Ready to Learn Math Framework 2.0 (FINAL)

	Numbers & Operations in Base Ten						Geometry & Spatial Sense						Measurement & Data					Algebraic Thinking	
	Numbers & Counting				Operations		Geometry			Spatial Sense			Measurement		Data Collection & Analysis				
	Counting & Cardinality	Place Value & Grouping	Comparing Sets & Numerical Relations	Reading, Writing and Representing Numbers	Operating with Numbers	Representing Operations	2D & 3D shapes	Shape Attributes	Composition & Decomposition	Position, Location, Direction	Understanding Maps	Spatial Visualizations & Transformations	Measureable Attributes (Length, Weight, Capacity, Temperature, Time, Money)	Estimation	Sorting & Classifying	Data Collection & Analysis	Graphing	Patterns	Algebraic Principles of Operations
Curious George	x		x	x	x	x											x		
SID	x			x	x	x	x		x	x	x		x		x	x		x	
Dinosaur Train							x	x					x	x	x	x		x	
Cat in The Hat	x		x	x		x	x	x	x	x	x	x	x					x	
Fizzy's	x			x			x	x	x	x	x		x	x		x			
Fetch					x	x						x	x						
Cyber Chase	x				x		x	x	x	x		x							
WildKratts					x						x	x	x		x	x	x		
TEC Prankster Planet	x	x		x		x				x		x	x				x	x	
Peg + Cat	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
Odd Squad	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

YEAR 1

YEAR 2

YEAR 3

YEARS 3 & 4 & 5

A NOTE ABOUT THE UPDATED MATH FRAMEWORK:

The PBS Kids Math Learning Framework has been updated with the goal to better align with the current developments and thinking around early childhood mathematics. The original framework was developed in the Fall of 2010 when The Common Core State Standards for Mathematics were first formally released to the public. Since 2010, there has been increased effort to raise awareness of the importance of preschool math learning and to better align with the Common Core and other state standards initiatives. This revised edition of the Math Framework is based on and tied together by "big ideas"—important notions that underlie various concepts and procedures within a domain such as Numbers and Operations and even across domains. Instruction that focuses on big ideas is important for three reasons. One is that they help children see that mathematics as a coherent field consisting of inter-related knowledge instead of disorganized accumulation of isolated facts, procedures, and definitions. A second reason is that learning big ideas empowers children by making related concepts and procedures easier to discover or otherwise learn. A third is that thinking about what big idea(s) might be applicable to a particular topic can help children understand that the essence of mathematics is looking beyond superficial differences to find underlying structure (commonalities or patterns). Additionally, we've included details regarding children's learning trajectories and guidance on children's mathematical thinking from ages 2 through 8 years old. In a sense, this update focuses on WHY and HOW math is important in addition to the discrete math concepts and skills that kids should be learning at specific age ranges.

The CCSS-M's Standards for Mathematical Practice describe expertise important for all mathematics learners and connect with the content domains of the Common Core State Standards for Mathematics. They cut across all of the content domains. As you develop mathematics content for PBS Kids and Ready To Learn, we encourage you to develop television content and design games and activities that provide children with opportunities to engage these mathematical habits of mind. The table below presents an edited version of the Practices with suggestions for content for both television and games.

Practices	Suggestions for Television Content	Suggestions for Game and Activity Content
Make Sense of Problems	Present characters regularly engaged in recognizing and solving problems	Pose meaningful problems in games that children are motivated to figure out.
Persevere in solving problems	Present characters showing persistence when "stuck."	Provide prompts and encouragement for extended attempts to solve a problem as well as getting the correct answer.
Reasoning	Show characters explaining their thinking and describing the approaches and strategies used and solutions they have found.	Provide more open-ended games that require kids to reason about solutions rather than quick 1-solution problems that have a simple correct answer.
Use appropriate tools strategically	Portray characters selecting and flexibly using manipulative materials, drawings, or technological tools, as appropriate, to visualize, explore, represent and solve problems.	Provide opportunities for kids to use tools in gaming content.
Attend to Precision	Show characters explaining their thinking using math vocabulary, and calculate or estimate with efficiency and precision.	Expose kids to math vocabulary and design games that help kids develop proficiency with precision and accuracy.
Patterns and Structure	Show characters looking for, developing, and generalizing relationships and patterns as well as evaluating the reasonableness of their results, and applying conjectures about patterns and properties to new situations and solving problems.	Provide content that presents a variety of patterns and prompts kids to look for patterns that may help solve the problem.

