Overview

In this lesson, students use coin flip results to take a simultaneous random walk on a number line. Using their own motions as a probability simulation, students investigate, develop, and analyze a probability model for compound events.

Prerequisite Knowledge

Students should be familiar with finding probabilities of events like coin flips. Students should also have experience with plotting numerical data in plots on a number line or histogram.

Lesson Goals for Grade 7

Students will:

- use a simulation to generate frequencies for compound events
- draw conclusions about probability models by examining frequencies in data
- find probabilities of compound events by creating representations of sample spaces
- use a probability distribution to generate and answer questions about probability

Lesson Day 1

Overview

A student takes a random walk on a number line determined by flips of a fair coin: one step forward with a flip of a heads, and one step back with a flip of tails. After students propose questions about an individual's random walk, the whole class, joined by other students if possible, conducts a simultaneous random walk. Students arrive in various locations on the number line and then walk into columns to create a human bar graph that records their final location. Students then investigate the theoretical probabilities involved in the random walk.

Introduction (5-10 minutes):

We're going to go for a walk today. But it's going to be a funny kind of walk… we're going to let coin flips be your guide.

Select a volunteer walker and another student to flip a coin. With the walker standing in a suitable location, explain to the class that the student will take one step forward when the coin lands heads, and one step back when the coin lands tails. With each flip, have the volunteer take a step forward or back. Repeat 6-10 times to interest students in the randomness of the student's motion.

If you were feeling random one day, and were to take a walk with coin flips as your guide like our volunteer just did, what would you wonder about? What would you be curious about? Record your thoughts on the handout. (Distribute the "Flip Trip" handout.) Allow students time to generate questions; circulate to see what students are writing.

As you distribute coins, ask students to share their thoughts. Possible responses:
How often will I get heads or tails?
Where will I arrive after a certain number of flips?
How long will it take me to get to the other end of the room?
What are my chances of getting to the other end of the room?
It would take forever just to walk a couple of feet!

Acknowledge students' responses. *To investigate your questions, you are all going to make instructions for a "6-Flip Trip". Right now, flip your coin 6 times and record your results on the handout.* Allow students time.

If a flip of heads means step forward one unit on the number line, and tails means step back one unit, what locations on the number line will you visit? Where will you arrive on the number line after your six flips? You can record this on your handout. Check students' work.

Ready to go for a walk? You will use the flips you recorded as your instructions. Before we go, imagine what you might observe when we all do this at once. What are you curious about? Talk about that as we walk to our location.

When we're done with our experiment we will come back to our classroom and investigate some of the questions you have written on your handout.

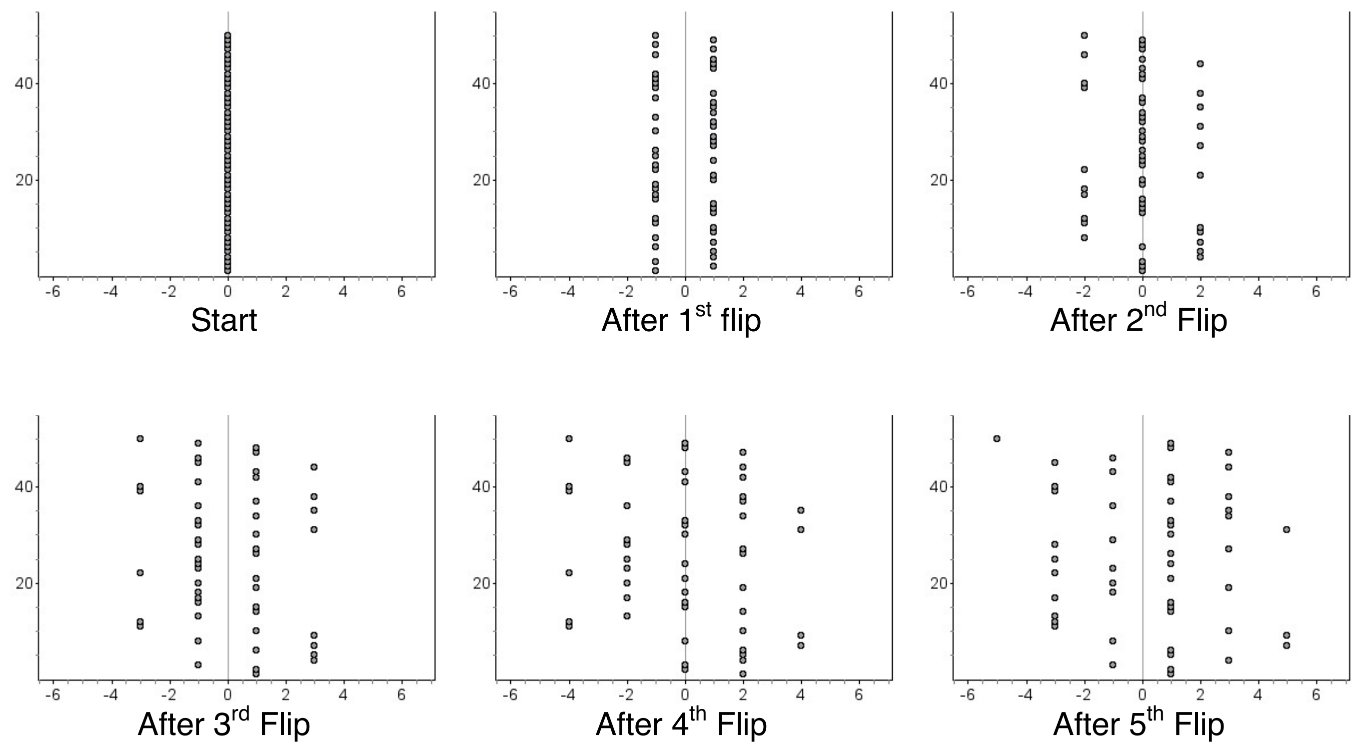
The Flip Trip (5-10 minutes):
Proceed to the location of your walk (hallway, gym, outdoors). Ask students to stand shoulder to shoulder along a line you have indicated, with all students facing in the same (positive) direction. You can have another class join your class to create a larger sample size for the experiment. (Provide the other class with slips of paper prerecorded with the results of 6 coin flips.)

