

# RTI IN MATHEMATICS: WHAT REALLY IS EFFECTIVE?

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NCTM

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# GOALS OF THE SESSION

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1. Obtain a sense of status of implementation
2. Orient participants to IES Practice Guide on RtI Mathematics as resource
3. Review and update consensus of experts based on recent research synthesis
3. Detail effective instructional practices
4. Articulate areas of confusion

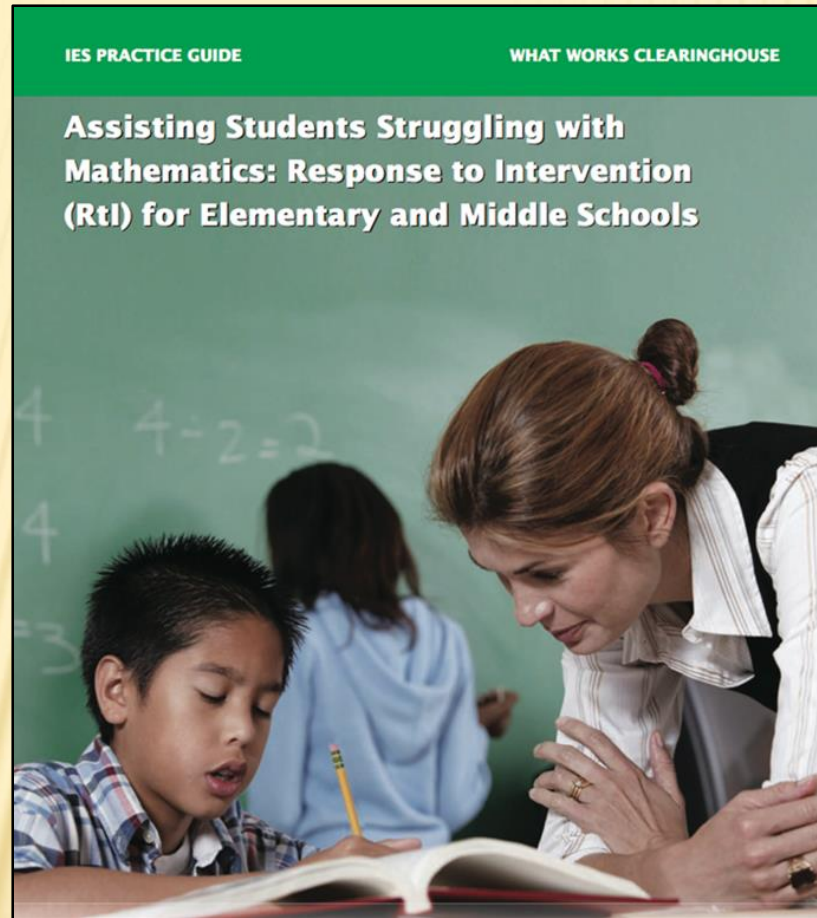
# THINK PAIR SHARE #1

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1. One or two successes in RtI mathematics implementation or any intervention services in mathematics

# RTI MATH PRACTICE GUIDE

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<http://ies.ed.gov/ncee/wwc/PracticeGuide>

Instructional Research Group

# EVIDENCE RATING

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1. Each recommendation receives a rating based on the strength of the research evidence.
  - Strong
  - Moderate
  - Minimal simply means no rigorous evidence, **not contradictory evidence**

# LEVELS OF EVIDENCE

Recommendation	Level of Scientific Evidence
1. Universal screening (Tier I)	Moderate
2. Focus instruction on whole number for grades k-5 and rational number for grades 6-8	Minimal
<b>3. Systematic instruction</b>	<b>Strong</b>
<b>4. Solving word problems</b>	<b>Strong</b>
5. Visual representations	Moderate
6. Building fluency with basic arithmetic facts	Moderate
7. Progress monitoring	Minimal
8. Use of motivational strategies	Minimal

# THINK PAIR SHARE #2

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Which level of evidence is the  
biggest surprise for you?

Why?