NUMBERS & OPERATIONS IN BASE TEN
Numbers and Counting

Big Ideas & Learning Progressions:

Numbers can tell us many different things about our world and have different roles and meanings:

- **Cardinal Meaning:** Numbers can specify how many or the total number of items in a collection of items.
- **Ordinal Meaning:** Numbers can indicate "more" or "less" and can help us compare or order amounts, especially when we can't readily see the difference (e.g. 2nd in line, 4th place).
- **Nominal Meaning:** Numbers can serve as a name so that we can identify an object (e.g. a house number or a uniform jersey number)
- **Measurement Meaning:** Numbers can tell us how much of a quantity we must measure in order to count. This meaning is relevant to the section on Measurement and Data.

Understanding the meanings of number provide the foundation for building mathematical knowledge across multiple content domains. Children with number sense can use numbers with flexibility, and they can solve problems using numbers, including determining how many (in counting), how big and how much (in measurement and data). Key bases for developing number sense are subitizing of small numbers and later one-to-one counting to determine how many total objects. Additionally, children learn to read, write and represent numbers in various ways as they count and demonstrate cardinality. Grouping and Place Value are two of the most foundational concepts in mathematics. Grouping involves the big idea that smaller units can be combined to former a larger unit. Arranging items in groups and skip counting can provide an easier way to count collections. Place value involves grouping by ten smaller units to make a larger base ten unit, and provides a critical foundation for understanding addition and subtraction.

2-3 Year Olds

Counting and Cardinality (total number in a set):

- Subitize—instantly and reliably recognize and label with an appropriate number word the cardinality (total) with collections of 1 to 3 items
- Verbally count by ones up to 10
- Count in a one-to-one fashion to determine the cardinality of a presented collection of up to 5 objects **(3 year olds)**
- Match a collection of objects to a number word and label set **(3s)**
- Produce by counting or putting out 1 to 3 objects upon request ("Give me N")

Place Value and Grouping: (2-3s are just beginning to learn 1-10. They don't understand place value at all.)

Comparing sets and Numerical Relations:

- Identify which of two sets up to 4 has the same, greater than, or fewer items

Pre-K and Kindergarten

Counting and Cardinality:

- Count in a one-to-one fashion to determine the cardinality of a presented collection of up to 10 haphazardly arranged objects **(4 and 5 year olds)** and up to 25 items in a row **(6 year olds)**
- Understands that the number of items is the same regardless of its arrangement or order in which they are counted **(older 4 and 5s)**
- Subitize (instantly see how many objects without counting) up to 5 objects **(4s and 5s)**
- Understands cardinality principle-- that the last number name said while counting is the total number of objects counted (**5s**)
- Count or put out 1 to 5 objects upon request ("Give me N")
- Count by ones up to 19 **(4s)**, 29 **(5s)**, 50 **(6s)**
- Count by tens to 100 **(older 5s and 6s)**
- Count by twos to 20 **(5s and 6s)**
- Count by fives to 50 **(5s and 6s)**
- Count backwards by ones from 10 **(5s and 6s)**
- Count-on by ones from number other than 1 **(6s)**

Grades 1-2

Counting and Cardinality:

- Count by ones to 120 **(Grade 1)** and 1000 **(Grade 2)**
- Subitize scrambled arrangements up to 5 objects.
- Rely more on common arrangement of items or spatial patterns (like on a dice) to subitize numbers up to 10 (Grade 1)
- Count by twos, fives, tens to 100 **(Grade 1)** and 1000 **(Grade 2)**
- Count on by multiples of 10 (e.g., Start at 3 and count by 10
- Use a number list or number line to visualize the positions of numbers relative to one another

