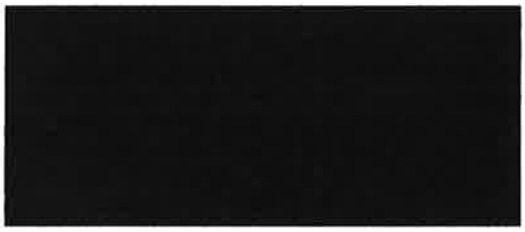
LET'S TALK MATH!

Creating effective mathematical discourse

GOALS

- Understand features of effective mathematical discourse at varying levels of implementation
- Learn to orchestrate productive classroom discussions by posing purposeful questions
- Identify components of the classroom community and environment that supports productive discussions
- Evaluate current practice and learn tips to increase classroom discourse

WHAT DOES THE DISCOURSE LOOK LIKE IN YOUR MATH CLASS?



WHAT IS MATHEMATICAL DISCOURSE?

- NCTM defines discourse as:
  - ways of representing, thinking, talking, agreeing, and disagreeing;
  - the way ideas are exchanged and what the ideas entail;
  - and as being shaped by the tasks in which students engage as well as by the nature of the learning environment.
- Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

WHY BEFORE THE HOW...

- Mathematical discourse is the primary mechanism to develop conceptual understanding
- Turn and talk....what is the typical teacher-student interaction in most classrooms

GOALS OF DISCOURSE

- Engaging students in the sharing of ideas, reasoning and approaches using varied representations
- Position students as the author of ideas
- Provide opportunities for students to explain and defend their approaches
- Ensure progress toward mathematical goal of the lesson by making EXPLICIT connections to student approaches and reasoning.

ALLOWS FOR STUDENTS TO:

- Have a purposeful exchange of ideas
- Clarify understanding
- Construct convincing arguments about how and why things work
- Develop a language for math
- Learn to see things from other perspectives

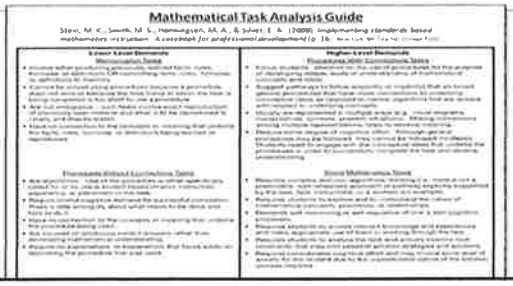
CHARACTERISTICS OF A DISCOURSE-RICH MATH CLASS

What are teachers doing?	*What are students doing?*
• Engaging students in purposeful sharing	• Presenting and explaining ideas
• Selecting and sequencing student approaches and solution strategies	• Listening carefully to and critiquing the reasoning of peers
• Facilitating discourse among students	• Seeking to understand the approaches used by peers
• Ensuring progress toward mathematical goals	• Identifying how different approaches to solving a tasks are similar or different

FOUR KEYS TO PRODUCTIVE DISCOURSE

- High level task
- Community
- Environment
- Questioning

SELECTING A HIGH LEVEL TASK



Mathematical Task Analysis Guide

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (2nd ed.). New York: Teachers College Press.

CREATE COMMUNITY USING ACCOUNTABLE TALK MOVES

- Talk moves are the tools for teachers to orchestrate meaningful mathematical discussion around the **MATHEMATI CAL GOAL OF THE LESSON**
- Ensuring Purposeful, Coherent Discussions
  - Revoicing: allows the teacher to check in with a student about whether what the student said was correctly heard and interpreted by the teacher or another student
    - Restating someone else's thinking, apply their own reasoning to someone else's
  - Challenging: Redirect the question back to the students.
  - Marking: drawing attention to important contributions by students.
  - Recapping: Pull all the ideas that have been shared together....can be done by teacher or student.
  - Wait time
  - These moves are essential for ensuring purposeful, coherent, and productive discussions—HOWEVER—THEY'RE NOT ENOUGH TO ADVANCE CONCEPTUAL UNDERESTANDING

