Mathematics Science Partnership - Student Level Problems Packet

Brad Forrest, Judy Vogel, Chia-Lin Wu

August 11, 2014

1. 計算 $12 \div (-3) - 2 \times (-3)$ 之值為何？

   (A) -18

   (B) -10

   (C) 2

   (D) 18

3. k、m、n 為三整數，若 $\sqrt{135} = k\sqrt{15}$， $\sqrt{450} = 15\sqrt{m}$ ，$\sqrt{180} = 6\sqrt{n}$ ，則下列有關 k、m、n 的大小關係，何者正確？

   (A) $k < m = n$

   (B) $m = n < k$

   (C) $m < n < k$

   (D) $m < k < n$

15. 計算 $(\frac{21}{26})^3 \times (\frac{13}{14})^4 \times (\frac{4}{3})^5$ 之值與下列何者相同？

   (A) $\frac{13}{3^3}$

   (B) $\frac{13^2}{3^3}$

   (C) $\frac{2 \times 13}{7 \times 3}$

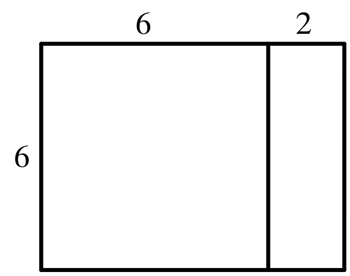
   (D) $\frac{13 \times 2^3}{7 \times 3^2}$

Problem 1(P).

a. How many $^1/_4$ cup servings are in $^2/_3$ of a cup of yogurt?

b. A rectangular box is 1 ft long, $^3/_4$ ft wide and $^1/_2$ ft high. What is it's volume in cubic inches?

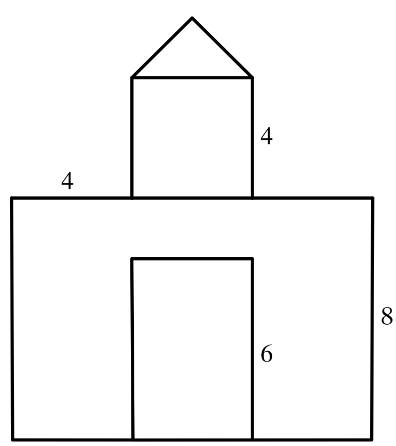
Problem 2(P). A quilter is using a pattern made up of a 6in square adjoined to a 2in by 6in rectangle. (See Below.) Assume that the pattern is repeated 5 times across the span of the quilt.



a. Write a mathematical expression that represents the width of the quilt. Provide at least 3 different representations of the expression.

b. What is the width of the quilt in feet?

Problem 3(P). A castle is being built for the scenery of a school play. The plans for the castle are provided below. The basic shape of the castle is a rectangle with a tower and a drawbridge(door). The tower is centered to the rectangle and is made up of an isosceles triangle on top of a square. The drawbridge and the tower are the same width and the same height. Everything on the castle will be painted white except for the drawbridge. (All units are in feet.)



a. What is the area covered by white paint? Explain how you arrived at your answer.

b. If one gallon of paint covers 400 square feet, how many quarts of white paint are needed?

Problem 4(P). A silk screening company charges $150 to create a design and $4 for each T-shirt produced. If the senior class has $430 budgeted for a T-shirt fundraiser, how many T-shirts can they purchase.

Problem 5(P). Carl wants to run at least 25 miles this week. He has already run 7 miles. If he can run $3/4$ of a mile in 10 minutes, how much additional time will he need to spend running this week?

Problem 6(P). Let *line 1* be the line representing the relationship between *x* and *y* expressed in the following table.

x	*y*
2	6
4	7
6	8
8	9

Let *line 2* be the line given by -2x + 3y = 6

a. Which line has the larger y - intercept?

b. Which line has the steeper slope?

Problem 7(P). Janet and Steven are discussing a homework problem to convert the repeating decimal $0.\overline{12}$ to a fraction. Steven says that there is a trick and all you have to do is put 12 over 99 to get $^{12}/_{99}$. Janet remembers that there was a much longer explanation to the solution and doesn't think the trick will work for all cases. Who is correct?