# RECOMMENDATION 2

## WHAT TO TEACH IN INTERVENTION

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Instructional materials for students receiving interventions should focus on:

- Whole numbers in K through 6
- **Rational numbers** in grades 4 through 8
- Applications to geometry and measurement

Level of Evidence: **Minimal**



# EMERGING CONSENSUS ON BEST WAY TO TEACH MATHEMATICS

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1. Instruction should include, and sometimes *integrate*
  - procedures
  - AND concepts
  - AND word problems
2. Whole number work consistently links operations to number properties
3. Same true for work with rational numbers (fractions/decimals)

# RECOMMENDATION 3

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Instruction during the intervention should be **systematic** and include models of proficient problem-solving, **verbalization of thought processes**, guided practice, corrective feedback, and frequent cumulative review.

Level of Evidence: **Strong**

# EVIDENCE (UPDATED)

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Some support in recent intervention for teaching more challenging material to students requiring mathematics intervention

- Teaching first graders to become fluent with the number line
- Teaching fourth graders requiring intervention to use the linear representation of fractions

# EVIDENCE FROM 6 RANDOMIZED CONTROL TRIALS

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- Extensive practice with feedback over several lessons \*
  - Need not be boring
- Very systematic in terms of introducing new mathematical ideas, cumulative review\*
- Let students provide rationale for their decisions
- Instructors model approaches to problem solving\*
- Fellow students think aloud and model

# ROADBLOCKS

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- (1) Intervention curricula may not have explicit instruction and may underestimate the amount of practice and review needed.
- (2) 2) Intervention curricula may not ask students to explain their reasoning through words or visual representations.

## Suggested Approaches:

1. Develop guidebooks for school staff to adapt the lessons.
2. Add new review problems and provide more practice.
3. Provide sample formats for interventionists to use

# EXPLICIT INSTRUCTION ALTERNATIVES

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**Question:** “What is involved in making success with complex mathematical work accessible for all students— **without doing the work for them?**”

**Answer:** *Explicit instruction* “seeks to make complex practice accessible” by **unpacking complex knowledge and mathematical practices”**

Ball, D. L. (2015, April). *With respect for teaching: Making the practice of mathematical instruction explicit*. Presented at the National Council of Teachers of Mathematics Annual Meeting, Boston, MA. Her role is acknowledged.

# WHAT DOES RESEARCH SAY?

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- ❖ Only one randomized trial examined efficacy of the alternative definition of explicit instruction.
  - ✓ This study showed positive impacts for first graders using mathematics Recovery
  - ✓ Drawbacks: 1:1 , costly, limited to first grade
    - Students did not maintain gains a year later.
- ❖ For systematic instruction, almost a dozen studies BUT no evidence of maintenance of effects either.
  - As a researcher, I conclude that the case is far from resolved. Explicit instruction requires higher degree of skill and knowledge than systematic instruction.

# THINK-PAIR-SHARE

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- ❖ Briefly share with a partner your ideas of potential for each method for at risk learners.



# RECOMMENDATION 5

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Intervention materials should include opportunities for the student to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Level of Evidence: **Moderate**

# THINK PAIR SHARE

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Shauntay spent  $\frac{2}{3}$  of the money she had on a book that cost \$26. How much money did she have before she bought the book?

1. Solve the problem
2. How did you find the answer?

# SUGGESTIONS

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1. Use visual representations such as number lines, arrays, and strip diagrams.
2. Consider use of concrete manipulatives before visual representations. The goal should be to move toward abstract understanding.

