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Productive Strategies for Engaging Students in Productive Struggle

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Redefining Student and Teacher Success

Expectations for students	Teacher actions to support students	Classroom-based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are "stuck" but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important but so is being able to explain and discuss how one thought about and solved particular tasks.	Ask students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.
Diagrams, sketches, and hands-on materials are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about one's thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students' reasoning, rather than on the way that the teacher is are thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students' thinking.

Principles to Actions, p. 49

Addition and Subtraction

Putting Essential Understanding of Addition and Subtraction into Practice Pre-K-2 (NCTM, 2014)

1. Task: Kendra's Stuffed Animals (p 43)
   Kendra has 7 stuffed animals. Asha has 14 stuffed animals. How many more stuffed animals does Kendra need so that she will have as many stuffed animals as Asha?

2. Task: Make It True (p 54)
   What number would make this number sentence true?

   $13 + 9 = 8 + 10 + ?$

3. Jack's Birthday Candles (p 127)
   Jack is 7 years old. How many candles has he had on his birthday cakes since he was born?

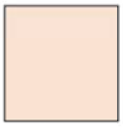
Fractions Problems

Putting Essential Understanding of Fractions into Practice 3-5 (NCTM, 2013)

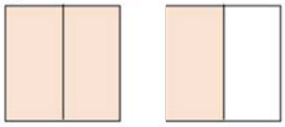
1. (p 136)

Read the thinking of the three students for the problem below.

1 brownie



What part is shaded for the brownies below?



(a) Sally: I think $\frac{3}{4}$ of the 2 brownies is shaded. The brownies are cut into 4 equal parts and 3 are shaded.

Is Sally correct? Explain your thinking.

(b) Marcus: I think $1\frac{1}{2}$ brownies are shaded. One of the brownies is shaded and $\frac{1}{2}$ of another brownie is shaded, so $1\frac{1}{2}$ brownies are shaded.

Is Marcus correct? Explain your thinking.

(c) Demetrius: I think that $\frac{3}{2}$ of a brownie is shaded. Each brownie is cut in half and 3 of the halves are shaded.

Is Demetrius correct? Explain your thinking.

2. (p 138)

(a) The shaded region below is $\frac{1}{4}$ of the whole. What does one whole look like?

(b) The shaded region below is $\frac{2}{3}$ of the whole. What does one whole look like?

(c) The shaded region below is $\frac{3}{2}$ of the whole. What does one whole look like?

Ratio and Proportion Problems

(*Putting Essential Understanding of Ratios and Proportions into Practice, 6-8*, NCTM, 2015)

1. Lee is having a party for many of his friends. He is making punch from a favorite recipe. However, with so many friends coming to the party, Lee realizes that the recipe will make only ¼ of the full amount he will need. The recipe calls for 3 cups of orange juice and 4 cups of pineapple juice. Lee makes the full amount that he will need. After tasting the completely mixed batch of punch, Lee decides that it doesn't have enough pineapple taste. To get the taste that he wants, he adds 4 cups of pineapple juice. He likes the taste of the resulting punch and decides to modify his recipe for punch. What is the ratio of orange juice to pineapple juice in Lee's new recipe? (p 25)

2. To make Luscious Lilikoi Punch, Austin mixes ½ cup lilikoi passion fruit concentrate with 2/3 cups water. If he wants to mix concentrate and water in the same ratio to make 28 cups of Luscious Lilikoi Punch. How many cups of lilikoi passion fruit concentrate and how many cups of water will Austin need? (p 74)

Functions Task: Grades 9-12

Putting Essential Understanding of Functions into Practice 9-12 (NCTM, 2014)

2. pp 97-98

Comparing Two Cars, Given Distance

The graph in figure 1 represents the distance, from a given location, of two cars, *A* and *B*, traveling in the same direction on the same road.

