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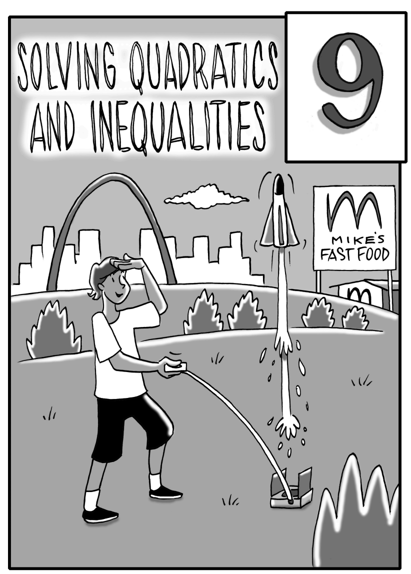
Engaging Inequalities



For more information about the materials you find in this packet, contact:

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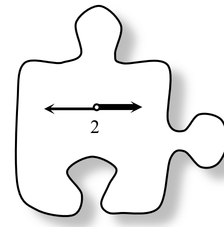
9.2.1 What if the quantities are not equal?



Solving Linear, One-Variable Inequalities

In this course, you have developed a variety of skills to find solutions to different kinds of equations. Now you will apply these equation-solving skills to solve inequalities.

9-44. As a class, create a "human number line" for each of the following mathematical sentences. You will be assigned a number to represent on the number line. When your number makes the equation or inequality true, stand up to show that your number is a solution. If your number does not make the equation or inequality true, remain seated.

a. $x \geq -2$ b. $x \leq 1$ c. $x = 3$ d. $x \geq 0$

e. $x = -2$ f. $-1 \leq x \leq 4$ g. $x^2 \geq 4$ h. $x < -3$

9-45. Based on your observations from problem 9-44, discuss the following questions with your class. Be sure to justify your responses.

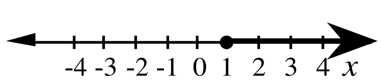
a. Compare the solutions to an inequality (like $x \geq -2$) with that of an equation (like $x = 3$). What is different? What causes this to happen?

b. How many solutions does an inequality such as $x \leq 1$ have?

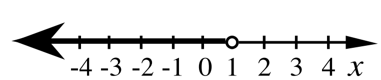
c. How is the result of $-1 \leq x \leq 4$ different from the other inequalities? What about the result of $x^2 \geq 4$?

9-46. Write an inequality that represents the solutions shown on each number line below.

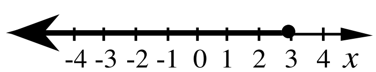
a.



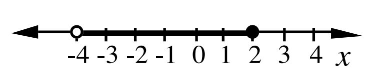
b.



c.



d.



9-47. SOLUTIONS TO A LINEAR INEQUALITY

With your study team, find at least five x-values that make the inequality below true:

$$2x - 5 \geq 3$$

a. How many solutions are there?

b. What is the smallest solution for x? This point is called a **boundary point**.

c. What is the significance of the boundary point? What is its relationship with the inequality $2x - 5 \geq 3$?

d. Write an inequality that represents the solutions for x. On a number line, highlight the solutions for x. Be ready to share your number line with the class.

9-48. SOLVING LINEAR INEQUALITIES WITH ONE VARIABLE

Analyze the process for solving an inequality, such as $3 - 2x < 1$, by addressing the questions below.

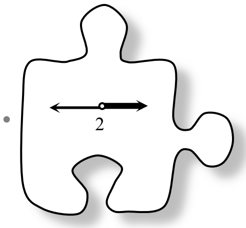
a. The key point to start with is the boundary point. How can you quickly solve for this point? Once you have determined your strategy, find the boundary point for $3 - 2x < 1$.

b. Decide if the boundary point is part of the solution to the inequality. If it *is* part of the solution, indicate this on a number line with a filled circle (point). If it is *not* a solution, show this by using an open circle as a boundary point.

c. Finally, to determine on which side of the boundary the solutions lie, choose a point to test in the inequality. If the point *is* a solution, then all points on that side of the boundary are part of the solution. If the point is *not* a solution, what does that tell you about the solutions? Write your solutions to $3 - 2x < 1$ as an inequality and represent the solutions on a number line.

9-49. With your study team, find all of the solutions to the inequality $3x + 1 < 7$. Decide how to represent these solutions on a number line and be prepared to justify your decisions to the class.

9.2.2 How can I use inequalities?



More Solving Inequalities

In Lesson 9.2.1 you learned how to solve inequalities with one variable. Today you will focus on special inequalities and learn how you can use inequalities to solve an application problem.

9-57. THE UNITED NATIONS

At the end of this chapter, your team will have the exci responsibility of representing a country at a special me of the United Nations (U.N.). The U.N. needs your he preparing for future large-scale disasters. You will nee help find a solution that not only works best for the cou you represent, but that also accommodates the needs o of the other countries. To prepare you for this task, the several lessons will present daily problems to familiari with the important issues and concerns of other countries.