Place Value and Grouping:

- Can identify tens and ones digits in 10-99 (68= 60+8 or 10, 20, 30, 40, 50, 60, 61, 62, 63, 64, 65, 66, 67, 68) **(Grade 1)**
- Understand that three digits of a 3-digit number represent numbers of hundreds, tens, and ones **(Grade 2)**

Reading, writing, representing numbers:

- Represent quantities up to 5 by drawing sets of objects or making tallies **(3s)**

- Use a number list (**see Appendix for example**) to place, order, and identify missing numbers up to 29 **(5s)** and to 50 **(6s)**

Place Value and Grouping:

- Compose and decompose numbers from 11 to 19 into ten ones and some "extra" ones by using objects or drawing **(5s and 6s)**
- Can see the 10 and the ones in each teen number (10+8=18, 10+3=13,etc…) **(6s)**

Comparing Sets and Numerical Relations:

- Understands that each consecutive number is 1 more than the preceding number (**4s and 5s**)
- Use counting to determine the larger of two collections up to 10 (**4s and 5s**)
- Determine which of two verbally stated numbers up to 10 is greater by counting or estimating **(4s)**
- Determine which of two number neighbors up to 10 is greater without counting represented with or without objects **(5s and 6s)**
- Show comparing situation with objects, in a drawing, by matching objects of two sets lined up in parallel rows, or counting
- Identify relationships between numbers of objects in two sets by identifying same number, greater number of, fewer number of, has more than, has less than

Reading, writing, representing numbers:

- Represent quantities in different ways – using real objects, drawings, words, and numerals read and write numerals 0 to 19 **(5s)**
- Read word names for numbers 0 to 19
- Read numerals 1 to 100 arranged in 10 rows, by decade, organized in rows 1-10, 11-20… This means that the 1 is offset and appears above the 11 **(6s)**

Comparing sets and Numerical Relations:

- Solve comparison word problems for two numbers up to 18 ("How many more or less has one group than another?")
- Locate numbers on a number line and determine where a number fits in relation to other numbers on a number line **(Grade 1– to 100, Grade 2— to 1000)**
- Round to the nearest 10 or 100

Reading, writing, representing numbers:

- Represent quantities using drawings, words, and numerals
- Write 3-digit numbers in expanded form
- Read and write numerals up to 120 **(Grade 1)** and 1000 **(Grade 2)**
- Read word names for numbers 0 to 100 **(Grade 2)**

NUMBERS & OPERATIONS IN BASE TEN

Operations

Big Ideas & Learning Progressions:

Operations in early childhood focus on addition and subtraction. Young children learn that at the most basic level, addition makes the initial amount larger and subtraction makes the initial amount smaller. Starting in Pre-K, children are developing strategies to add and subtract to work towards the understanding of and fluency with these operations. Place value, as mentioned previously, is an important foundation for the development of counting and in developing understandings and eventually fluency with addition and subtraction. Composition and decomposition of numbers is also an effective strategy when solving more complex addition and subtraction problems and underlies mathematical concepts such as equal partitioning, multiplication, and early work with fractions. Applying algebraic principles, particularly counting patterns and the commutative and associative properties, helps children solve complex arithmetic problems more quickly and fluently.

2-3 Year Olds

Operating With Numbers:

- Help children recognize the effects of adding and subtracting items from a collection in concrete or even semi-concrete situations with totals up to 4 *(Concrete = initial collection, transformation, and results with objects all visible; semi-concrete = like concrete, except results not visible)*
- Identify one more up to 4
- Name the total when 1 is added on or subtracted from, up to 4

Representing Operations: Generally starts at Pre-K

Pre-K and Kindergarten

Operating With Numbers:

- Solve word addition and subtraction word problems with numbers up to 10 by using objects, fingers, mentally, or by making a drawing, or counting to determine the sum or difference
- Decompose numbers up to 10 in various ways (e.g., 8 = 4+4, 8 = 6+2, 8 = 7+1, etc…) by using objects or drawings, and record each decomposition by a drawing or an equation **(5s and 6s)**
- For any number 1 to 9, find the number that makes 10 **(5s and 6s)**
- Add and subtract within 5 **(5s and 6s)**
- Mentally add and subtract 0 and 1 **(5s and 6s)**
- Decompose a set up to 12 into two or three equal groups (equal partitioning or fair sharing) **(5s and 6s)**

Representing Operations:

- Use counters to represent objects
- Understand and say *this is/has less/fewer than that*
- Identify and use the = symbol (**5s & 6s)**
- Identify and use the + and – symbols **(5s & 6s)**

Grades 1-2

Operating with numbers:

- Add and subtract within 10 **(Grade 1)** and 20 **(Grade 2) using mental strategies**
- Solve addition and subtraction problems using objects, fingers, mentally, or by making a drawing or counting for totals up to 20 **(Grade 1)** and 100 **(Grade 2)**
- Add and subtract two-digit numbers within 20 **(Grade 1) and 100 (Grade 2) using one or more of the following strategies:**
  - Counting on
  - Use place value tools including using base-10 blocks and counting sticks
  - For addition problems, decomposing a complex problem into an easier addition problem (examples: near doubles: 5+6=5+5+1=10; make a double :5+3 = 4+4; make-10: 9+7=9+1+6=10+6)
- Use equal partitioning and group objects into same size groups to explore the concept of multiplication **(Grade 2)**
- Determine whether a group of objects up to 20 has an odd or even number of members
- Add and subtract on open number lines or with sets of objects
- Recognizes mathematical patterns to solve more challenging problems (see Algebraic Thinking for examples)
- Beginning understanding of Fractions:
  - Partition simple, regular shapes such as rectangles and circles into halves, fourths, **(Grade 1)** and thirds **(Grade 2)**
  - Recognize that 1 of 2, or even 2 of 4, equal parts is

“one half”

Representing operations:

- Use counters to represent objects
- Understand and say *this is/has less/fewer than that*
- Identify and use the = and ≠ symbols
- Identify and use the + and – symbols
- Begin to work with equations
- Write and solve addition and subtraction sentences
- Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers (e.g., 8 + _ = 11)

GEOMETRY & SPATIAL SENSE

Shapes

Big Ideas & Learning Progressions:

Geometry and spatial sense are important because they connect mathematics with the physical world and lay the foundation for other scientific disciplines (engineering, physics, etc…). Children should quickly progress from merely identifying shapes to analyzing and discussing the properties that distinguish one type of shape from another. While it's important that children learn important attributes of each shape, such as counting sides and angles, it's essential to understand the idea of particular shape. For instance, understanding how a square is similar to how it differs from another shape like a rectangle (that is not a square). Just as in work with numbers, composition and decomposition of shapes play a key role in a child's growing mathematical and spatial development. Children in PreK and Kindergarten begin using trial and error to put shapes together into pictures or designs, but as their knowledge of shape attributes develop, they begin composing shape pictures more intentionally. They also start to understand that shapes can be combined to make new shapes. By grades 1 and 2, children learn that shapes can be decomposed to make smaller different shapes (e.g. a pentagon is comprised of a triangle on top and square on the bottom), and there can be several possibilities in how shapes can be decomposed (e.g. a pentagon can also break down into 3 or 4 smaller triangles).

2-3 Year Olds

2D and 3D shapes:

- Recognize circles, squares, and triangles
- Recognize shapes in different orientations and sizes
- Use shape vocabulary
- May name a regular 3D shape by the name of one of its faces (e.g., calling a cube a square). Depending on the development of the child, this common misconception can provide a teachable moment to introduce 2D vs. 3D shapes and their attributes.

Shape attributes:

- Recognize that some shapes have curved sides (like a circle) and some have straight sides (rectangle)
- Identify some of the attributes of shapes, such as number of sides and number of corners (angles).