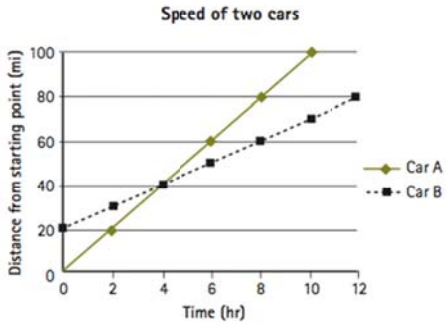


Fig. 1. Distance traveled by two cars

Questions

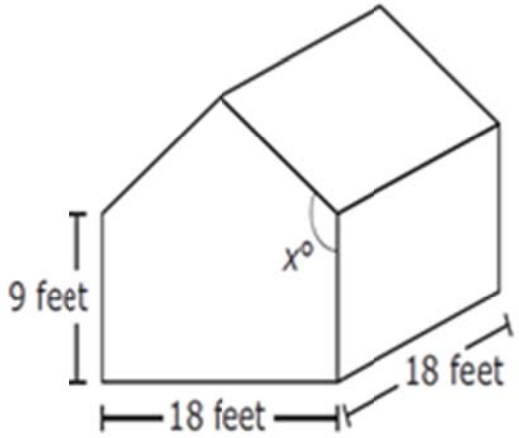
1. Describe what is happening at $t = 2$. Which car is going faster? How can you tell? Which car has gone farther? How far apart are the two cars?
2. Are the cars ever moving at the same speed? At what time? Explain how the graph represents this event.
3. Describe location of the cars at $t = 0$. What can be said about the speeds of the cars at this time?
4. Does one car ever pass the other? At what time?
5. Have the two cars traveled the same distance at any point in time? If so, when?

Focus in High School Mathematics: Fostering Reasoning and Sense Making for All Students (NCTM, 2011)

4. **The Cookie Problem (p 78)**

   A bakery currently sells 1000 large chocolate chip cookies each week for $0.50 each. A survey convinced the manager that for every $0.10 increase, the bakery would sell 70 fewer cookies each week. At what price should the cookies be sold to maximize the revenue to the bakery.

Designing a Shed



The base of the shed will be a square measuring 18 feet by 18 feet. The height of the rectangular sides will be 9 feet. The measure of the angle made by the roof with the side of the shed can vary and is labeled as X°. Different roof angles create different surface areas of the roof. The surface area of the roof will determine the number of roofing shingles needed in constructing the shed. To meet drainage requirements, the roof angle must be at least 117°.

Two Teachers' Responses to Students' Struggles to Solve a Multi-step Word Problem Involving Fractions

Ms. Flahive and Ms. Ramirez teach fifth grade and plan their lessons collaboratively. Their current instructional unit focuses on fractions. They have selected the Shopping Trip task shown below because they think it will be accessible to their students, yet provoke some struggle and challenge, since a solution pathway is not straightforward. The mathematics goal for students is to draw on and apply their understanding of how to build non-unit fractions from unit fractions and to use visual representations to solve a multi-step word problem:

Shopping Trip Task
Joseph went to the mall with his friends to spend the money that he had received for his birthday. When he got home he still had $24 remaining. He had spent $^{3}/_{5}$ of his birthday money at the mall on video games and food. How much money did he spend?
How much money had he received for his birthday?

When Ms. Flahive and Ms. Ramirez present the problem in their classrooms, both teachers see students struggling to get started. Some students in both classrooms immediately raise their hands, saying, "I don't get it," or "I don't know what to do."

Ms. Flahive is very directive in her response to her students. She tells them to draw a rectangle and shows them how to divide it into fifths to represent what Joseph had spent and what he had left. She then guides her students step by step until they have labeled each one-fifth of the rectangle as worth $12, as shown below. Finally, she tells the students to use the information in the diagram to figure out the answers to the questions.

Ms. Ramirez approaches her students' struggles very differently. After she sees them struggling, she has them stop working on the problem and asks all the students to write down two things that they know about the problem and one thing that they wish they knew because it would help them make progress in solving the problem. Then Ms. Ramirez initiates a short class discussion in which several ideas are offered for what to do next. Suggestions include drawing a tape diagram or number line showing fifths, or just picking a number, such as $50 and acting it out through trial and error. Ms. Ramirez encourages the students to consider the various ideas that have been shared as they continue working on the task.

Principles to Actions, p. 51

Resources

- Illuminations: https://illuminations.nctm.org

- *Putting Essential Understanding into Practice* series
  - Addition and Subtraction, Pre K-2;
  - Multiplication and Division, 3-5;
  - Ratio and Proportions, 6-8;
  - Functions, 9-12; Statistics, 9-12

- Journals:
  - *Teaching Children Mathematics*
  - *Mathematics Teaching in the Middle School*
  - *Mathematics Teacher*

- Student Explorations in Mathematics: https://www.nctm.org/sem/

- *Principles to Actions: Ensuring Mathematical Success for All Students* (2014) http://www.nctm.org/PtA/