Start by writing and solving an equation (or system of equations) that represents the problem below. Be sure to define any variables you use.

Turkey has a population of 66 million people and is made up almost entirely of two ethnic groups: Turks and Kurds. There are four times more Turks than Kurds. Write an equation and solve it to find out how many Kurds live in Turkey.

9-58. In 1912, Japan gave the United States several thousand flowering cherry trees as a symbol of friendship. Similarly, the nation of Cameroon plans to give flowering Satta trees to other countries this year. When asked how to decide which Satta trees make good gifts, Cameroon's chief arborist explained:

"We plant Satta trees when they are 6 cm tall, and they grow 9 cm every year. The trees only flower when they are taller than 150 cm."

It is very important that the trees Cameroon gives flower this year! It would be considered an insult to receive a tree that did not bloom. Luckily, Cameroon has many groves of Satta trees from which to select its gifts. How old must the trees be so that they will flower within the year?

a. Discuss with your study team whether an inequality or an equation is appropriate for this situation. Be prepared to share your reasoning.

b. Write and solve a mathematical sentence to determine how old the trees can be so that they flower this year.

c. Later, the arborist added:

   "I almost forgot to tell you! When the trees become very old, they stop flowering. Make sure you choose trees that are no more than 240 cm tall!"

   Discuss with your team how you can use this additional information to make sure you choose trees that will flower. Be prepared to share your answer with the class.

9.3.1 What if the inequality has two variables?

Graphing Linear, Two-Variable Inequalities

In Section 9.2, you learned how to use an inequality with one variable to help solve a word problem. You also discovered that a one-variable inequality can have zero, one, or more solutions and that these solutions can be represented on a number line. But what if an inequality has two variables? What is a solution to a two-variable inequality? How could these solutions be represented graphically?

9-65. EXAMINING THE SOLUTIONS OF A LINEAR EQUATION

Find your graph of $y = -2x + 3$ from problem 9-61. Compare your graph with the poster graph provided by your teacher.

a. Is the point (–1, 5) a solution to the equation $y = -2x + 3$? How can you tell by looking at the graph? How can you tell by using the equation?

b. Is the point (2, –1) a solution? What about the point (0, 0)? Justify each conclusion with both the graph and the equation.

c. What determines if a point lies on the line? What is the difference between the points on the line and the points not on the line?

9-66. GRAPHING A LINEAR, TWO-VARIABLE INEQUALITY

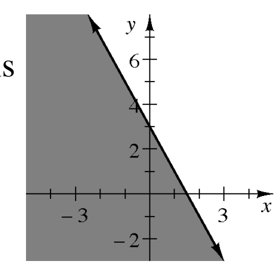
In problem 9-65, you found that the points on the line are the *only* points that make the equation $y=-2x+3$ true. But what if you want to graph the solutions for the inequality $y \geq -2x+3$? How will that graph differ from the graph of $y=-2x+3$? Consider this question as you follow the steps below.

a. Your team will be given a list of points to test in the inequality $y \geq -2x+3$. For each point that makes the inequality true, place a sticky dot on that point on the class graph.

b. Now examine the solutions shown on the graph. With your team, discuss the questions below. Be ready to share your conclusions with the class.

- Are there any points on the graph that you suspect are solutions but do not have a sticker?
- Are there any stickers that you think may be misplaced? If so, verify these points so that you can have a complete graph of the solutions.
- What about the points on the line? Are they all solutions to the inequality $y \geq -2x+3$? Why or why not?
- How many solutions are there?
- Why aren't any of the solutions located below the line?

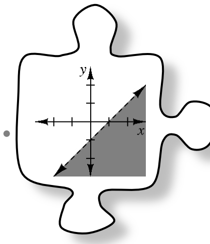
9-67. What else can you learn about solutions of linear inequalities? Think about this as you answer the questions below with your team.



a. What if the graph were shaded like the one at right? What inequality would correspond with this graph?

b. Heidi asks, "What if I changed the inequality to be $y < -2x+3$? Now what would the graph look like?" Discuss this with your teammates and decide the best way to represent the solutions to the inequality $y < -2x+3$. Be prepared to share your graph with the class.

9.3.2 What if the inequality is not linear?



Graphing Linear and Non-Linear Inequalities

In Lesson 9.3.1, you discovered that the solutions of a linear inequality with two variables can be represented by a shaded region on one side of the line. But how can the graph of an inequality help solve a problem? And what happens when the inequality is not linear? Consider these questions as you complete the following problems with your study team.

9-77. Review what you learned about graphing inequalities in Lesson 9.3.1 by graphing the inequality below on graph paper.

$$y \geq -\frac{5}{3}x - 3$$

a. What is the minimum number of points you need to test in order to know which side of the line the solution falls on?

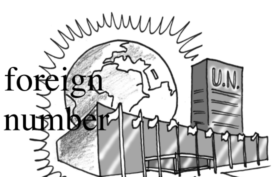
b. Orville thinks that using the point (0, 0) to test this inequality is a great idea. Why is using this point so convenient?

c. Anita decided to use the point (–3, 2) to test the inequality. Test the inequality with her point. Does this point help her decide which side to shade? Why or why not?