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The following slides are adapted from

*Evidence-Based Fraction Intervention at 4<sup>th</sup> Grade: Evolution of Curriculum in relation to Common Core* by Robin F. Schumacher,  
Instructional Research Group

And research studies such as:

*Improving At-Risk Learners' Understanding of Fractions* (Fuchs et al., 2013)

# BIG IDEAS UNDERLYING 4<sup>TH</sup> GRADE FRACTIONS INTERVENTION: FRACTIONS FACE OFF

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1. Build understanding of a fraction as a number (because some students think a fraction is two different numbers)
2. Primary focus: Linear representations (aka Measurement Interpretation) (typical Asian curriculum)
  - Number Lines
  - Fraction tiles or strip diagrams as a transition tool
  - Magnitude: ability to reason about size and relative size
3. Secondary focus: Part-Whole Understanding (typical US curriculum)
  - Shaded Regions of one or more Units (e.g. pizzas etc.)

# INSTRUCTIONAL DESIGN

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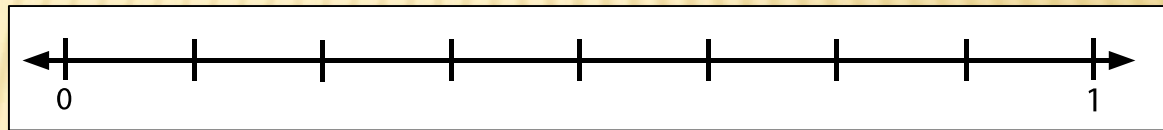
1. Introduce concept with manipulatives
  - Fraction Circles or Fraction Tiles
2. Provide “real-life” context
  - Equal Sharing example
3. Provide strategies for solving each task
4. Build on existing knowledge base to maintain conceptual Foci
  - Part-Whole
  - Measurement (to expand concept of fractions)

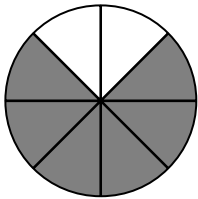
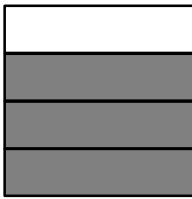

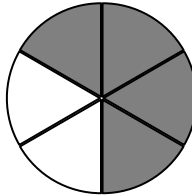
# EARLY SKILLS:

## UNIT FRACTIONS AND NAMING FRACTIONS

- Introduce unit fractions with shaded regions
- Show how *unit fractions* make larger fractions with manipulatives, number lines, and numbers
- Name fractions from shaded representational regions (see example below)