THE ROLE OF COMMUNITY, REASONING AND KNOWLEDGE IN MATHEMATICAL DISCOURSE

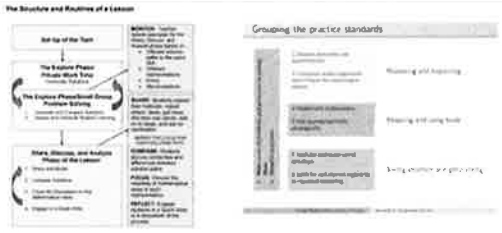
- Students must also be accountable to the community, rigor and knowledge during productive discussions.
- Community Goal: To listen attentively and be active participants in the discussion elaborating and clarifying ideas
- Knowledge Goal: To be specific and accurate always supporting claims with evidence
- Rigor Goal: to synthesize information from several sources and construct explanations and test understanding while challenging the quality of evidence and reasoning
- Look over the AT handout

Talk Move	Function	Example
	To Ensure Purposeful, Coherent, and Productive Group Discussion	
Marking	Direct attention to the value and importance of a student's contribution.	It is important to say describe to compare the size of the pieces and then to look at how many pieces of that size.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	You said 3, yes there are three columns and each column is 1/3 of the whole.
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?

	To Support Accountability to Community	
Keeping the Channels Open	Ensure that students can hear each other, and remind them that they must hear what others have said.	Say that again and louder. Can someone repeat what was just said?
Keeping Everyone Together	Ensure that everyone not only heard, but also understood, what a speaker said.	Can someone add on to what was said? Did everyone hear that?
Linking Contributions	Make explicit the relationship between a new contribution and what has gone before.	Does anyone have a similar idea? Do you agree or disagree with what was said? Your idea sounds similar to his idea.
Verifying and Clarifying	Revoice a student's contribution, thereby helping both speakers and listeners to engage more profitably in the conversation.	So are you saying ..? Can you say more? Who understood what was said?

	To Support Accountability to Knowledge	
Pressing for Accuracy	Hold students accountable for the accuracy, credibility, and clarity of their contributions.	Why does that happen? Someone give me the term for that.
Building on Prior Knowledge	Tie a current contribution back to knowledge accumulated by the class at a previous time.	What have we learned in the past that links with this?
	To Support Accountability to Rigorous Thinking	
Pressing for Reasoning	Elicit evidence to establish what contribution a student's utterance is intended to make within the group's larger enterprise.	Say why this works. What does this mean? Who can make a claim and then tell us what their claim means?
Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

DISCOURSE REVOLVES AROUND:



CREATING AN ENVIRONMENT CONDUCIVE FOR PRODUCTIVE DISCOURSE

- Explicitly teach/model expectations of the Structures and Routines of a Lesson and the math practices—USE ANCHOR CHARTS
- Model Accountable Talk expectations
- Establish routines for solving problems
- Establish a classroom environment supportive of risk taking
- Deep content knowledge
- Listening and patience
- Good questioning skills
- Use a wrong answer in pedagogically productive ways
- Keep the mathematical goal of the lesson in mind

TO ENSURE PURPOSEFUL DISCUSSIONS TEACHERS MUST....

- Anticipate
- Monitor
- Select
- Sequence
- Connect

The effectiveness of a lesson depends significantly on the care with which the lesson plan is prepared.

Brahler, D.J. (2000). Planning for instruction. In *Teaching secondary and middle school mathematics*, (pp. 111-134). Needham Heights, MA: Allyn & Bacon

