

Demystifying Division: Whole Numbers and Fractions

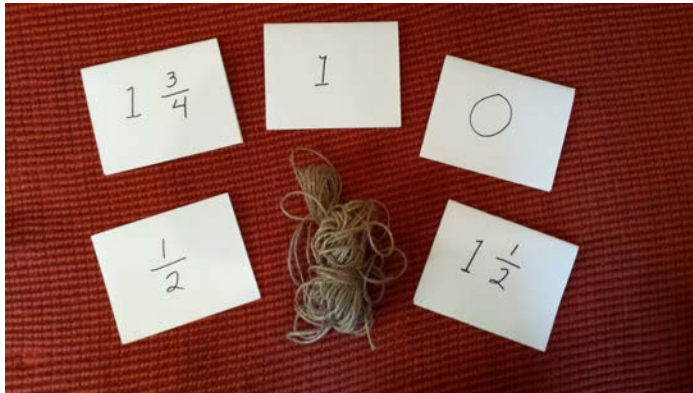
NCTM Conference Atlantic City
Thursday October 22, 2015

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Agenda



- 1.) Draw an area model to solve $96 \div 6$. Explain how you can use what you know from that problem to solve for the quotient of $100 \div 6$.

$$\begin{array}{r} 10 \div 6 = 16 \\ 6 \overline{) 96} \\ \underline{60} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

I know that $96 \div 6 = 16$ but, your asking For $100 \div 6$. $96 + 4 = 100$ so that means we have a remainder because 4 is less then 6 and nothing multiplied by 6 equals 4 so our answer would be 16 R4



Academic Language of Division

Dividend

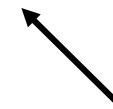


$$12 \div 2 = 6$$

Divisor



Quotient
(unknown)



Multiplicative Inverse

Product
(unknown)

$$12 = 2 \times 6$$

Multiplicand

Multiplicand

A Simple Problem



A Simple Problem



$$12 \div 2 = 6$$

Another Simple Problem



$$12 \div 2 = 6$$



Another Simple Problem



$$12 \div 2 = 6$$



$$12 \div 2 = 6$$



Are They the Same?

- Partitive: Sharing things equally



$$12 \div 2 = 6$$

- Quotative (Measurement): How many times one number fits into another



$$12 \div 2 = 6$$



Two Kinds of Division

- Our class got 20 goldfish. Wow!
 - We had to separate them equally into 5 fish bowls. How many went into each bowl?
 - We had to put them into groups of 5 and place them into separate bowls. How many bowls will we need?

$$20 \div 5 = 4$$

Two Kinds of Division

- Partitive: Sharing things equally
 - We know the starting value.
 - We know how many groups.
 - The size of each group is unknown.

Two Kinds of Division

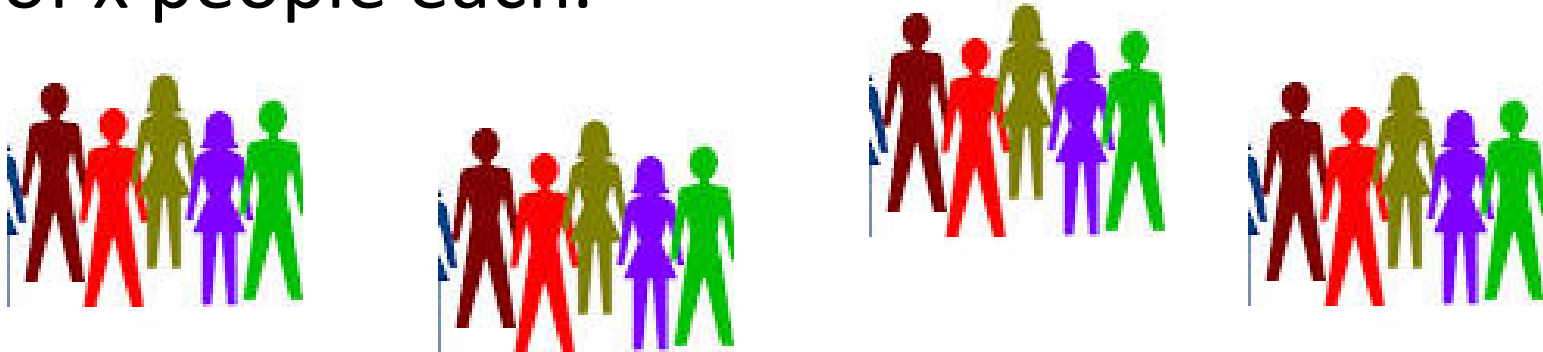
- Partitive: Sharing things equally
 - We know the starting value.
 - We know how many groups.
 - The size of each group is unknown.
- Quotative (Measurement): How many times one number fits into another
 - We know the starting value.
 - We know the size of each group.
 - The number of groups is unknown.