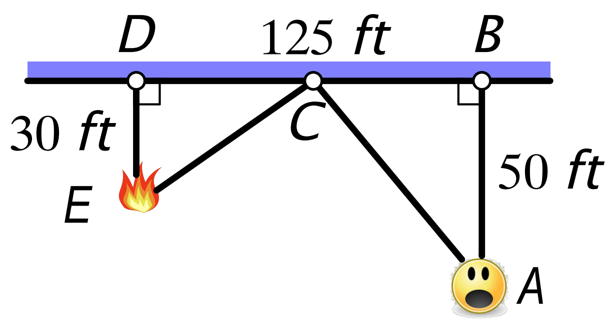
Problem 8(P). Maxwell has rented a storage unit 4ft by 6ft by 6ft. He plans to store extra stock from his shoe store in the storage unit. Each of his shoe boxes measures 4in by 3in by 10in. Maxwell computes the volume of the unit and the volume of each shoe box and reasons that he should be able to store 2073 pairs of shoes in the storage unit. Is his reasoning correct? Explain.

Problem 9. The current I, in amperes, flowing through an ac (alternating current) circuit at time t is

$$I = 120 sin\left(30\pi t - \frac{\pi}{3}\right), \qquad t \geq 0$$

What is the period? What is the amplitude? What is the phase shift? Graph this function over two periods.

Problem 10. As you are gathering leaves for a science project, you look back at you campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



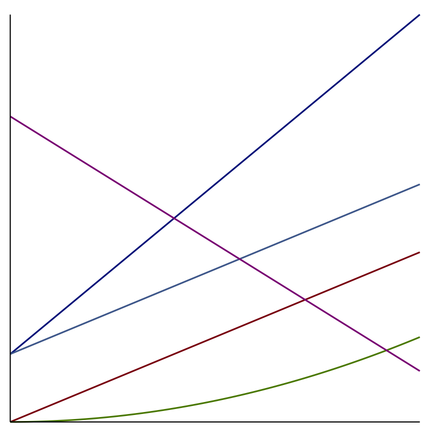
a. Make a table with columns labeled BC, AC, CE, and AC + CE. Enter values of BC from 10 to 120 in increments of 10.

b. Calculate AC, CE, and AC + CE for each value of BC, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. Draw an accurate picture to scale of the shortest distance.

Problem 11. Given below are five functions and at the right five graphs. Without doing any calculating or graphing yourself, match each function with the graph that most likely represents it. In each case, explain the clues that helped you make the match.

(A.) $y = x$ (B.) $y = 2x + 2$ (C.) y= $0.1x^2$ (D.) $y = x + 2$ (E.) $y = 9 - 1.5x$



Problem 12.

a. Find the slope of the line represented by the equation $y = -2x + 3$

b. Make a table of x and y values for the equation $y = -2x + 3$. How is the slope related to the table entries?

Problem 13. The table shows the number of golf facilities in the United States during the period 1997-2001.

Year	Golf facilities (thousands)
1997	14.6
1998	14.9
1999	15.2
2000	15.5
2001	15.7

a. Make a scatter plot of the data where x is the number of years since 1997 and y is the number of golf facilities (in thousands).

b. Write an equation that models the number of golf facilities (in thousands) as a function of the number of years since 1997.

c. At about what rate did the number of golf facilities change during the period 1997-2001?

d. Use the equation from part (b) to predict the number of golf facilities in 2004.

e. Predict the year at which the number of golf facilities reached 16,000. Explain how you found your answer.

Problem 14. It is often possible to solve problems that involve linear equations without the use of tables, graphs, or computer algebra systems. Solving equations by symbolic reasoning is called solving algebraically. For example, to solve $3x + 12 = 45$ algebraically you might reason like one of these students.

Natasha: I need a value of x making $3x + 12 = 45$, so the left and right sides are balanced. If I subtract 12 from both sides, the sides will remain balanced. So, $3x = 33$. If I divide both sides by 3, the sides will remain balanced. So,$x = 11$.