SELECTING AND SEQUENCING SOLUTION PATHS IN ORDER TO CREATE PRODUCTIVE DISCUSSIONS

- Must select and sequence student solution paths appropriately to create a productive discussion to move students toward the mathematical goal of the lesson
- Tips for selecting and sequencing:
  - Show incomplete work to engage discussion and clarify what needs to happen next
  - Sequence them from basic (concrete) to complex (abstract)
  - Illustrate efficient and inefficient methods and discuss in what situations one is more preferable over the other
  - Share at least one complete response
  - Address misconceptions that can lead to understanding but avoid profound misunderstandings that can't advance the discussion
  - Show most frequently use method first to provide entry points for the majority
  - Show solutions with a range of representations
  - Order so that the solutions build on the previous solution
  - Keep the goals in mind so that students can leave with rich understanding
  - Have a reason for the sequencing you choose
  - Make sure you get to the generalization (if there is one)

ORCHESTRATING DISCUSSIONS WITH INTENTIONAL QUESTIONING

QUESTIONS...THE VESSEL TO DEVELOPING TRUE UNDERSTANDING

- What kinds of questions should I ask?
- Questions should assess and then advance student understanding of the mathematical goal of the lesson.
- Question Types:
  - Gathering information—recall
  - Probing thinking—explain, elaborate, clarify
  - Making the math visible—connections made
  - Encouraging reflection and justification—making an argument for the validity of their work

POSING PURPOSEFUL QUESTIONS

Teacher

- Advance students' understanding by asking questions that build on but don't take over or funnel student thinking
- Go beyond gathering info to probing thinking and requiring explanation and justification
- Be intentional in order to make the mathematics more visible and accessible in order for students to examine and discuss
- Allowing sufficient wait time
- Questions MUST be planned before instruction occurs

Students

- Expecting to be asked to clarify ideas or justify solutions
- Listening to others reasoning
- Questioning that reasoning and making connections between solutions

QUESTIONS THAT PROMOTE MATHEMATICAL THINKING AND DISCOURSE HELP STUDENTS...

- **work together to make sense of mathematics:**
  "What strategy did you use?"
- **rely more on themselves to determine whether something is mathematically correct:**
  "Is this a reasonable answer?"
- **learn to reason mathematically:**
  "How did you begin to think about this problem?"
- **learn to conjecture, invent, and solve problems:**
  "Do you see a pattern?"
- **learn to connect mathematics, ideas and applications:**
  "What is the relationship of this to that?"
- **with problem comprehension:**
  "Would you please explain that in your own words?"
- **focus on the mathematics from activities:**
  "What was one thing you learned (or two, or more)?"
- **persevere:**
  "Have you tried making a guess?"
- **evaluate their own processes, activities, and program:**

FOCUSING VS. FUNNELING

Funneling

- occurs when a teacher asks a series of questions to guide students through a procedure or to a desired result.
  - *Teacher engages in cognitive activity*
  - *Student merely answering questions – often without seeing connections*

Focusing

- requires the teacher to listen to student responses and guide them based on what students are thinking rather than how the teacher would solve the problem.
  - *Allows teacher to learn about student thinking*
  - *Requires students to articulate their thinking*

LEVELS OF CLASSROOM DISCOURSE

HUFFORD-ACKLES, FUSON AND SHERIN'S (2014)

- Hufford-Ackles, Fuson and Sherin's (2014) created a framework around which discourse can be evaluated and moved. The framework is describes growth on the following principles:
- 1. How the teacher supports student engagement
- 2. Who serves as the questioner and what kinds of questions are posed
- 3. Who provides what kinds of explanations
- 4. How mathematical representations are used
- 5. How much responsibility students share for the learning of their peers and themselves

EVALUATE YOUR CURRENT PRACTICE

- Think about the conversations that you're currently having in your classroom.
- Take a look at Hufford-Ackles, Fuson and Sherin's (2014) chart and analyze your current practice.
- What is your strongest area? What are needs improvement?
- So…what can we do to improve?

REFLECTION

- The best way to eat an elephant is one bite at a time!
- Summary:
  - Begin with a high level task and a mathematical goal
  - Create community with Accountable Talk moves
  - Create an environment conducive to talk
  - Ask purposeful, preplanned questions all geared toward moving students thinking toward that mathematical goal
- What is one step you can take tomorrow to improve the discourse in your classroom?
- Questions??