Whole Group Activity

- Partitive: Divide yourself into x number of groups.



- Quotative (Measurement): Divide into groups of x people each.



Which is Which?

Partitive –
of groups
is known

- Our class got 20 goldfish. Wow!
 - We had to separate them equally into 5 fish bowls. How many went into each bowl?
 - We had to put them into groups of four and place them into separate bowls. How many bowls will we need?

$$20 \div 5 = 4$$

Quotative –
the size of
each group is
known

What Happens with the Remainder?







We can fit 12 of the 14 students in the three cars. For the other two . . .



. . . we'll get a 4th car?
. . . you get to go to the beach instead!

Partitive Model: Whole Numbers



conceptua[®] MATH Equal Shares

 We have 24 students in the class, and we get to divide up into four teams. How many will be on each team? 

  10

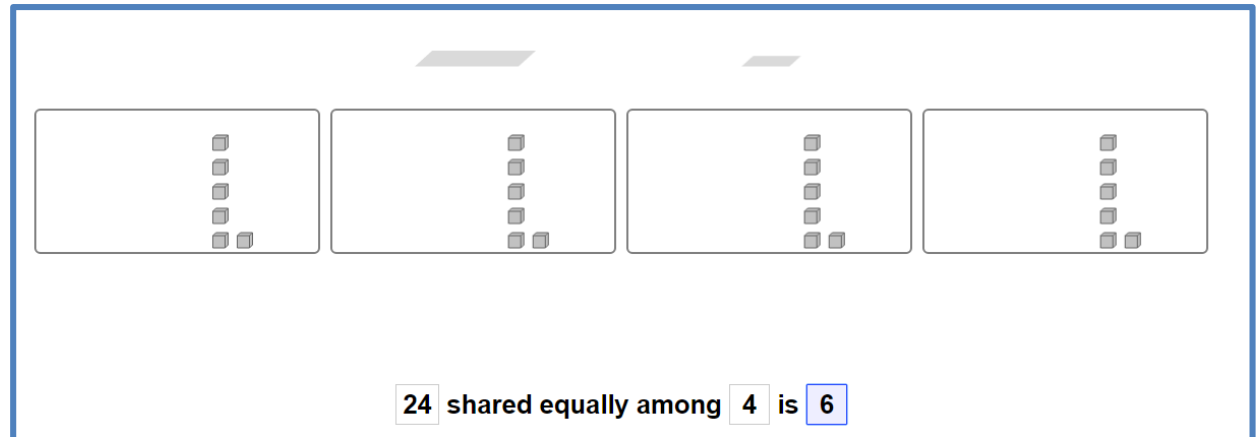
 

shared equally among is

0 1 2 3 4 5 6 7 8 9  

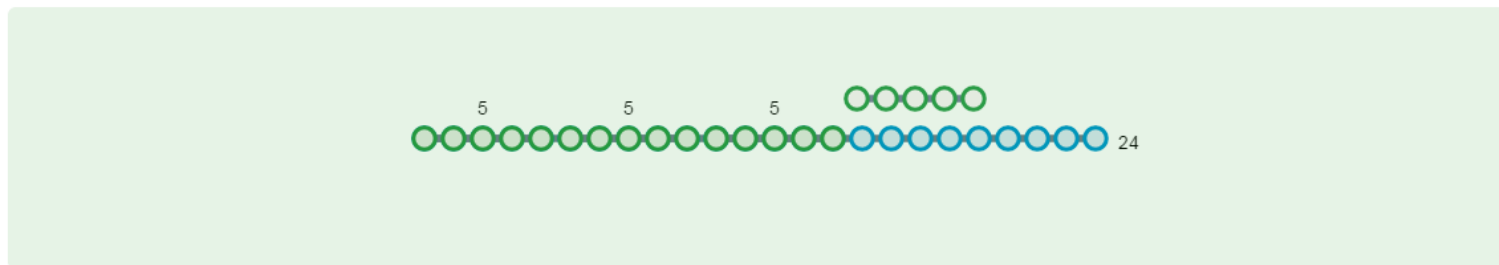