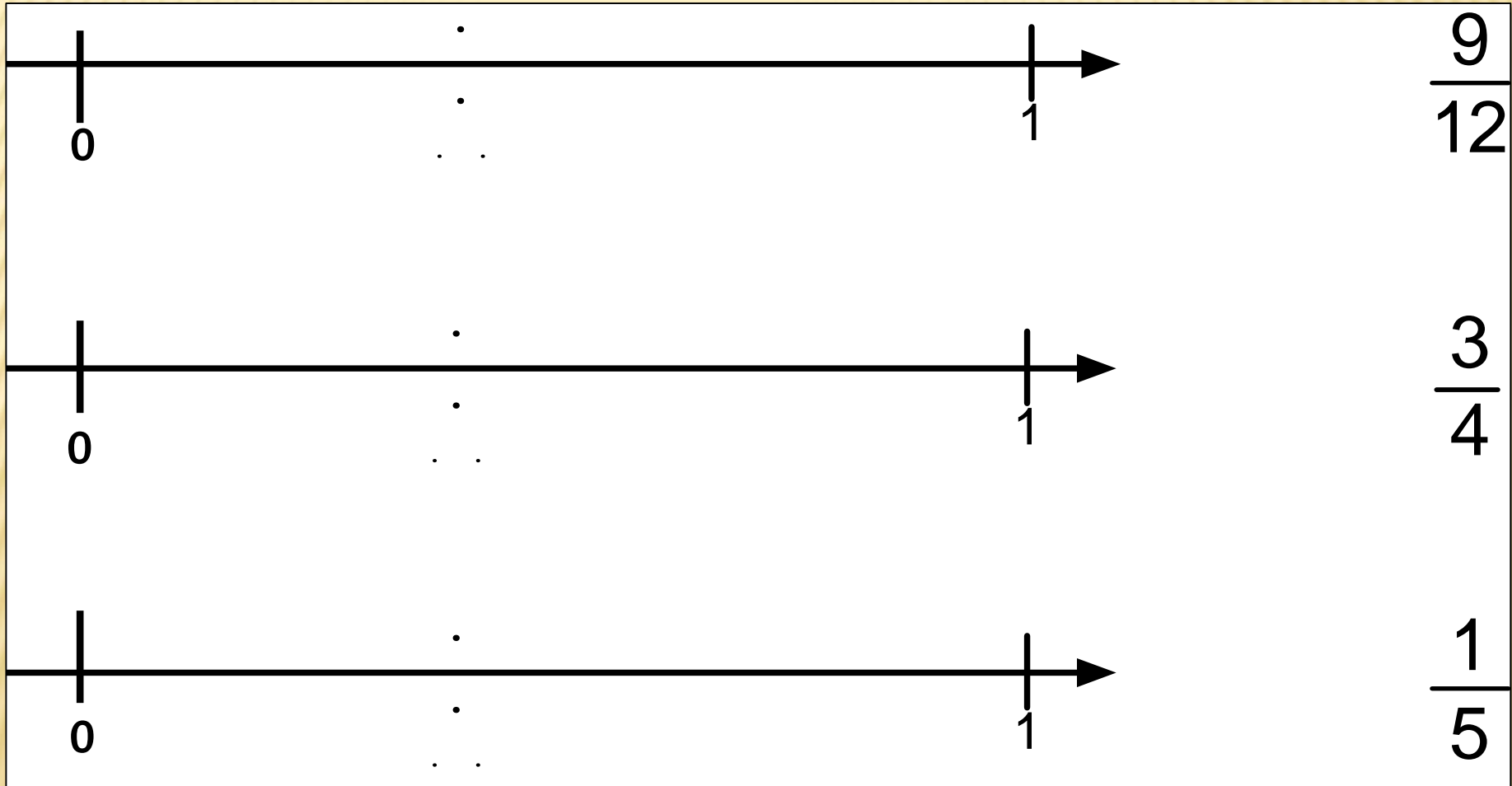
$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$$



A.		_____
B.		_____
C.		_____
D.		_____

# COMPARE THESE FRACTIONS USING NUMBER

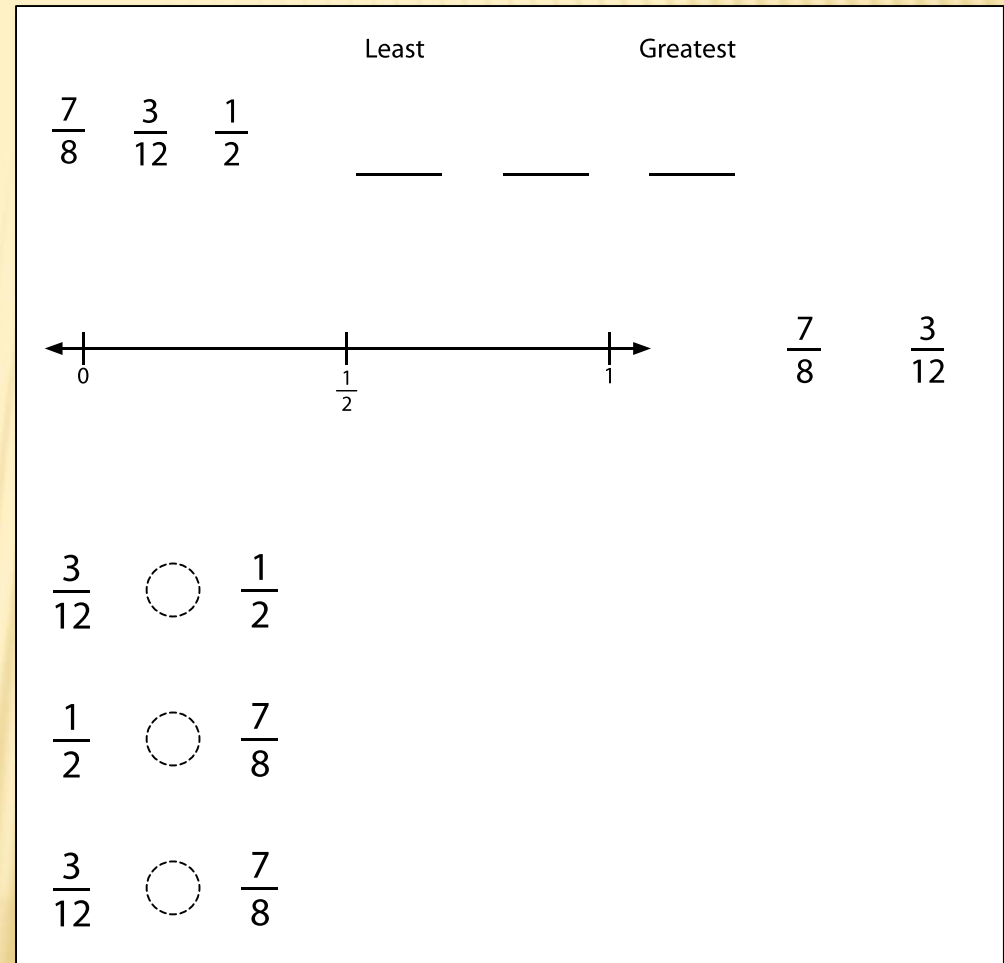
LINES:  $\frac{3}{4}$ ,  $\frac{9}{12}$ ,  $\frac{1}{5}$