9-78. FOREIGN AID

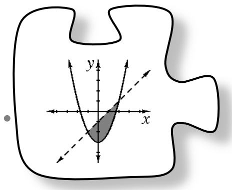
One of the purposes of the United Nations is to have nations work together to help each other. Recently, the members of the U. N. decided to give grants to poor countries to help reduce poverty. However, the United Nations only has the resources to help those countries with the greatest need. Therefore, it was decided that only countries in which the number of people in poverty is *more than* one-half of its total population would receive foreign aid.

a. A **constraint** is a limitation or a restriction. Write an inequality that represents the **constraints** on receiving foreign aid. Let x represent the population and y represent the number of people in poverty.



b. On the Lesson 9.3.2 Resource Page provided by your teacher (also available at www.cpm.org), find the graph that shows the number of people in poverty per the population for each of the countries being considered for foreign aid. Carefully graph your inequality from part (a) on this data graph. Which countries should receive foreign aid?

9.4.1 How can I represent it?



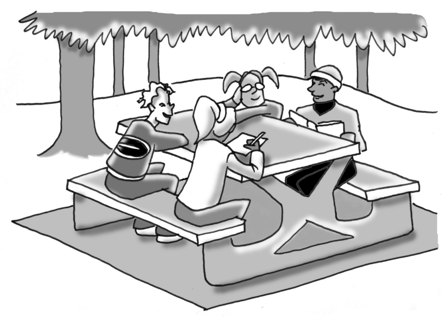
Systems of Inequalities

In Chapter 4 you learned that the solution to a system of equations is a point that makes both equations true. But what about the solution of a system of two inequalities? How can you represent these solutions on a graph? How many solutions can a system of inequalities have?

Consider these questions as you learn how to graph a system of inequalities.

9-89. Graph these inequalities.

$$y \le -x+5$$
$$y > \tfrac{2}{3}x-1$$



a. Compare your solution graphs for $y \le -x +$ $y > \frac{2}{3}x-1$ with those of your teammates. Correct any errors. Be sure to focus on wl the boundary line should be included in ea graph.

b. What would the graph of the system of inequalities look like? Consider the system of inequalities below. Which points are solutions to this system (that is, which points make *both* inequalities true)?

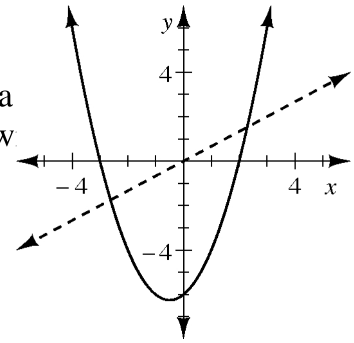
c. If you have not done so already, verify your solution region from part (b) algebraically by substituting the coordinates of a point from your solution region into each inequality.

d. How can you be sure that this region is the only set of points that makes both inequalities true?

9-90. Draw a graph of the region satisfying both constraints below. Start by graphing the boundary lines and then test points to find the region that makes both inequalities true.

$$y < x+2$$
$$y \le 10-\tfrac{3}{4}x$$

9-91. HOW MANY REGIONS?



When graphing the system of inequalities below, Reyna started with the boundary graph of each constraint shown at right.

$$y \le x^2 + x - 6$$
$$y > \frac{2}{3}x$$

a. Why is the line dashed while the parabola is not?

b. Find a copy of Reyna's graph on the Lesson 9.4.1B Resource Page provided by your teacher. How many possible solution regions are there? Carefully count each region with your teammates.

c. Pick a point in each region and test it in the system of inequalities. Shade any regions that contain solutions to both inequalities. How many regions make up the solution to this system?

d. Why is $(0, 0)$ not a good point to use to test for this solution?

9-92. How does changing the inequality affect the solution graph? Notice that each system of inequalities below uses the same boundary graphs as Reyna's graph from problem 9-91. However, notice that this time the constraints are slightly altered.

With your teammates, devise a method to determine which region (or regions) are solutions for each system. Shade the appropriate regions on your resource page.

a. $y \ge x^2 + x - 6$, $y > \frac{2}{3}x$

b. $y \ge x^2 + x - 6$, $y < \frac{2}{3}x$

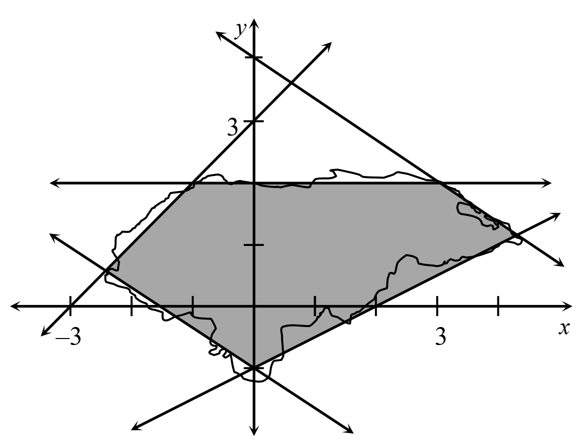
c. $y \le x^2 + x - 6$, $y < \frac{2}{3}x$

9-93. The United Nations asked every nation to write constraints that best approximate its country's shape (the U.N. thinks this will help find each country's area). Honduras sent in its inequalities, but some of the information is unreadable. With your study team, determine the missing parts of the inequalities and rewrite them on your paper.