5 hr 03 min (90%) remaining

Partitive Model: Whole Numbers



Quotative Model: Whole Numbers

There were 24 students in the class. We needed to make basketball teams of 5 people each. How many teams could we make?



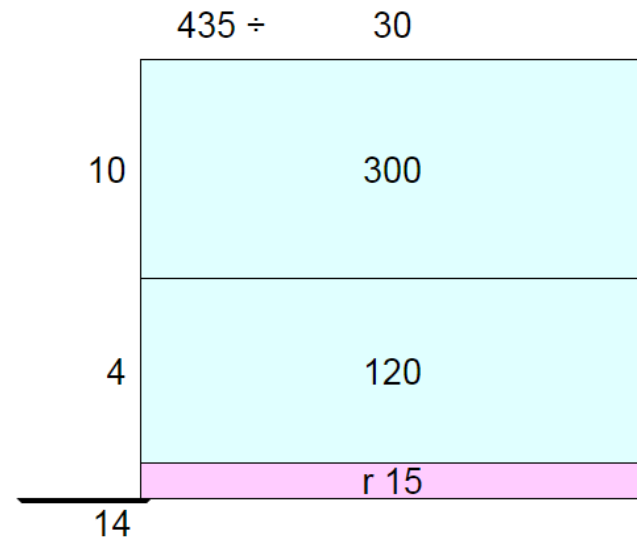
24 contains 4 groups of 5 with 4 remaining



Modeling Abstractions



$$\begin{array}{r} 10 \times 30 \\ + 4 \times 30 \\ \hline 14 \end{array} \quad \begin{array}{r} 14 \overline{) 435} \\ \underline{300} \\ 135 \\ \underline{120} \\ 15 \end{array}$$



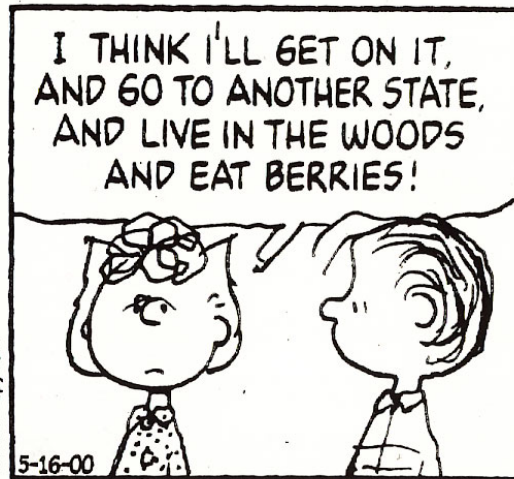
435 contains 14 groups of 30 with 15 remaining



The Open Array Model

Working with Fractions

CLASSIC PEANUTS



Partitive Division : Fractions

- Arjan left his hotel, went to a jazz club, and then walked back later, a very happy person. When he checked his GPS, he saw that he had walked a total $1 \frac{3}{4}$ miles.
- How far did he walk to go to the jazz club?
How long was his one-way trip?