On your signal, have each student make his or her first step. After students have stepped, walk along the starting line between the two rows now created to help students align themselves at the +1 or -1 positions.

Ask your class to look around to see if this is what they expected.

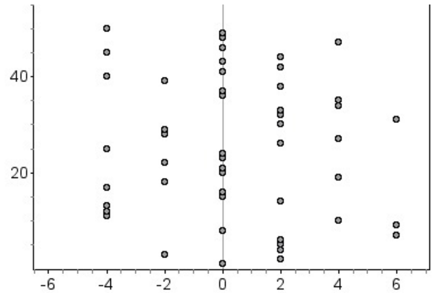
Before the second step, let students know they can check with their fellow classmates to align themselves after each step, keeping track of their location on the number line. (Tiles in a hallway, yard marks on football field, or cones in a gym provide suitable means of alignment.)

See the figure below for an illustration of possible locations of students after each step. Each dot represents a student. After 1 step students are standing on -1 or 1;
…after 2 steps, -2, 0, or 2;
…after 3 steps, -3, -1, 1, or 3;
…after 4 steps, -4, -2, 0, 2, or 4;
…after 5 steps, -5, -3, -1, 1, 3, or 5.

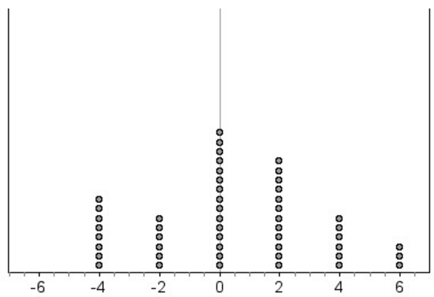


As time permits, ask selected students how they got to their location. Highlight how different students can arrive in the same location with a different sequence of coin flips.

When everyone is in their final position after the 6th flip (-6, -4, -2, 0, 2, 4, or 6), ask students to turn 90 degrees to form single file columns. Ask students to march forward, staying in the column on the number line in which they have arrived.



After 6th Flip



After the lead person in each column has reached a position you have designated for the horizontal base of the histogram, and the bars in the histogram formed, ask all students to turn around and examine the human bar graph they have created. *What do you notice? What do you wonder?* "Interview" a few students and ask them how they got to their final position. Highlight again that there was more than one way to get to a particular final position on the number line. Return to your classroom.

Wondering about the walk (5 minutes)

You've all participated in what is called a random walk.

At the beginning, you thought only about yourself; but now you have seen what happened when a large group of people did it, and the bar graph (histogram) that resulted. What were some of your noticings and wonderings about the human bar graph we created? Record them on your handout and then share them with a partner.

Circulate around the room to read what students have written and listen to their discussions. Students may be curious as to why so few students arrived on the far ends, or why the shape of the bar graph was symmetric (or not). Students may be curious why everyone didn't arrive back where they started, given that there is a 50-50 chance of moving forward or back. Share some of students' wonderings with the larger group.