Michael: The equation tells me to multiply x by 3, then add 12 to get 45. To find out what value of x gives me 45, I have to undo those operations. That means starting with 45, I can subtract 12 and then divide by 3 to get $x = (45 - 12)/3 = 33/3 = 11$

As you work on the problems in this investigation, think about these questions: Why does solving linear equations by reasoning like that of Natasha and Michael make sense? How can reasoning like that of Natasha and Michael be used to solve other linear equations algebraically?

Analyze the reasoning strategies used by Natasha and Michael by answering the following questions.

a. Why did Natasha subtract 12 from both sides? Why didn't she add 12 to both sides? What if she subtracted 10 from both sides?

b. Why did Natasha divide both sides by 3?

c. What did Michael mean by undoing the operations?

d. Why did Michael subtract 12 and then divide by 3? Why not divide by 3 and then subtract 12?

e. Both students found that $x = 11$. How can you be sure the answer is correct?

Problem 15. In (a) - (g), solve the equation by using the symbolic method. Check your solution by making tables or graphs with your calculator.

a. $6x + 10 = 4x + 18$

b. $3x + 47 = 7x + 7$

c. $105x = 10x + 5(x - 6)$

d. $x(x - 7) = 0$

e. $x^2 + 6x + 8 = 0$

f. $x^2 + 2x + 1 = 0$

g. $8x - x^2 = 0$

Problem 16. As a biology project, Nicole is investigating how fast a particular beetle population will grow under controlled conditions. She started her experiment with 5 beetles. The next month she counted 15 beetles.

a. If the beetle population is growing linearly, how many beetles can Nicole expect to find after 2, 3, and 4 months?

b. If the beetle population is growing exponentially, how many beetles can Nicole expect to find after 2, 3, and 4 months?

c. Write an equation for the relationship between the number of beetles and the number of months if the beetle population is growing linearly.

d. Write an equation for the relationship between the number of beetles and the number of months if the beetle population is growing exponentially.

e. If the beetle population is growing linearly, how long will it take the population to reach 200?

f. If the beetle population is growing exponentially, how long will it take the population to reach 200?

Problem 17. Use properties of exponents to write each of the following expressions in equivalent simplest form, using only positive exponents.

a. $(p^3)(p^7)$

b. $(p^3)^2$

c. $(p^2q^3)(p^{-1})(q^2)$

d. $\frac{p^4q^3}{q^2p^2}$

e. $\left(\frac{p^2q}{pq^3}\right)^4$

Problem 18. Whether they knew it or not, Conrad, Jenna, and Andrea solved the system of Linear equations using a strategy called the elimination method. That method is based on two key properties of equations:

- If both sides of an equation are multiplied or divided by the same (nonzero) number, then the solutions of the new equation are identical to those of the original.
  For example, the solutions of 2p + 2d = 7.50 are identical to those of p+d = 3.75.
- If you find the sum or difference of two equations in a system, the result often gives useful new information about the unknown values of the variable.
  For example, if p+d = 3.75 is subtracted from 4p + d = 10.50, the result is 3p = 6.75. From this, we can conclude that p=2.25 and then that d= 1.50.

The challenge in using these ideas is finding the multiples, sums, and differences of given equations that lead to a single equation revealing part of the solution.

Consider the system of equations:

$$\begin{cases} 3x - y = 6 \\ x + 2y = 5.5 \end{cases}$$

a. For each of the steps below, explain what actions have been taken since the previous step. Justify the actions using previous mathematical knowledge and the two properties stated above.

Start $\begin{cases} 3x - y = 6 \\ x + 2y = 5.5 \end{cases}$ Step 1 $\begin{cases} 6x - 2y = 6 \\ x + 2y = 5.5 \end{cases}$ Step 2 $7x = 17.5$

Step 3 $x = 2.5$ Step 4 $2.5 + 2y = 5.5$ Step 5 $2y = 3$

Step 6 $y = 1.5$ Check $\begin{cases} 3(2.5) - 1.5 = 6 \\ 2.5 + 2(1.5) = 5.5 \end{cases}$

b. Look back closely at the start of the solution in Part a.

   i. Why was 2 chosen as the constant to multiply both sides of the first equation in the original system to obtain the system in Step 1?