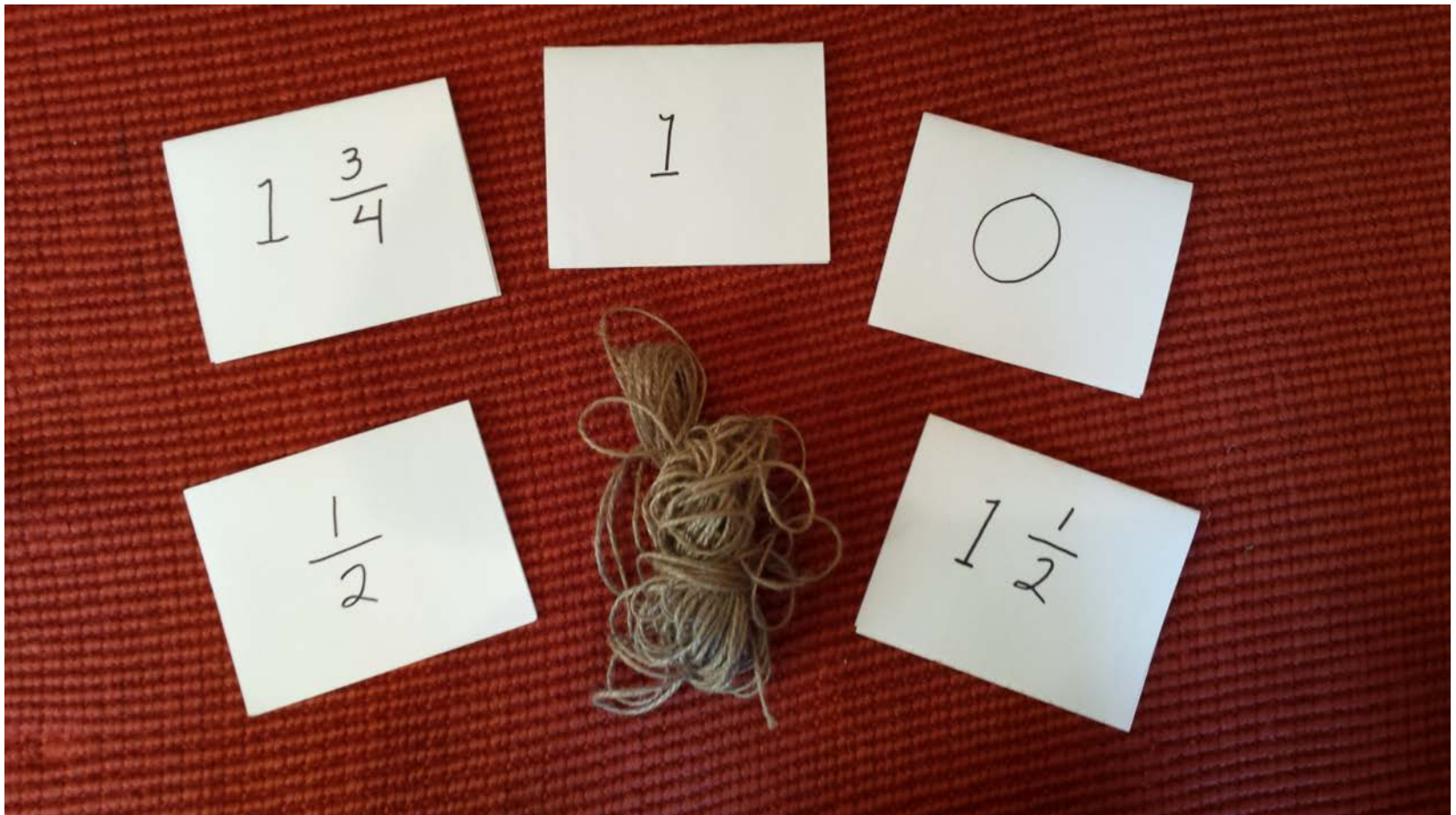
$1 \frac{3}{4}$ shared equally among 2

Quotative Division: Fractions

- Arjan has $1 \frac{3}{4}$ lb. of chocolate and needs $\frac{1}{2}$ lb. to make his favorite fudge recipe. He is willing to prepare partial batches.
- How many batches of fudge can he make with the chocolate he has?

