You, too, Can Model with Mathematics

Cathy Martin (cathy_martin@dpsk12.org)

NCTM Regional Conference and Exposition
Atlantic City, October 21-23, 2015

Math Practice 4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Tasks to Consider

Gotham City Taxis (7.RP.A.3, 7.EE.B.3, 7.EE.B.4—Illustrative Mathematics)

The taxi fare in Gotham City is $2.40 for the first ½ mile and additional mileage is charged at the rate of $0.20 for each additional 0.1 mile. You plan to give the driver a $2 tip. How many miles can you ride for $10?

The Dairy Queen Dilemma (7.RP.2—MTMS, Vol. 20, No 6, p 352)

I love ice cream, and I love bargains. One day, I went to Dairy Queen to have a Blizzard. I saw four sizes listed with the following prices: The Mini Blizzard was $2.55, the Small Blizzard was $3.25, the Medium Blizzard was $3.80, and the Large Blizzard was $4.65. If I wanted to get the most Blizzard for my money, then which size should I buy? Prove that you selection is the best buy by justifying your answer.

Dogs and Cats (K.OA.2)

There are nine animals in the yard, some are dogs and some are cats, how many of each could there be?

Water Conservation Task (6.RP.2-3, 7.RP.1-2, 7.EE.3-4--MTMS, Vol. 20, No 6, p 345)

Some water conservationists say that showering uses less water than bathing. Others say that this is not true! Keep in mind that older showerheads have a flow rate of up to 3.4 gallons/minute whereas energy-saving showerheads have a flow rate as low as 1.9 gallons/minute. Bathtubs also vary in size. Provide a method to determine if a shower or a bath uses more water and explain your approach.

Ice Cream Van (N-Q.A.1—Illustrative Mathematics)

You are considering driving an ice cream van during the summer vacation. Your friend, who "knows everything" tells you that "It's easy money." You make a few inquiries and find that the van costs $600 per week to rent. Each ice cream cone costs 50 cents to make and sells for $1.50.

For each of the questions below, show all work and include an explanation of your method of solution.

a. How many ice cream cones would you have to sell each week just to cover the cost of renting the van?

b. In order to sell the ice cream cones, you have a choice of driving the van through neighborhoods or parking the van in a public area. Typical selling data is that one can sell an average of 35 ice cream cones per hour at each of your planned stops if driving through neighborhoods, while you can sell an average of 30 ice cream cones per hour if one parks the van in a public area.

i. If you choose to drive the van, you will have to consider the time spent driving the van, which will depend on the average speed from stop to stop on your route, as well as the cost of gasoline, which will depend on the number of miles per gallon the van gets. Make reasonable estimates for these and any other costs you feel would be relevant. If you drive an average of 180 miles per week, how many ice cream cones would you have to sell just to cover the cost of driving the van for a week (not including rental costs)?

ii. If you choose to park the van, you will have to pay a one-time seasonal permit fee and weekly space rental. If the seasonal permit costs $90.00 and space rental ranges from $140 to $150 per week, how many ice cream cones would you have to sell just to cover the cost of parking the van for a week (again, not including rental costs)? Identify any assumptions you make.

c. How many hours a week will you have to work in order to make this "easy money"? After how many hours would the amounts you earned under each of the two options be the same? How much money might you be able to make if you were willing to work really hard? Identify and take into account any additional expenses for the additional hours. Explain your reasoning clearly.

Comparing Money Raised (4.OA.A.2—Illustrative Mathematics)

a. Helen raised $12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

b. Sandra raised $15 for the PTA and Nita raised $45. How many times as much money did Nita raise as compared to Sandra?

c. Luis raised $45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise?

Birthday Cakes (G-GMD.A.3)

Would all the birthday cakes eaten by all the people in Texas in one year fit inside the Dallas Cowboys football stadium? Justify your answer mathematically.

High School Graduation (8.F.B.4—Illustrative Mathematics)

The SLV High School graduation started at 1:00PM. After some speeches, the principal started reading off the names of the students, alphabetically by last name. When he finishes, the graduation will end.

a. Use the bulletin shown below to estimate when the graduation will end.