All of you were behaving randomly, and independently of each other. Yet you have noticed some structure in the results of a large group trip. Now we're going to do some analysis. To study what happened and investigate the questions you have posed, we're going to look at what we will call a "4-Flip Trip" – a four-step random walk.

Analyzing the "Flip Trip" (15 minutes)

You saw the actual outcome of an experiment with 6 flips. To analyze it, I'd like you to *__predict__* *what the human bar graph would look like for a 4-Flip Trip. Be as specific as possible. It would be great if you were able to show* *__where__* *people could arrive after 4 flips, and what* *__fraction__* *or* *__percentage__* *of the people would arrive in each column of the bar graph (histogram.) We'll use your analysis to answer your questions from the start of class – and ask some new questions!*

Encourage students to form hypotheses about the human bar graph (histogram) for a 4-Flip Trip, and discuss their ideas about the probability of arriving at each of the 5 final locations. Students may focus first on explaining why a person could arrive at -4, -2, 0, 2 or 4 only. Allow students to create their own representations of the sample space to assess what prior knowledge of probability they believe is relevant to this situation.

Students may create a tree diagram to show the different possible outcomes of flipping a coin, or make an exhaustive list of possible outcomes (HHHH, HHHT, HHTH, etc.) You may suggest that students imagine that 100, 1000, or any large number of people are participating in a Flip Trip and that their goal is to determine how many people would arrive in each final position.

Students may believe that since there are 5 different columns, there is an equal probability of arriving in each, and that it does not matter "how you got there". Whether this occurs or not, you may suggest that students progressively analyze a 1-Flip Trip, a 2-Flip Trip, and a 3-Flip Trip. A full analysis of the sample space and associated probabilities will likely extend into Day 2. Allow students time to debate their ideas; extensions for students who move quickly through this phase of the lesson are included on page 8.

Closure for Day 1

People have found that even if behaviors – of molecules, microscopic organisms, animals, or even people driving cars or buying stocks – might be individually random, there is still a lot one can say about what might happen, as you have seen today! We'll continue to research your questions about random behavior in Day 2 of our Flip Trip investigation.

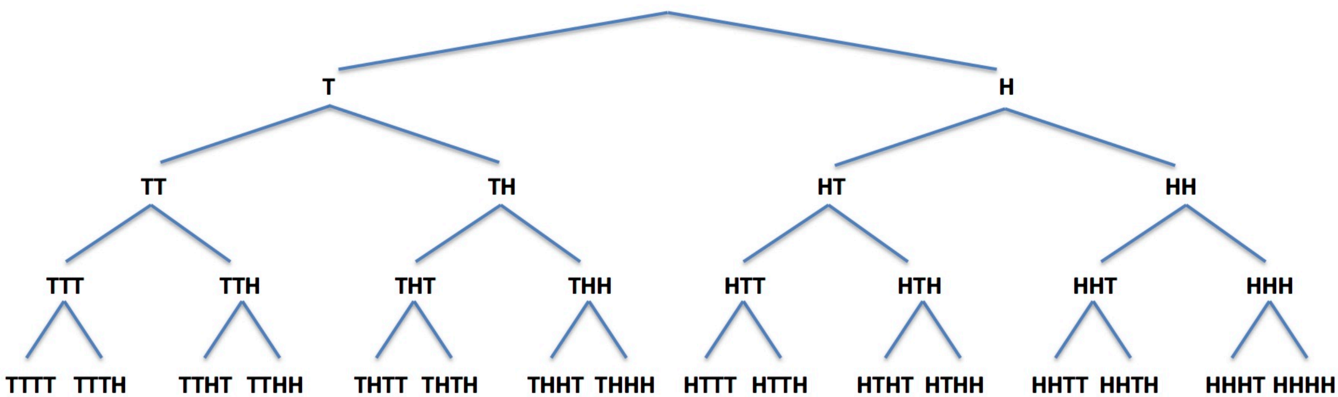
Ask students to record any conclusions they believe they have firmly established and collect their handouts to examine before Day 2.

Day 2

Studying the Probability Model (20 minutes)

Your assessment of students' progress towards describing the outcomes of the 4-Flip Trip on Day 1 will determine how to proceed on Day 2. On Day 2, examine different aspects of the probability model and work towards drawing conclusions about Flip Trips in general.

Students should discuss why each unique sequence of coin flips in the 4-Flip Trip has 1/16 probability of occurring. Students can imagine that 1000 people are about to take a Flip Trip. Half would flip heads, half tails; of those who flipped heads, half of those would flip heads again, and half tails. Reasoning in this way, students can establish that each different possible sequence of 2 flips has a ¼ probability. A tree diagram representation may be helpful in establishing the equal probabilities of the 16 different outcomes of the 4-Flip Trip.