How many $\frac{1}{2}$ s are in $1 \frac{3}{4}$?

Fraction Tents



Partitive Model: Fractions

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Arjan left his hotel, went to a jazz club, and then walked back later, a very happy person. When he checked his GPS, he saw that he had walked a total $1\frac{3}{4}$ miles. How far did he walk to go to the jazz club? How long was his one-way trip?

$1\frac{3}{4}$ shared equally among

Dividend \div Divisor

$$\frac{7}{4} \div \frac{2}{1} = \frac{7}{4} \times \frac{2}{2} \div \frac{2}{1} = \frac{7}{4} \times \frac{1}{2} = \frac{7}{8}$$

Rename

Invert & Multiply

Quotient

Divide Fractions - Partitive

Quotative Model: Fractions

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Arjan has $1 \frac{3}{4}$ lb. of chocolate and needs $\frac{1}{2}$ lb. to make his favorite fudge recipe. He is willing to prepare partial batches. How many batches of fudge can he make with the chocolate he has?

How many groups of $\frac{1}{2}$ are in $1 \frac{3}{4}$?

When the models show the dividend and divisor, shade and drag groups of or parts from the divisor at the right onto the dividend below to fill it. The quotient is the number of groups and/or parts of the divisor that fit in the dividend.

Dividend \div Divisor = Quotient

7	\div	1	=	
4		2		

Navigation icons: Home, Check, Close, Chat, Settings

Quotative Model: Fractions

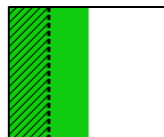
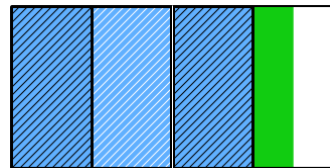
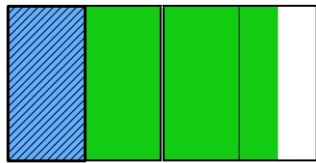
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Divide Fractions - Measurement



Arjan has $1\frac{3}{4}$ lb. of chocolate and needs $\frac{1}{2}$ lb. to make his favorite fudge recipe. He is willing to prepare partial batches. How many batches of fudge can he make with the chocolate he has?

How many groups of $\frac{1}{2}$ are in $1\frac{3}{4}$?

Dividend \div Divisor

$$\frac{7}{4} \div \frac{1}{2} = \frac{7}{2} = 3\frac{1}{2}$$

Quotient

Making Your Own Problems

x groups, or x in each group
whole numbers or fractions

- **Collections** (objects, school supplies, food, animals)
 - We received 43 fish, and we needed to divide them into 4 tanks, or we were told to put no more than 8 into a tank.
 - There are 16 people. How many teams of 5 can I make? Or, I need to make 5 teams, how many can I have on each team?
- **Time** (traveling, reading, playing sports)
 - The teacher read our homework for $3\frac{1}{2}$ hours. Each paper took $\frac{1}{4}$ hour to read, or she read 14 papers.
 - The trail is $7\frac{1}{2}$ miles long. It takes me $\frac{3}{4}$ hour to hike 1 mile, or how fast must I hike to make it in 4 hours?

Making Your Own Problems

x groups, or x in each group
whole numbers or fractions

- **Rate** (pages read per minute, miles traveled hour, \$ earned per day)
 - I ride x miles a day, I ride an average of 14 miles per hour and I rode 30 miles
 - I earn \$40/ hour and I need \$300
- **Capacity** (acres of land, square feet of a room, box can hold x)
 - The land is $2\frac{3}{4}$ acres. We will divide it into x parcels, or we can divide it into parcels that are x acres in size.