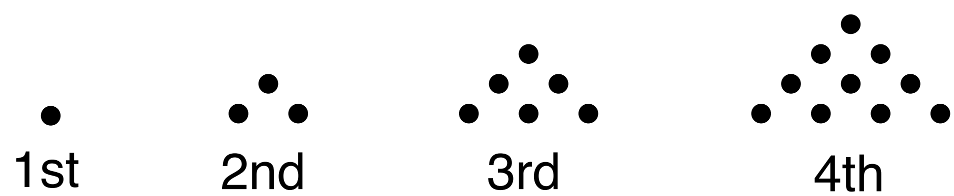
   ii. How could you start the solution process by leaving the first equation as $3x - y = 6$ and multiplying both sides of the second equation by a number that makes it easy to eliminate the x variable? Show the solution steps that would follow from choosing that multiplier.

Problem 19. The frets on a guitar are the small metal bars that divide the fingerboard. The distance d (in inches) between the nut and the first fret or any two consecutive frets can be modeled by the function $d = 1.516(0.9439)^f$ where f is the number of the fret farthest from the nut.

a. What is the distance between the nut and the first fret?

b. The distance between the 12th and 13th frets is about half the distance between the nut and the first fret. Use this fact to find the distance between the 12th and 13th frets. Use the model to verify your answer.

Problem 20. How many dots do you predict will be in the 5th figure? In the nth figure?



The numbers of dots in the figures above are called triangular numbers. The first triangular number is 1, the second triangular number is 3, the third is 6, the fourth is 10, and so on.

a. What two variables are important in this situation? Which is the independent variable, and which is the dependent variable?

b. Look for a pattern in the figures above. Use the pattern to help you make a table of the first ten triangular numbers.

c. Describe the pattern of change from one triangular number to the next.

d. How can you use this pattern of change to predict the 15th triangular number without making a drawing?

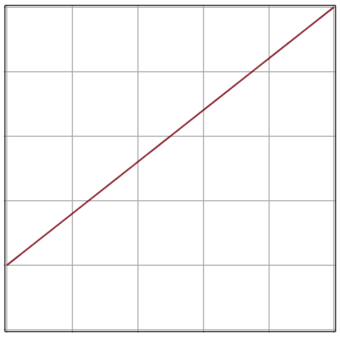
e. Write an equation that can be used to determine the nth triangular number.

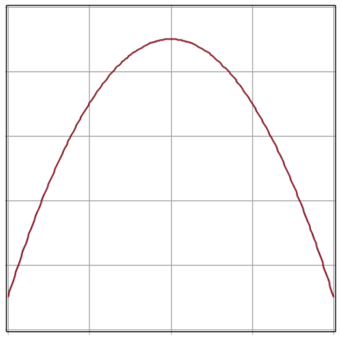
f. Does your equation represent a quadratic relationship? Explain your answer.

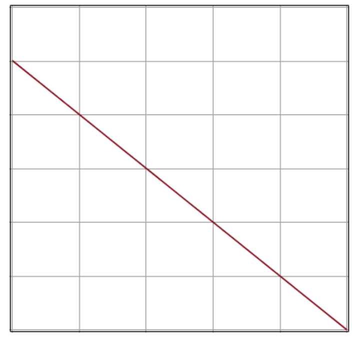
Problem 21. There are many important situations in which variables are related by a function. Below are graphs and descriptions of several such situations. Match the descriptions in Parts i-vi to graphs a-f that seem to fit them best. Then for each situation:

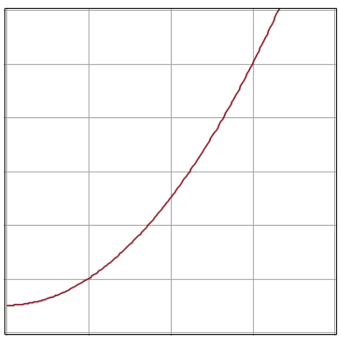
I. Explain why the graph makes sense as a model of the relationship between variables.