Earning Money Task

Flora, Frankie, Diana, and Derrick are waiters in a restaurant. The table below shows the amount of money each of them made in tips on Friday night. Each waiter earned 3 times more money on Friday than he or she did on Thursday.

Waiter	Thursday's Tips	Friday's Tips	Total Tips for the Week
Flora		$93	
Frankie		$54	
Diana		$42	
Derrick		$84	

1. How much money did each waiter make in tips on Thursday night?

2. Each person's total for the week was 3 times more than the amount each received on Friday. How much did each person earn for the week?

3. What is the same in Part 1 and Part 2? What is different?

Accountable Talk® Moves

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Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

Michaels, S., O'Connor, M.C., & Hall, M.W., with Resnick, L.B. (2010). Engagement with learning through talk. *Accountable Talk® sourcebook: For classroom conversation that works* (pp. 1-6). Pittsburgh, PA: University of Pittsburgh, Learning Research and Development Center.

Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

Reasoning and explaining

4. Model with mathematics

5. Use appropriate tools strategically

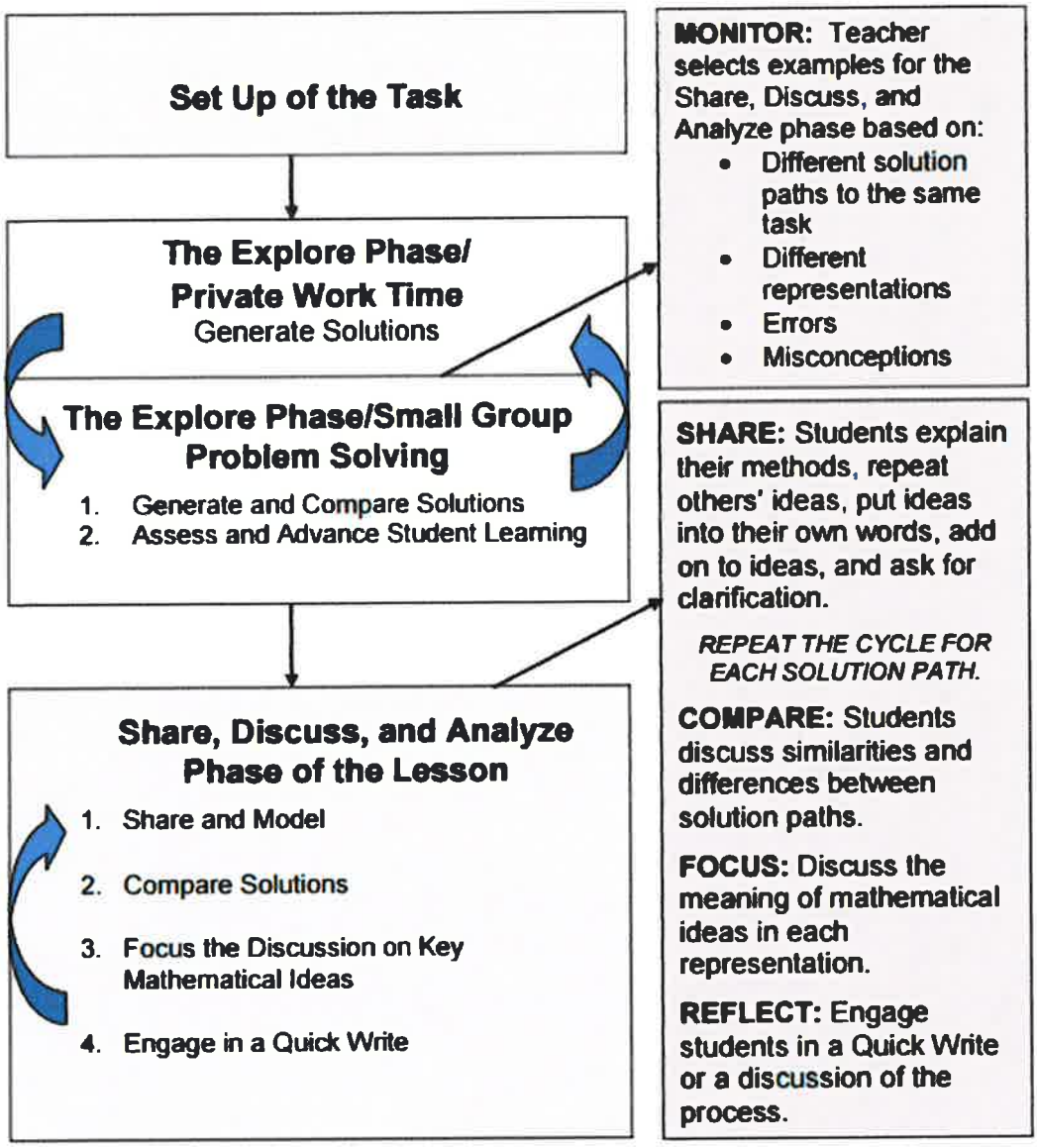
Modeling and using tools

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Seeing structure and generalizing

The Structure and Routines of a Lesson



	High attention to understanding	Attention to meaning of procedures with some transition to developing understanding	Attention to procedures and facts only
Facilitating Meaningful Discourse	Engaging students in purposeful sharing of ideas, reasoning, and approaches, using varied representations or from multiple texts/sources	Engaging students in purposeful sharing of ideas underlying procedure or skill using a teacher chosen representation of idea.	Engaging students in practice with skill and practice is more important than discussion.
	Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	Engaging in exchanges with one or two students with the goal of assessing what students do or do not understand.	Inviting no explanations
	Ensuring progress toward goal of the lesson by making explicit connections to student approaches and reasoning.	Ensuring progress toward goals that are procedural in nature by making connections with one or two student approaches and reasoning related to the procedure.	Demonstrating a method for solving a type of problem and ensuring progress toward the use of that procedure.