Notice and Wonder



**What fractions do you see?
What division problems can you generate from this picture?**

Inspired by the Math Forum at Drexel, part of NCTM

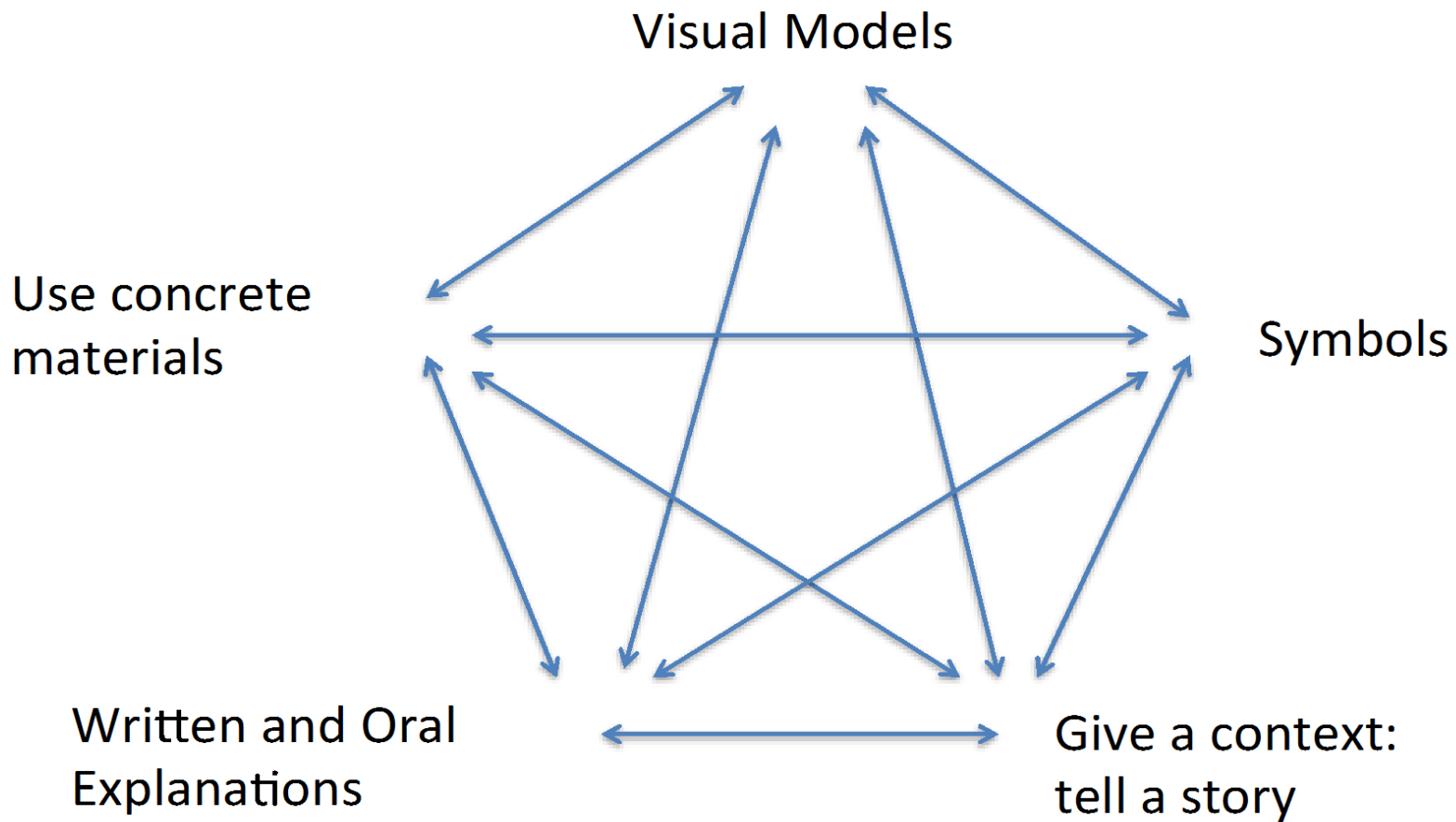
Arijan Khalsa, October 22, 2015 | akhalsa@conceptuamath.com | [@arijanKhalsa](https://twitter.com/arijanKhalsa)
<http://mathforum.org/pow/support/activityseries/understandtheproblem.html>