# BUILDING MAGNITUDE UNDERSTANDING

- Relating Magnitude Activities
- Use the same three fractions for each magnitude activity
  - Comparing
  - Ordering
  - Number Line



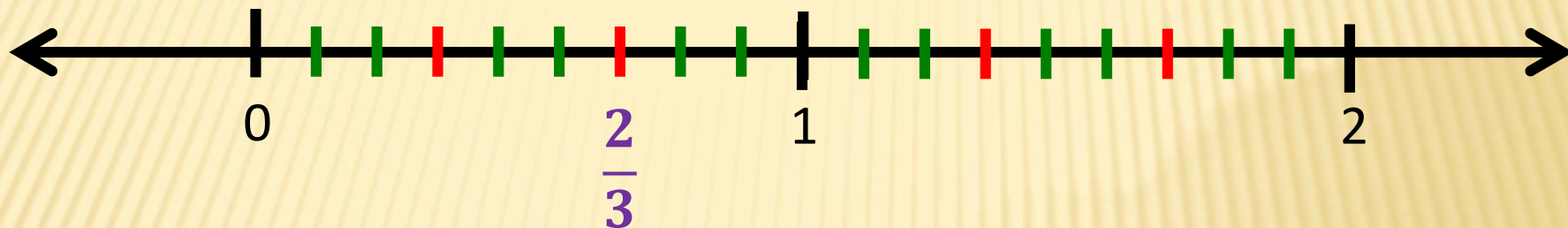
# THINK PAIR SHARE

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1. Assignment: Solve this problem using a number line

$$\frac{2}{3} + \frac{5}{9}$$

# THE NUMBER LINE



- ❖ Where is the location of  $\frac{2}{3} + \frac{5}{9}$  ?
- ❖ What's the answer?  $1 \frac{2}{9}$  or  $\frac{11}{9}$

# RESULTS FROM 3 YEARS OF RESEARCH

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Assessment	Tutoring vs Control (Y1)
Comparing	1.82
Number Line	1.14
NAEP	0.94
Calculations	2.51

# RECOMMENDATION 4

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Interventions should include instruction on solving word problems that is based on common underlying structures.