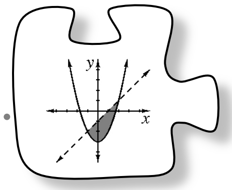
y [illegible] $x + 3$ $\quad$ $y \ge \frac{1}{2}x -$ [illegible]

[illegible] 2 $\quad$ y [illegible] $-\frac{2}{3}x + 4$

y [illegible] $-\frac{2}{3}x - 1$



9.4.2 How can I apply it?



More Systems of Inequalities

9-101. Review what you learned about systems of inequalities in Lesson 9.4.1 by graphing the system of inequalities at right on graph paper. Carefully shade the region of points that make *both* inequalities true.

$$y \le |x| + 4$$
$$-x + 4y \ge 4$$

9-102. SEARCH AND RESCUE

"*I'm completely lost… water everywhere I can see… both burners have failed… Wait! I see land. I'm going to try to land. I think it's…*"

Those were the last words heard from Harold in his hot-air balloon. The last time the balloon showed up on radar, it was near the Solomon Islands in the Pacific Ocean.

Your Task: Your team must determine where to send the search-and-rescue teams! Use the following reports along with the map on the Lesson 9.4.2 Resource Page and look carefully for information that will help you draw constraints on where the balloon might be found. Give the search constraints to the search-and-rescue team as a system of inequalities. Be sure to identify the probable landing site on the map.

Basic facts of the case:

The balloon departed from the airport at the very northern tip of the Philippines. The flight was supposed to follow a straight path *directly* to an airport in French Polynesia.

The balloon's last known location was at (–1000, 1000) near the Solomon Islands.

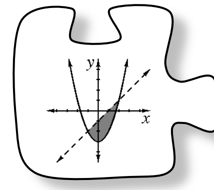
Pilot's report from a nearby airplane:

"*We were on our way from Australia, when we saw a hot-air balloon sinking rapidly. I am certain that it crashed south of our flight path. When we left Australia, we traveled 2000 km north for every 3000 km east that we flew.*"

Phone call received today:

"*I was a passenger on a flight that flew directly from French Polynesia to Indonesia. I was looking out my window when I saw the balloon going down to the north of where we were flying.*"

9.4.3 How can I use inequalities to solve problems?



Applying Inequalities to Solve Problems

Today you will pull together all of the mathematics you have studied in this chapter and apply it to solve an application problem.

9-111. UNITED NATIONS TO THE RESCUE

As a representative of your country, you have been sent the following letter and given an important task:

> *Dear Representative to the United Nations:*
>
> *A critical matter has come to the attention of the United Nations. In the past, when a catastrophe struck a part of the world, the U. N. gathered supplies to give to people in need. Unfortunately, because the U. N. had to collect supplies from each country at the time of the catastrophe, it was always quite a few days before the supplies could be sent to the areas that needed them the most.*
>
> *A recommendation has come before the U. N. to create a supply of food and medicine packages for future emergencies. Each food package will be able to feed several hundred people, while each medicine package will supply one first-aid station. I am asking each country to donate the same number of packages so each country shares the burden equally.*
>
> *I am asking each country to determine how many food and medicine packages they are able to give. You will present your findings at today's United Nations meeting. Please be certain to use the information that your country's Budget Committee has prepared to help you decide how many packages you can afford.*
>
> *Best of luck, and may our efforts make our world a better place!*
>
> *Sincerely,*
> *The Secretary General of the United Nations*

After consulting with your country's Budget Committee, your teacher will supply you with some information that will help decide how many food and medicine packets your country can afford.

Problem continues on next page. →

9-111. *Problem continued from previous page.*

Your Task: To communicate your country's budget constraints, write an inequality expressing how many food and medicine packages your country is able to give. Let *x* equal the number of food packages and *y* equal the number of medicine packages.

On the Lesson 9.4.3B Resource Page provided by your teacher, graph the solution region representing the number of medicine and food packets that can be donated by your country. Be prepared to share your graph with the other countries of the United Nations.

9-112. As a member of the United Nations, you must consider each of the following proposals. In each case, assume that the United Nations would like to receive as many emergency supplies as possible, while still having each nation give equally.

a. One proposal is that each country gives 185 medicine packages. How many food packages should the United Nations require from each country in this case? Explain how you made your decision.

b. Another proposal is to get the largest number of medicine packages possible. What is the largest number of medicine packages that each country can offer? How did you find your answer?