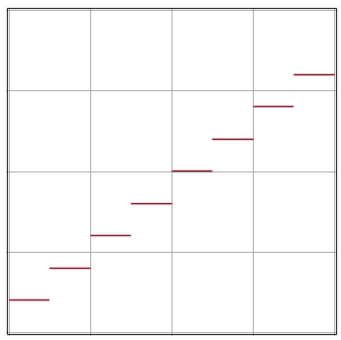
II. Describe the function family (if any) that would probably provide a good modeling rule.

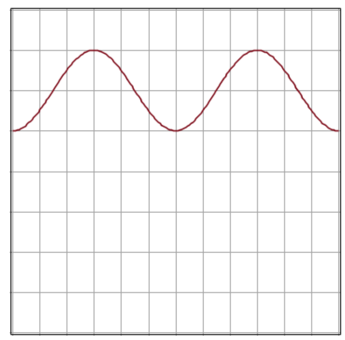

(a)


(b)


(c)


(d)


(e)


(f)

i. When a football teams punter kicks the ball, the balls height changes as time passes from kick to catch. What pattern seems likely to relate time and height?

ii. The Senior class officers at Lincoln High School decided to order and sell souvenir baseball caps with the school insignia and name on them. One supplier said it would charge \$150 to create the design and then an additional \$6 for each cap made. How would the total cost of the order be related to the number of caps in the order?

iii. The number of hours between sunrise and sunset changes throughout the year. What pattern seems likely to relate time and hours of sunlight?

iv. In planning a bus trip to Florida for spring break, a travel agent worked on the assumption that each bus would hold at most 40 students. How would the number of buses be related to the number of student customers?

v. When the Riverside High School sophomore class officers decided to order and sell T-shirts with the names of everyone in their class on the shirts, they checked with a sample of students to see how many would buy a T-shirt at various proposed prices. How would sales be related to price charged?

vi. The population of the world has been increasing for as long as records have been available. What pattern of population growth has occurred over that time?

Problem 22. A circular walkway is to be built around a statue in a park. There is enough concrete available for the walkway to have an area of 600 square feet.

a. Let the inside and outside radii of the walkway be x feet and y feet, respectively. Draw a diagram of the situation. Then write an equation relating x and y.

b. Give four possible pairs of dimensions x and y that satisfy the equation from part (a).

c. Graph the equation from part (a). What portion of the graph represents solutions that make sense in this situation?

d. How does the width of the walkway, $y - x$, change as both x and y increase? Explain why this makes sense.

Problem 23. An odd number can be expressed in the form $2m + 1$ for some whole number m. Any even number can be expressed in the form $2m$ for some whole number m.

a. Show how to express 5, 17, and 231, in the form $2m + 1$.

b. Show how to express 6, 18, and 94 in the form $2m$.

c. Prove that the product of an odd number and an even number, $(2m + 1)(2n)$, is always an even number.

d. Prove that the product of any two odd numbers, $(2m + 1)(2n + 1)$, is always an odd number.

e. Why are different letters, m and n, used to represent whole numbers in parts c and d?

Problem 24. This table shows the fees charged for campsites at one of the campgrounds on the Ocean and History Bike Tour:

Number of Campsites	1	2	3	4	5	6	7	8
Total Campground Fees	\$12.50	\$25.00	\$37.50	\$50.00	\$62.50	\$75.00	\$87.50	\$100

a. Make a coordinate graph of these data.

b. Would it make sense to connect the points on your graph? Why or why not?

c. Using the table, describe the pattern of change you find in the total campground fee as the number of campsites needed increases. How is this pattern shown in your graph?

*Many problems in this packet were taken from a presentation at the May 2014 AMTNJ conference at The College of New Jersey.