Composition and Decomposition:

- Solve simple puzzles
- Create pictures by representing single objects, each with a different shape
- Build 3-D structures using one type of shape (stacking with cubes)

Pre-K and Kindergarten

2D and 3D Shapes:

- Recognize, name, and identify common attributes of some 2D shapes: circles, squares, triangles, rectangles, pentagons and hexagons
- Recognize, name and identify common attributes of some 3D shapes: cubes, cones, cylinders, rectangular prisms, and spheres (**5 and 6s**)
- Use shape vocabulary—becomes more mathematical (rhombus instead of diamond, angles instead of corners)
- Recognize squares as special rectangles

Shape attributes:

- Recognize that some shapes have curved sides (like a circle) and some have straight sides (rectangle)
- Count sides and corners
- Analyze shapes according to key attributes
- Distinguish and describe the difference between 2D and 3D shapes
- Understand that the number of sides of shapes are important when identifying shapes (e.g., the difference between a triangle and a square)

Grades 1-2

2D and 3D Shapes:

- Recognize, name, and identify common attributes of additional 2D shapes: circles, squares, triangles, rectangles, pentagons, hexagons, octagons, parallelograms
- Recognize, name, and identify common attributes of additional 3D shapes: cubes, rectangular prisms, cylinders, spheres, square pyramids, triangular prisms
- Recognize similarities and differences of squares and rectangles
- Recognize cubes as special rectangular prisms
- Match shapes and names
- Identify faces and shapes of faces of three-dimensional shapes
- Use shape vocabulary

Shape attributes:

- Count sides and corners
- Recognize that some shapes have curved sides (like a circle) and some have straight sides (rectangle)
- Notice that the length of the sides of shapes are important when identifying shapes (e.g., the difference between a square and a rectangle)

Composition and Decomposition:

- Investigate and predict the results of putting together, taking apart, moving, and rotating 2D and 3D shapes (e.g., shape puzzles, designs, patterns, block constructions, tangram puzzles)
- Find “hidden” shapes in arrangements where figures overlap
- Construct more complex shapes from basic shapes (e.g. create a square out of 2 rectangles)

Composition and Decomposition:

- Investigate and predict the results of putting together, taking apart, moving, and rotating 2D and 3D shapes (e.g., shape puzzles, designs, patterns, block constructions, tangram puzzles)
- Find “hidden” shapes in complicated arrangements (e.g., shapes inside of other shapes)
- Construct more complex shapes from basic shapes (e.g. create a pentagon out of 3 triangles)

GEOMETRY & SPATIAL SENSE

Spatial Sense

Big Ideas & Learning Progressions:

The goal of spatial reasoning is to think and reason through the comparison, manipulation, and transformation of mental pictures. As previously stated, the task of composing and decomposing shapes lays the foundation for spatial reasoning. Starting in preschool, children should be encouraged to solve spatial problems, which may include tangrams, pattern blocks and reproducing block constructions. They should also be exposed to and encouraged to use related spatial vocabulary as they interpret relative positions in space. Additionally, children should deepen their knowledge of specific concepts beyond a simple definition or identification. For instance, in addition to merely learning where to place a line of symmetry, it's important to learn about the different ideas around symmetry and how symmetry has an important aesthetic quality that many children instinctively use in their everyday life (through drawing, organizing, etc.).

2-3 Year Olds

Position, Location, Direction & Understanding Maps: Generally too abstract at this level. Concepts usually begin around 4 or 5 years old.

- Can develop understanding of relative position words, such as *on top of* and *under*

Spatial Visualizations and Transformations:

- Engage in simple transformations, such as rotating and flipping blocks to make a tower, stacking cups in a sequence, or matching, flipping, and fitting shapes into holes or making a train with blocks

Pre-K and Kindergarten

Position, Location, Direction:

- Describe, name, and interpret relative positions in space and apply ideas about relative position using words, such *as, next to, above, below, behind, between*
- Orient lines vertically or horizontally

Understanding Maps:

- Interpret a model of a room to identify where an object is hidden in a real room
- Place objects in correct relative positions to make a map of a room
- Build 3D maps of the environment using recycled materials (e.g., build a model of a neighborhood or a town or a farm)

Spatial Visualizations and Transformations:

- Apply transformations and use symmetry to analyze mathematical situations, for example, solving spatial visualization problems with tangrams and pattern blocks
- Match halves at line of symmetry
- Identify lines of symmetry in 2D shapes and in common figures such as snowflakes
- Recognize and apply slides, flips, and turns

Grades 1-2

Position, Location, Direction:

- Describe, name, and interpret relative positions in space and apply ideas about relative position using words, such as, *next to, above, below, behind, right and left*
- Recognize 2D and 3D geometric shapes in environment and specify location
- Give and follow directions for moving in a physical space and on a map

Understanding Maps:

- Understand that maps provide answers to questions about direction, distance and location
- Build 3D maps of the environment using recycled materials (e.g., build a model of a neighborhood or a town or a farm)
- Use simple 2D maps to locate landmarks, to navigate through territory, and to find the shortest route to a location (e.g., a treasure hunt on the playground or neighborhood)
- Use coordinates to locate points on a graph

Spatial Visualizations and Transformations:

- Apply transformations and use symmetry to analyze mathematical situations
- Identify lines of symmetry of simple polygons
- Recognize and create shapes that have symmetry
- Recognize and apply slides, flips, and turns
- Recognize and represent shapes from different perspectives
- Use visualization, spatial reasoning, and geometric modeling to solve problems

MEASUREMENT & DATA

Measurement

Big Ideas & Learning Progressions:

In measurement, children assign number to continuous quantities (the length of a stick) rather than discrete quantities (3 paper clips). Assigning a number to a continuous quantity requires the understanding of the concept of measurement, which is dividing an object into equal size parts ("units") so that the parts (units) can be counted. Before actually assigning a number to a quantity, very young children engage in direct comparison and use their growing vocabulary of measurement words to describe relative measurement. As children's knowledge of composition and decomposition in number and geometry develop, their understanding of measurement units deepens -- first as non-standard units (1 stick = 3 paper clips laid one in front of the other with no gaps or overlaps) and then standard units . Children are exploring various measurement attributes, but it should be understood that the basic concept of measurement and the measurement unit is the same for all attributes.

2-3 Year Olds

Length:

- Compare two objects directly, noting same or different lengths or heights
- Informally recognizing length ("This train track is long")
- Can compose lengths intuitively ("I can make a long road")
- Use comparison vocabulary (*bigger/smaller/larger, longer/shorter, taller/shorter)*

Weight:

- Use a pan balance to compare weights of two objects
- Explore methods for achieving balance
- Use comparative vocabulary (*heavier/lighter)*

Capacity:

- Identify which of two containers holds more or les
- Use comparative vocabulary

Temperature:

- Use comparative vocabulary (*hot, warm cool, cold)*
- Associate outdoor events with hot or cold temperatures

Pre-K and Kindergarten

Length:

- Compare height and length of two or more objects
- Order objects by length or height
- Use comparative vocabulary (*same length/height; tall, taller, tallest, near, nearest, far, farther, farthest*)
- Recognize relationship between height and length (i.e. the longest of two objects will be the tallest)
- Use standard and nonstandard units to measure and compare length
- Understand that measuring objects (using either standard or nonstandard measurement) means to break up into equal size pieces that can then be counted
- Add two lengths to obtain the length of a whole

Weight:

- Use a pan balance to compare weights
- Explore methods for achieving balance
- Compare weight and height of two or more objects
- Order up to three objects by weight
- Use nonstandard units to measure and compare weight

Capacity:

- Identify which of 2 or 3 containers holds more, most, less or least, or the same amount

Grades 1-2

Length:

- Compare height and length of two or more objects
- Order objects by length or height
- Recognize relationship between height and length
- Use standard and nonstandard units to measure and compare length
- Understand that measuring objects (using either standard or nonstandard measurement) means to break up into equal size pieces that can then be counted
- Use feet, inches, or yards **(Grade 2)** to measure and compare length
- Make comparisons between nonstandard measurement estimates and the actual number of units used to measure the length of an object

Weight:

- Use a pan balance to compare weights
- Compare weight of two or more objects
- Explore methods for achieving balance
- Order more than 3 objects by weight
- Compare weights using weight scales
- Measure and compare weights using pounds and ounces
- Use comparative vocabulary (*same weight, heavy, heavier, heaviest, light, lightest etc*)

Time:

- Names parts of a day: morning, afternoon, and night
- Associate activities with part of day

- Use comparative vocabulary

Temperature:

- Use comparative vocabulary *(hot, warm, cool, cold)*
- Associate outdoor events with hot or cold temperatures

Time and Calendar:

- Identify and understand morning, afternoon, and night
- Associate activities with time of day
- Identify and understand today, tomorrow, yesterday
- Use a calendar and count by days of the week, number of days, weeks, etc.
- Use temporal vocabulary and ordinal numbers to order events over time *(happens first, second… last, before, after, between)*

Estimation:

- Make informal comparisons and estimates (of length, weight, capacity, and time)

Capacity:

- Identify which of 2 or 3 containers holds more or less
- Order 3 or 4 containers by capacity
- Use comparative vocabulary
- Use standard units to measure and compare capacities
- Explore relationships among quarts, cups, ½ cups, pints, gallons, and half gallon

Temperature:

- Compare temperatures
- Use comparative vocabulary *(hot, warm ,cool, cold)*
- Associate outdoor events with hot or cold temperatures
- Use a Fahrenheit thermometer to measure temperatures

Time:

- Associate activities with time of day
- Tell time to the hour on analog and digital clocks
- Tell time in intervals of hours, half hours, quarter hours and to the nearest 5 minutes
- Determine elapsed time to the hour
- Explore what you can do in a minute to develop benchmarks for longer periods of time-- an hour, half hour, and quarter hour
- Use a calendar to identify specific events, such as birthdays
- Order events over time
- Use temporal vocabulary and ordinal numbers to order events (*happens first, second… last, before, after, between*)

Money:

- Identify and name pennies, nickels, dimes, and quarters
- Identify the values of pennies, nickels, dimes, and quarters
- Identify total value of a set of dimes, nickels, pennies, and quarters
- Trade pennies for nickels
- Trade nickels for dimes
- Given a total value, identify coins that would have that total value (“I have 50 cents – what coins could I have?”)

- Determine if you have enough money to buy an object

Estimation:

- Estimate using standard length measurements

MEASUREMENT & DATA

Data Collection and Analysis

Big Ideas & Learning Progressions:

Data collection and analysis lays the foundation for the more serious study of statistics and probability, which is used widely in many scientific disciplines and within many real-life contexts. Preschool children are introduced to data through sorting objects by attribute and assigning a classification to those objects (e.g. red blocks, blue blocks). As children's number sense and concept of measurement develops, they begin to collect specific data on these sorted and classified objects, describing which classified groups have more and have less. Starting in Kindergarten, children learn to visually represent data through graphing. In later grades, children go beyond collecting, representing and analyzing data already provided to them and begin to conduct simple surveys to collect, represent and analyze original data (e.g. "In what month were you born?").

2-3 Year Olds

Sorting and Classifying:

- Begin sorting and classifying objects according to their attributes
- Explore found objects and sort arbitrarily

*****At this age they will not have complete categorization based on color, shape, size, etc.***

Data Collection & Analysis and Graphing: Begins in Pre-K

Pre-K and Kindergarten

Sorting and Classifying:

- Sort and classify objects according to their attributes

Data Collection and Analysis:

- Collect relevant data for addressing a question or making a decision
- Pose questions, gather data about their surrounding, then organize data by various attributes
- Describe parts of the data and/or the set of data as a whole to determine what the data show
- Learn that some events are more likely to occur than others
- Make predictions for everyday events

Graphing:

- Represent data using concrete objects and picto-graphs
- Use picto-graphs or real graphs to organize, describe, and analyze data and compare numbers of objects/people/pockets

Grades 1-2

Sorting and Classifying:

- Sort and classify based on several attributes concurrently (e.g. red circles, green circles, red squares, green squares, etc.)
- Determine if an object belongs in a group based on its attributes

