9.42
6	889	10.08
7	874	9.45
8	510	6.73
9	529	7.24
10	420	6.12
11	679	7.63
12	872	9.43
13	924	9.46
14	607	7.64
15	452	6.92
16	729	8.95
17	794	9.33
18	844	10.23
19	1010	11.77
20	621	7.41