Fractions and Division



- Fractions
 - $\frac{1}{2}$ gallon, $\frac{1}{4}$ gallon (1 quart)
 - $\frac{3}{4}$ of the container
 - $\frac{1}{2}$ of a quart
 - $1 \frac{1}{2}$ bottles
- Division
 - Share ($\frac{1}{2}$ gallon, $\frac{1}{2}$ bottle, $1 \frac{1}{2}$ bottles, $\frac{3}{8}$ gallon) between x people
 - How many glasses of $\frac{1}{4}$ of a bottle are in the glass bottles?
 - $\frac{3}{8}$ gallon is how many quarts?

Ways to Demonstrate Mathematical Understanding



Sample Student Work

- 1.) Draw an area model to solve $96 \div 6$. Explain how you can use what you know from that problem to solve for the quotient of $100 \div 6$.

$$10 + 6 = 16$$

6	$\begin{array}{r} 96 \\ -60 \\ \hline 36 \end{array}$	$\begin{array}{r} 36 \\ -36 \\ \hline 0 \end{array}$
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I know that $96 \div 6 = 16$ but, you're asking for $100 \div 6$. $96 + 4 = 100$ so that means we have a remainder because 4 is less than 6 and nothing multiplied by 6 equals 4 so our answer would be 16R4

Sample Student Work

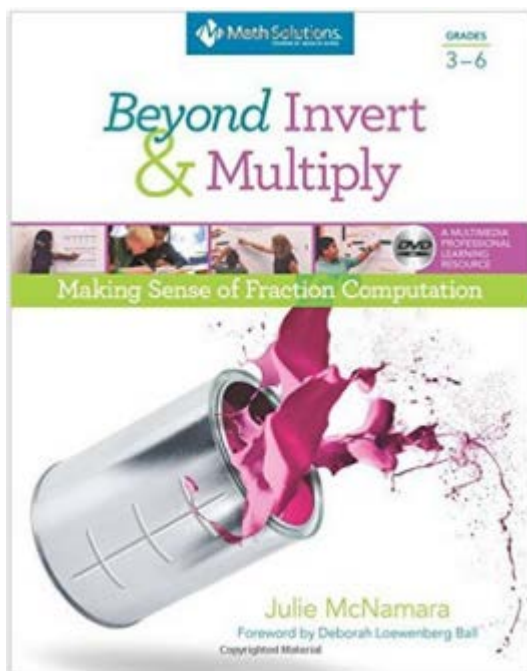
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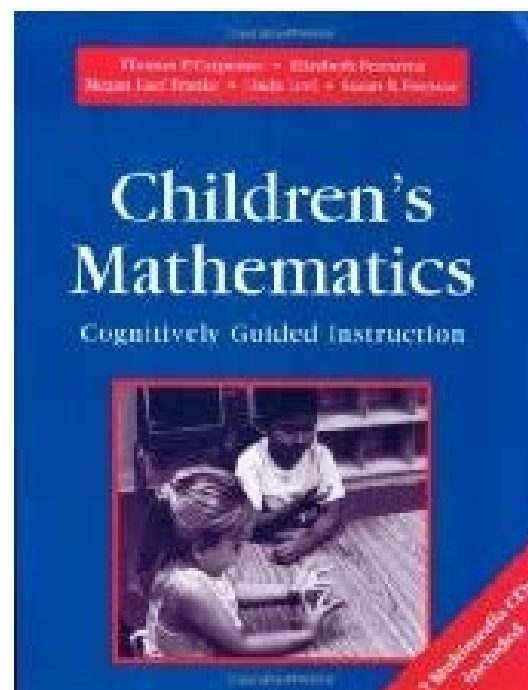
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Sample Student Work - Videos



Beyond Invert and Multiply,
by Julie McNamara, PhD.



Children's Mathematics: Cognitively Guided Instruction
By Carpenter, et al.

Stay in Touch

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