Data Analysis and Collection:

- Conduct surveys and experiments and collect data to answer questions.
- Distinguish among categorical ("names") versus quantitative data ("numbers")
- Recognize the usefulness of a sample instead of getting everyone's information
- Use tally marks to record data
- Make predictions for everyday event
- Understand uncertainty and chance in everyday situations
- Informally experiment with simple probability problems
- Begin to use probabilistic vocabulary like *certain, sure, uncertain, unsure, likely, unlikely, maybe, possible, must happen, impossible, can't happen, might happen*

Graphing:

- Use bar graphs and picto-graphs

ALGEBRAIC THINKING

The Common Core State Standards for Mathematics at the (K-2) levels approach Algebraic Thinking through Operations. Please see Key Concepts in Operations to learn more about the importance of using algebraic principles to develop fluency with addition and subtraction. We have included other types of patterns in the PBS Kids framework, like shape, color, and sound for younger kids (ages 2-5), who are not accounted for in the Common Core.

Big Ideas & Learning Progressions:

Young children are fascinated with patterns. Preschoolers begin to recognize patterns as items that repeat in a certain order, however, young children may begin to recognize that patterns, especially numerical patterns, can also "grow", which lays the foundation for counting by ones and skip counting by 2s, 5s, and 10s. These growing patterns and developing deductive reasoning skills set the stage for the study of algebraic functions in operations. Kindergarteners also start to understand that patterns can be represented abstractly by use of letters that can transfer across different domains. For example, an ABAB pattern can be a sound pattern, such as clap, snap, clap, snap, or it can be a visual pattern such as red, green, red, green, etc… This is a very useful for the understanding of why we use letters such as x and y in algebra to represent different things, be they sounds, shapes, letters, or numbers.

2-3 Year Olds

Patterns:

- Recognize and copy simple AB patterns involving colors, shapes, sounds, and motions

Algebraic Principles of Operations:

Not applicable for this age group.

*****At this age they will not have complete categorization based on color, shape, size, etc.***

Pre-K and Kindergarten

Patterns:

- Identify, create, copy, fix, and extend AB patterns and then somewhat more complex repeating patterns such as ABB patterns with sounds, color, shapes, letter and numbers
- Identify the "core" or "unit" of a repeating pattern and label it as an AB or ABB pattern
- Identify, create, and extend simple growing and decreasing patterns
- Identify position of objects within a pattern
- Predict what comes next in a pattern

Alebraic Principles of Operations

- Use clues, rules, and patterns to solve operation problems.
  Examples:
  - Understands if 8 comes after (or is greater than) 7, then an unfamiliar number such as 38 comes after (or is greater than) 37 **(6s)**
  - Applies the addition-subtraction inverse principle (10 + 4 - 4 = 10 - 0= 10) **(6s)**
  - Applies the additive/subtractive identity (7 + 0 = 7 or 7 – 0 = 7) and subtractive negation (7- 7 = 0) **(6s)**

Grades 1-2

Patterns:

- Identify, create, copy, extend sound, color, shape, letter and number patterns
- Identify patterns with color and shape
- Identify and extend letter patterns
- Identify, create, and extend growing and shrinking patterns
- Identify patterns with doubles
- Identify positions of objects within a pattern
- Identify and extend skip counting patterns (which lays foundation for multiplication, and the study of factors and multiples in later grades)

Algebraic Principles of Operations:

- Use more complex clues, rules, and patterns to solve mathematical problems.
  Examples:
  - Applies the commutative property of addition (if 9+7 =16, then the sum of 7+9 must be 16) **(Grade 1)**
  - Applies the associative property of addition (2+6+4=2+(6+4) = 2+ 10=12) (**Grade 1**)
  - Applies the successor principle to near addition doubles (if 7+7 =14, then 7+8 is one more than 14— that is 15) **(Grade 1)**
  - Applies 10+n=n-teen knowledge to combinations involving 8 or 9 (e.g., if 10+7 = 17, then 9+7 is one less than 17 or 16) **(Grade 2)**

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