9-113. EXTENSION

A last-minute proposal suggests balancing the number of food and medicine supplies. For instance, if a country gives 150 food packages, then they would also give 150 medicine packages. How many food and medicine packages should the United Nations require from each country in this case? Explain how you determined your solution.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
- **Find meaning in problems**
- **Look for entry points**
- **Analyze, conjecture and plan solution pathways**
- **Monitor and adjust**
- **Verify answers**
- **Ask themselves the question: "Does this make sense?"**

2 Reason abstractly and quantitatively.
- **Make sense of quantities and their relationships in problems**
- **Learn to contextualize and decontextualize**
- **Create coherent representations of problems**

3 Construct viable arguments and critique the reasoning of others.
- **Understand and use information to construct arguments**
- **Make and explore the truth of conjectures**
- **Recognize and use counterexamples**
- **Justify conclusions and respond to arguments of others**

4 Model with mathematics.
- **Apply mathematics to problems in everyday life**
- **Make assumptions and approximations to simplify a complicated situation**
- **Identify quantities in a practical situation**
- **Interpret results in the context of the situation and reflect on whether the results make sense**

5 Use appropriate tools strategically.
- **Consider the available tools when solving problems**
- **Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)**
- **Make sound decisions of which of these tools might be helpful**

6 Attend to precision.
- **Communicate precisely to others**
- **Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes**
- **Calculate accurately and efficiently**

7 Look for and make use of structure.
- **Discern patterns and structures**
- **Can step back for an overview and shift perspective**
- **See complicated things as single objects or as being composed of several objects**

8 Look for and express regularity in repeated reasoning.
- **Notice if calculations are repeated and look both for general methods and shortcuts**
- **In solving problems, maintain oversight of the process while attending to detail**

- **Evaluate the reasonableness of their immediate results**

Lesson 9.3.1 Resource Page
Problem 9-66

$(-1, 6)$	$(2, 4)$	$(3, 3)$	$(-2, 2)$	$(1, -3)$,	$(3, -3)$
$(2, 7)$	$(-1, 1)$	$(2, -1)$	$(2, -2)$	$(3, 5)$	$(4, 6)$
$(-1, 5)$	$(3, 1)$	$(-2, -2)$	$(-3, 4)$,	$(1, -4)$	$(0, 5)$
$(5, 4)$	$(0, -1)$	$(-2, 7)$	$(-1, -4)$	$(1, 4)$	$(5, 6)$
$(3, -1)$	$(5, 2)$	$(-3, -3)$	$(-2, 5)$	$(1, 1)$	$(4, 3)$
$(1, 6)$	$(1, -2)$	$(4, -2)$	$(2, 2)$	$(-3, 1)$	$(0, 3)$
$(0, 0)$	$(1, 3)$	$(4, -5)$	$(4, -1)$	$(5, 3)$	$(-2, -5)$
$(4, -4)$	$(-1, 3)$	$(-1, -2)$	$(3, 6)$	$(2, 0)$	$(-1, 7)$

 Core Connections Algebra

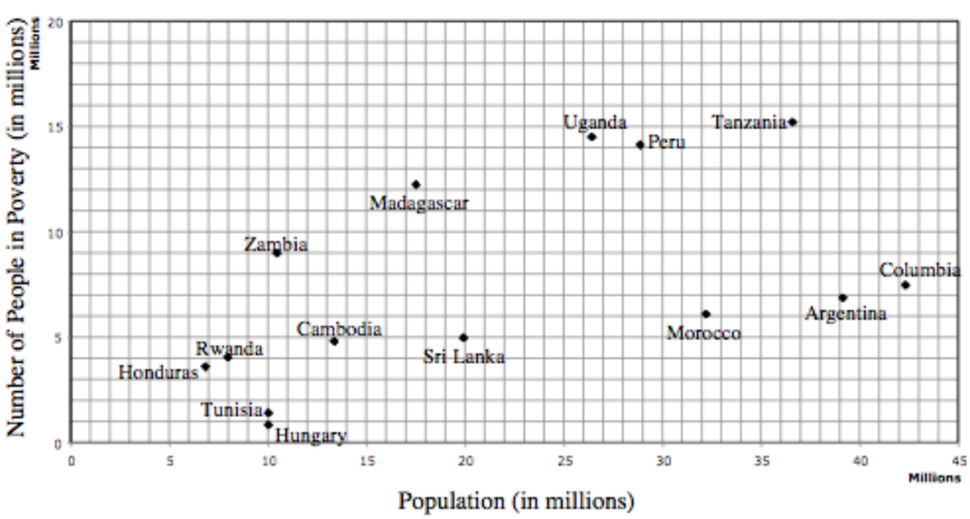
Lesson 9.3.2 Resource Page
Problem 9-78

Foreign Aid

One of the purposes of the United Nations is to have nations work together to help each other. Recently, the members of the U. N. decided to give grants to poor countries to help reduce poverty. However, the United Nations only has the resources to help those countries in the greatest need. Therefore, it was decided that only countries in which the number of people in poverty is *more than* one-half of its total population would receive foreign aid.

a. Write an inequality that represents the criteria to receive foreign aid. Let x represent the population and y represent the number of people in poverty.