Posing Purposeful Questions	Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.	Advancing student understanding by asking questions that require students to explain the procedures used and the choices made	Advancing student use of prescribed procedures by asking questions about steps in procedures.
	Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.	Asking questions about the meaning of the procedures as they relate to the goal of the lesson.	Asking questions that focus solely on gathering information about the procedures students used to arrive at answers.
	Asking intentional questions so connections are more visible and accessible for student examination and discussion	Asking intentional questions that make the relationships between the procedures and meaning more visible.	Asking questions that are only procedure based or recall in nature.

Funneling	Focusing
T: What do you notice about the pattern of each person's tips? (waits briefly) How much are they multiplying by each time? S: You multiply by 3 to find each person's tips. T: Right because it says each waiter earned three times more on Friday than Thursday. How would you find out the total for the week? S: Multiply by 3 T: How is that the same as what you did to find Thursday's tips? S: You multiply by 3 on both T: Can you show me that in an equation? S: Writes __31___X3=93 and 93X3=__279__ T: So what was missing in the first equation? S: the factor in the first and the product in the second T: When the factor is missing what operation can you use? Can you show me how to find 31 using division?	T: What do you notice or wonder about the waiters' tips? S1: I know that to find the week I just multiply Thursday's total by 3 T: How did you know to do that? S1: In the problem it says that the total for the week is 3 times more than Thursday's total so you multiply by 3 for each. T: What else do you know? S2: I know that Thursday's is three times more than Wednesday's. T: So can't we just multiply by 3 again? S2: well yeah but we don't know what to multiply by? T: I'm not sure I understand. Can someone explain? S4: She means that we don't have Wednesday's total so we don't know what to multiply 3 by. T: Can you show me what you're saying in an equation? S4: wrote _____ X 3= 93. T: Ok so what do we know about multiplication? S3: We know that it is repeated addition of equal size groups so in this one we know there are 3 groups. S4: so if we make three groups, we just need to decide how many goes in each one. S5: We're dividing them up! T: Oh, can you show me what you mean?

Adapted from NCTM's *Principles to Action: Ensuring Mathematical Success for All*

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.