Level of Evidence: **Strong**

# SUGGESTIONS

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1. Teach students about the structure of various problem types, **how to categorize problems**, and how to determine appropriate solutions.
2. Middle step, is it:
  - Quantity (compare)
  - Change (over time)

# EXPLICITLY TEACH THE UNDERLYING STRUCTURE

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## Addition and Subtraction Story Problems

### 1. Change problems

A quantity is increased or decreased

### 2. Group/Combine Problems

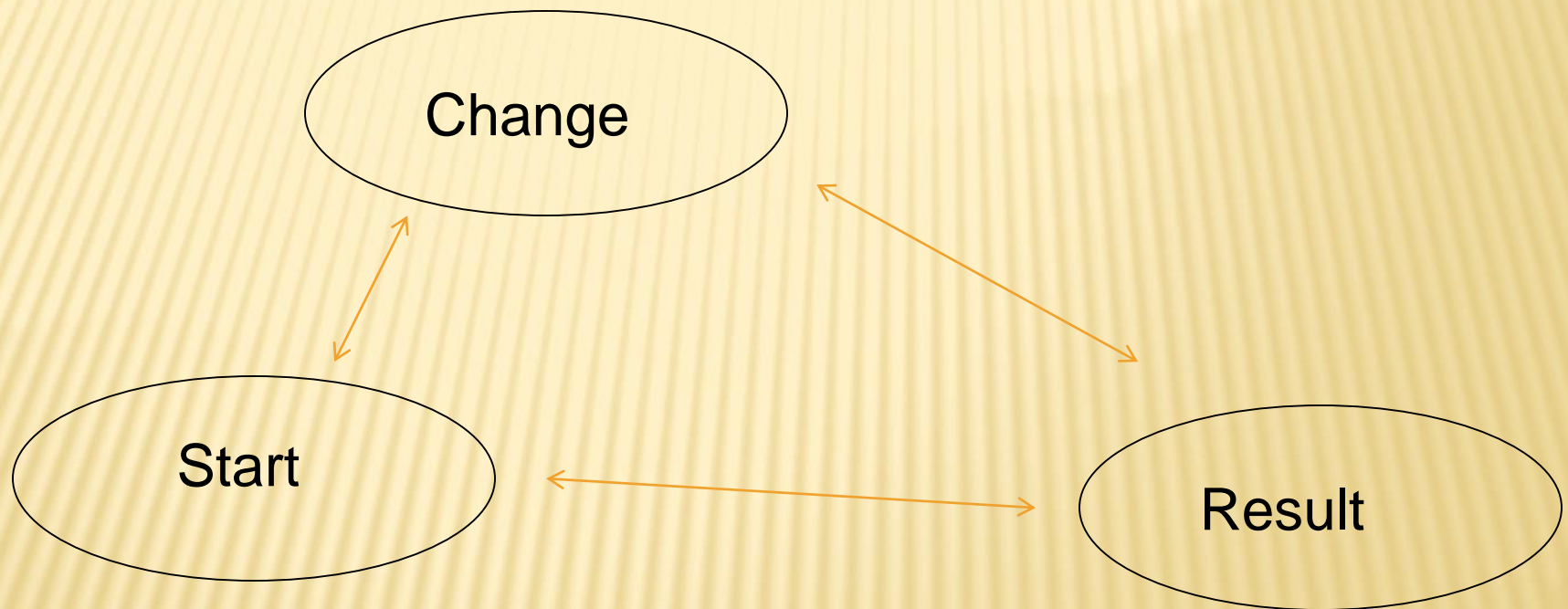
Two groups are combined to form a large group

### 3. Compare Problems

Two things are compared to find the difference

# Explicit Schema for Additive Structures

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# CCSSM Appendix - Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>1</sup>
Put Together/ Take Apart <sup>2</sup>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

# CHANGE, GROUP, OR COMPARE?

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## *Think Pair Share #???*

1. Dillon leaped 32 inches. Marcus leaped 27 inches. How many more inches did Dillon leap?  
(Everyday Math 4)
2. Uranus has 11 rings. Neptune has 4 rings. How many rings do they have altogether? (SF/AW 3)
3. There are 18 ducks. Then 5 more swim over. How many ducks are there now? (Math Expressions 1)

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# Questions?

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# Thank You