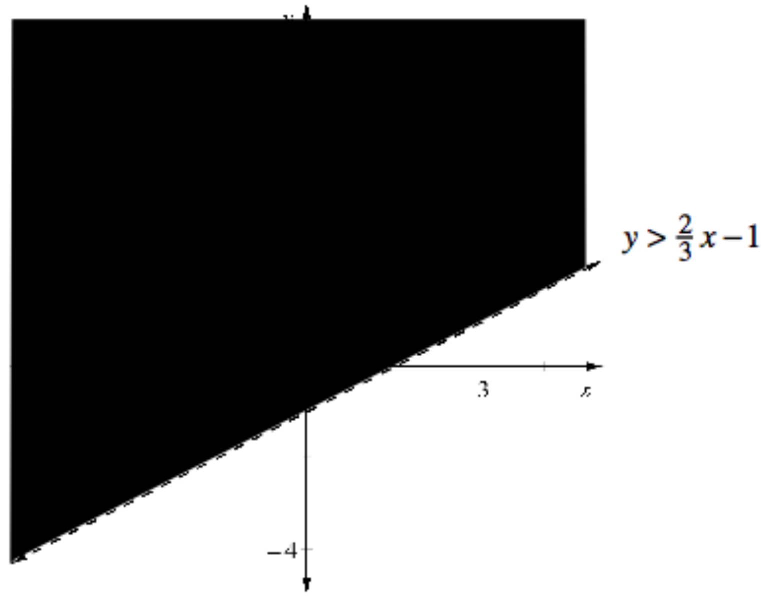
b. The graph below shows the number of people in poverty per the population for each of the countries being considered for foreign aid. Carefully graph your inequality from part (a) on the data graph below. Which countries should receive foreign aid?

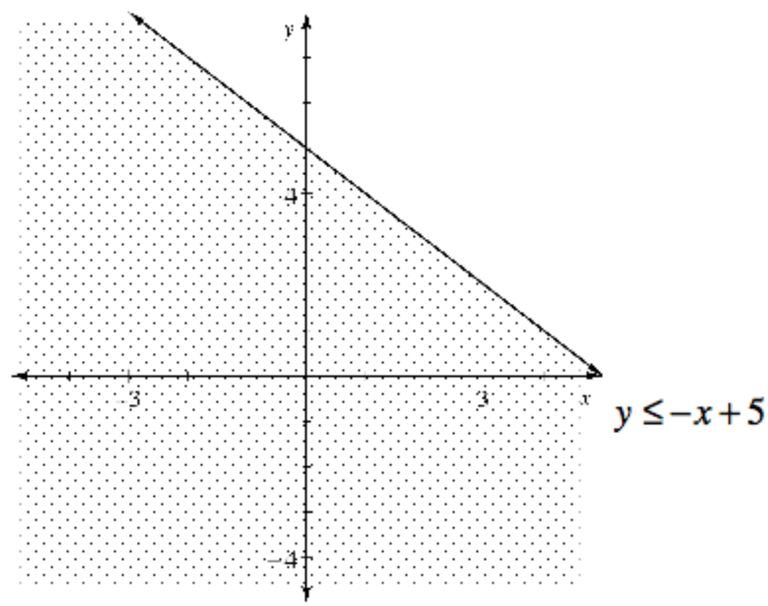


Data from The World Bank Group, 2004

 Core Connections Algebra

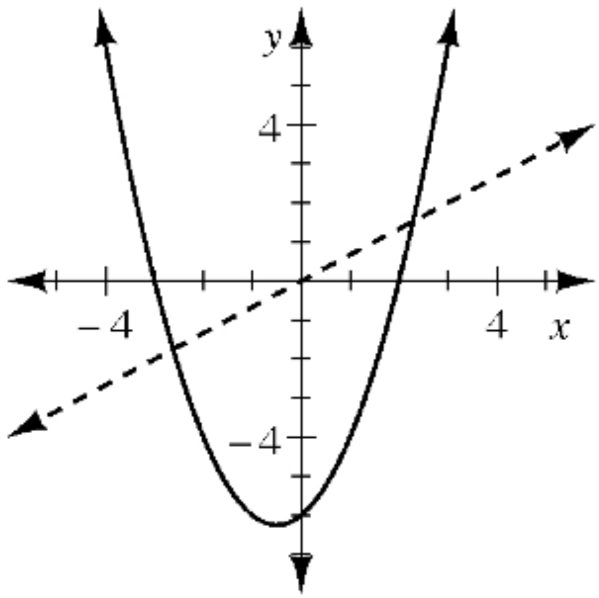
Lesson 9.4.1A Resource Page
Problem 9-89





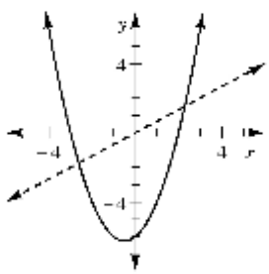
Lesson 9.4.1B Resource Page

Problem 9-91



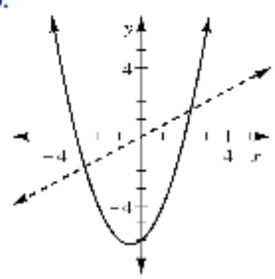
Problem 9-92

a.