San Lorenzo Valley High School
Class of 2011

1:28pm

Micah Stephen Adams
Xavier Josue Alvin
Connor James Antisdel
David Zachary Baker
Bradley Garrett Barnard
Joseph Barnes
James Alexander Barnes
Michael Barton
Sarah Raisha Beasley
Allison Rose Beasley
Emily Anne Bechtle
Brandon Scott Beevers
Patrick Fereva Belardi
Kaitlyn Ella Benson
Jordan Starr Thomas Bereman
Taylor Renee Foster Berritto
Ashley Michelle Ann Billington
Morgen King Biswa
Jaclyn Kate Black
Xitlali Borreson
Chere Nicole Brandon
Sydney Lauren Garcia Breli
Curtis Martin Brewer
Katherine Evelyn Brown
Annika Bruce
Amanda Lynn Bruce
David Michael Burgler
Jourdan Donovan Burk
Nicholas Ryan Burks
Joseph Thomas Burton
McLean Avery Camacho
Alexander Austin Campbell
Jason William McGregor
Taylor James Casey
Kory Daniel Chadwick
Audrey Elizabeth Chapin

Amber Anne Chase
Faye Elise Chavez
David Thomas Chegus
Zoe Darlene Chinn
Emily Elizabeth Clements
Claire Elizabeth Cloud
Steven Dennis Connelly
Conor Thomas D'Amato
Cassandra Suzanne Davis
Lauren Marie Dempewolf
Blake Bradley Dennis
Jake Madison Debreuil
Shannon Elaine Eisner
Emily Anne Engel
Samantha Marie Ferguson
Dominique Angelina Jones
Cherina Marie Freitas
Anju Friend
Nathaniel Justin Fruzza
Mariah Nichole Galmez
Robin Lavender Garcia
Ronja Andrea Franklin
Mason Garrett
Kayle Anne Genis
Melanie Lauren Gleim
Andrea Serena Godbout
Angela Gonzales
Chloe Chandler Goodreau
Dakota Makua Gorman
Dondie Jessie Greer
Michael Marie Grindy
Erin Cassidy Groewird
Krista Marie Grunberger
Tyler Alan Hagen
Hazel Gladys Jane Haikkila
Marcus Taylor Halversen

1:33pm

Nicholas Gregory Hargraves
Alexander Samuel Harnett
David Michael Hart
Melissa Lorraine Harvey
Olivia Louise Herrera
Kasia Adair Hill
Ludmila Eliana Hlosley
Jazelle Gina Hooper
Mariah Donna May Hopkins
Lindsey Marie Hoopin
Bryn Kathryn Horton
Thomas John Housek
Ellen Joanne Innis
Jordan Isaacson
Joseph Clifford Jansen
Allison Kirstine Janus
Nettie Pearl Mitchell Johnson
Lacey Marie Johnson
Felipe Johnson
Shelby Danielle Johnson
Zachary David Johnson
Dakota Shea Jones
Tanner William Jones
Clara Joy Kamau
Mia Anne Kellogg
Maxine Elaine Kelly
Ryan Scott Kennedy
Jenny Rae Kersten
Andrew Benjamin King
Russell Theodore Klair
Zachariah Weya Tsgifi Klus
Christina Rose Knoll
Jessica Danielle Kraft
Jessica Ann Lacy
Joseph Allan Landry
Abbie Mae Leveque

Matthew Alexander Lippert
Madeline Elizabeth Lopez
Emily Marie Madison
Rebecca Lynn Makita
Savanna San Mangelsen
Zane Michael Markham
Alfredo Martinez
Tyler Cameron Mattson
Wayne Thomas McCarthy
Theos Christopher McClish
Annelise Marie McFarland
Alexander Michael McIntosh
Dominique Sharee McIntosh
Victoria McKenzie
Violett Josette McNally
Kendra Dee Messimer
Marisa Grace Brudnick
Sage Nicole Monack
Thomas Ocean Moreno
Rainbow Roxanne Muchamuel
Nicholas Jefferey Newberry
George Austin Norfleet
Travis James Nugent
Joseph Carl Olson
Lacie Marie Orlando
Mary Elaine Ivy Orr
Austin Leonard Overton
Haley Lauren Pace
James Evan Paolini
Timothy William Parker
Carson Taylor Paynter
Zachary Alan Peabody
Katrina May Pearce
Samuel Jackson Pelphrey
Chandler Elizabeth Perazzo
Sebastian Thomas Peterson

A Wikipedia Claim (N-Q.A.1-3)

Wikipedia reports that 8% of all Americans eat at McDonald's every day. Data reveal approximately 311 million Americans in 2012 and 12,800 McDonald's restaurants in the United States. Make a conjecture as to whether or not you believe the web release to be true, and then create a mathematical argument that justifies your conclusion.

Rock Concert (N-Q.A.1-3 --2003 PISA)

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. How many fans attended the concert? Justify your answer mathematically.