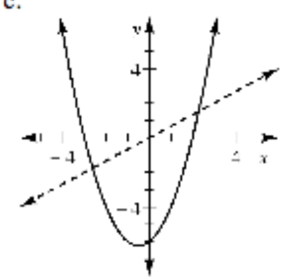
$y \geq x^2 + x - 6$
$y < \frac{2}{3}x$

b.



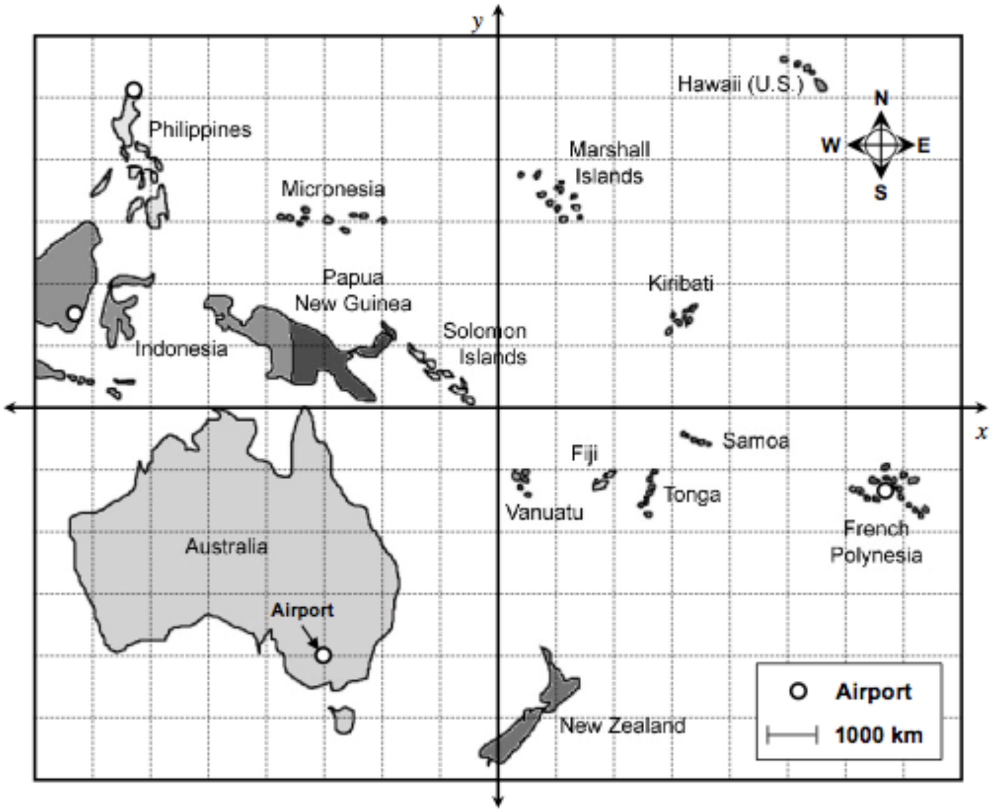
$y \geq x^2 + x - 6$
$y < \frac{2}{3}x$

c.



$y \leq x^2 + x - 6$
$y < \frac{2}{3}x$

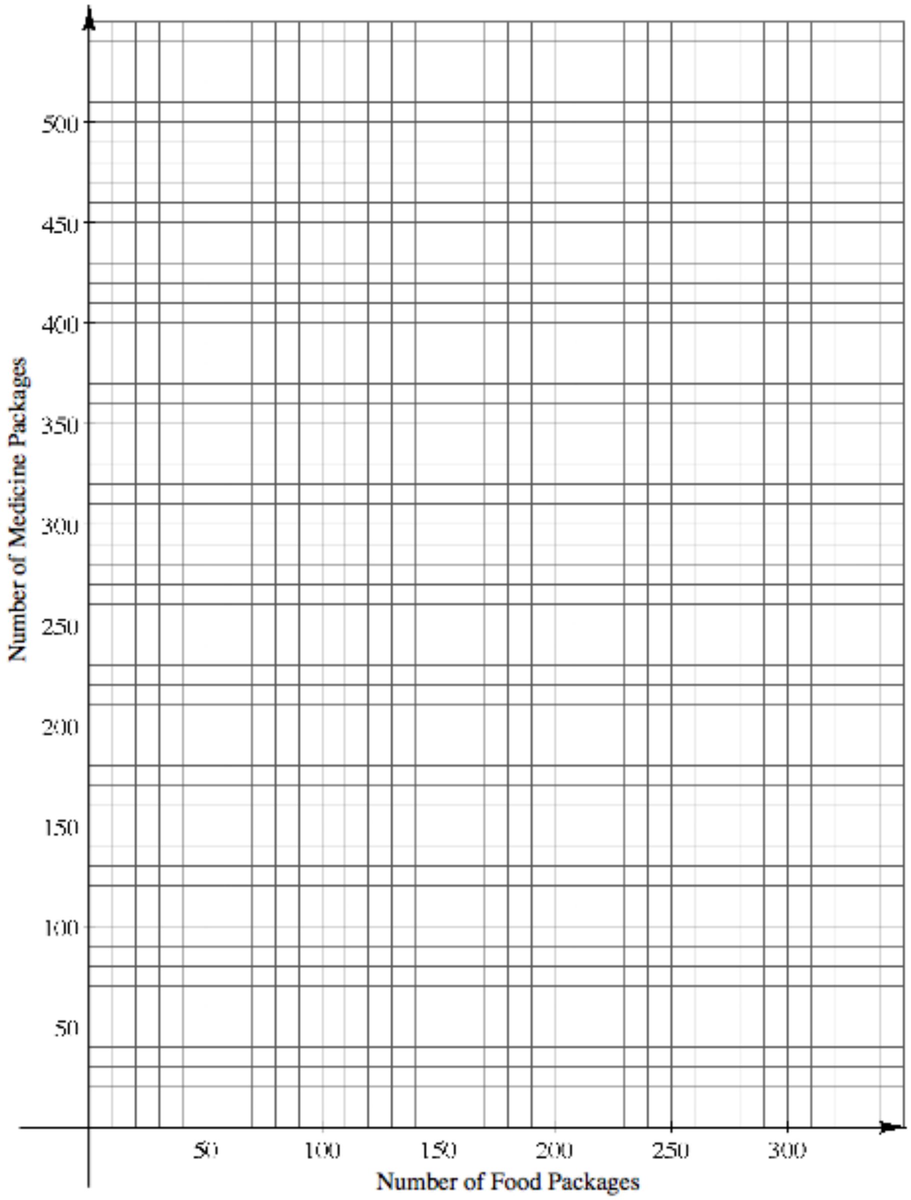
Search and Rescue



Country A	Country B
Memo from your country's Budget Committee: *Because we are spending so much money on improving our hospitals right now, our funds are very limited. We are constrained to spending* ***up to, but not more than*** *\$300,000. Each food package will cost our country \$900, and each medicine package will cost \$600.*	**Memo from your country's Budget Committee:** *Because we are spending so much money on improving our schools right now, our funds are very limited. We are constrained to must spending* ***no more than*** *\$600,000 total. Each food package will cost our country \$500, and each medicine package will cost \$2000.*
Country C	**Country D**
Memo from your country's Budget Committee: *Since our country is doing so well financially, our citizens voted that we must spend* ***more than*** *\$540,000 in aid to other countries. Each food package will cost our country \$2000, and each medicine package will cost \$3000.*	**Memo from your country's Budget Committee:** *Since our country is doing so well financially, our citizens voted that we must spend* ***more than*** *\$900,000 in aid to other countries. Each food package will cost our country \$5000. Each medicine package will cost \$2000.*
Country E	**Special Assignment**
Memo from your country's Budget Committee: *Luckily, our country has many medical supplies, so there is no limit to the number of medicine packets we can offer. However, due to the farming restrictions in our country, we are constrained to donating* ***fewer than*** *250 food packages in order to feed our own people.*	**A special note from the Secretary General:** *Since your country is a member of the Emergency Fund Committee, I have a special job for you.* *We are expecting food shortages in the near future because of the severe drought in Sudan. Each country must give* ***at least*** *100 food packages.*

Lesson 9.4.3B Resource Page
Problem 9-111

United Nations to the Rescue



Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

9 Make sense of problems and persevere in solving them.

- **Find meaning in problems**
- **Look for entry points**
- **Analyze, conjecture and plan solution pathways**
- **Monitor and adjust**
- **Verify answers**
- **Ask themselves the question: "Does this make sense?"**

10 Reason abstractly and quantitatively.

- **Make sense of quantities and their relationships in problems**
- **Learn to contextualize and decontextualize**
- **Create coherent representations of problems**

11 Construct viable arguments and critique the reasoning of others.

- **Understand and use information to construct arguments**
- **Make and explore the truth of conjectures**
- **Recognize and use counterexamples**
- **Justify conclusions and respond to arguments of others**

12 Model with mathematics.

- **Apply mathematics to problems in everyday life**
- **Make assumptions and approximations to simplify a complicated situation**
- **Identify quantities in a practical situation**
- **Interpret results in the context of the situation and reflect on whether the results make sense**

13 Use appropriate tools strategically.

- **Consider the available tools when solving problems**
- **Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)**
- **Make sound decisions of which of these tools might be helpful**

14 Attend to precision.

- **Communicate precisely to others**
- **Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes**
- **Calculate accurately and efficiently**

15 Look for and make use of structure.

- **Discern patterns and structures**
- **Can step back for an overview and shift perspective**
- **See complicated things as single objects or as being composed of several objects**

16 Look for and express regularity in repeated reasoning.

- **Notice if calculations are repeated and look both for general methods and shortcuts**
- **In solving problems, maintain oversight of the process while attending to detail**
- **Evaluate the reasonableness of their immediate results**