To help students understand the probability model, you can have your entire class take a 2-Flip Trip and form a human bar graph. Students will see experimentally that approximately 50% of students will arrive at 0 after 2 flips, and 25% each at -2 and 2, respectively.

Ask students to list the 4 different outcomes of two flips and their associated locations on the number line to create the chart shown below.

		TH		
TT		HT		HH
-2	-1	0	1	2

This representation of the theoretical probability of arriving at each of the 3 final locations in the 2-Flip Trip is also a representation of what their human bar graph (histogram) may have looked like. Use the chart and the experiment with the 2-Flip Trip to discuss the notions of theoretical probability and experimental probability with the class.

Students may be ready to analyze the 4-Flip Trip without looking explicitly at the 2- or 3-Flip Trips. However, the charts may be useful as objects for discussion.

		TTH		HHT		
		THT		HTH		
TTT		HTT		THH		HHH
-3	-2	-1	0	1	2	3

				TTHH				
				HTTH				
		TTTH		HHTT		HHHT		
		TTHT		HTHT		HHTH		
		THTT		THTH		HTHH		
TTTT		HTTT		THHT		THHH		HHHH
-4	-3	-2	-1	0	1	2	3	4

The listing of different coin flip outcomes for the 4-Flip Trip, arranged in this format, may vividly remind students of the shape of the human bar graph (histogram) in the 6-Flip Trip from Day 1. Ask if students believe this is a coincidence or whether it might help explain what happened.

Assess students' understanding of the concepts under investigation through verbal questioning and opportunities to record conclusions in writing. Students should use their charts or tree diagrams to justify conclusions like, "There is a 6/16 probability that a person will arrive at 0 at the end of a 4-Flip Trip." Emphasize as needed that each of the unique sequences of flips has an equal probability of occurring.

Emphasize how probabilities of compound events can be analyzed by creating representations of sample spaces such as a tree diagram or charts of the possible outcomes.

Reflections and Extensions (20 Minutes)
Return to some of the students' original questions. There are a variety of ways to conclude the investigation at the end of Day 2, using the charts created, while leaving some questions open for future study later in the year or at later grade levels. There are also multiple avenues for exploration during Day 2 and/or at later dates.

- Questions such as "*How often will I get heads or tails?"* can be answered using the experimental data and the theoretical probability models. An individual may flip heads more often than tails, but a large sample will produce approximately equal percentages of heads and tails.

- Students may notice that the number of total possible outcomes when flipping a coin *N* times doubles as the number of flips grows by 1. Drawing a tree diagram may explain why this is the case.

- Ask students to respond to the prompt, *"Where will I arrive after a certain number of flips?"* in their own words. Examine students' different responses to the question as they record them and ask students to share their statements with others. The statement, "There is a 3/8 chance I will arrive back at zero after 4 flips," for example, may be surprising to some students.

- What is the average distance a person will be from zero after 4 flips? Introduce students to the idea of expected value.

- Students may be curious about the probability of everyone in the class flipping heads at the same time and moving in the same direction. What are the chances of that event?

- Return to students' experience with the 6-Flip Trip. *Do you think the chance that a person will return to zero is greater than or smaller than 3/8?* List all the possible ways to arrive at zero after 6 flips and determine the probability. Students may be interested in exploring how to find the total number of unique sequences of *M* heads within *N* coin flips.

- By listing the number of ways to arrive at each location in Flip Trips of different length, students may notice that the number of ways to arrive at a location in an *N*-Flip Trip is the sum of the ways of arriving at the two adjacent locations in an (*N*-1)-Flip Trip. Why is this so? (For example, there are 6 ways to arrive at 0 in a 4-Flip Trip, and 4 ways of arriving at 2. There will be 10 ways of arriving at 1 in a 5-Trip Flip. Why?) Students can begin to investigate some of the properties of Pascal's Triangle.

- What if we used dice and made a walk with a 2/3 probability of moving forward and 1/3 probability of moving back? How would the probabilities of arriving at the different final positions change?

Materials, Preparation, Modifications, Advice
This lesson is intended for Grade 7 but is suitable for use in Grades 6 or 8.

Teachers will need to create sets of coin flips and print them for students if additional classes join in for the group random walk. Supplies needed are coins or other random number generators, and a suitable space to conduct the Flip Trip. Having multiple classes participate adds to the spectacle of the probability simulation.

For very small classes or schools, or if other students are not available, a single class can repeat the experiment multiple times. Each time they form the human bar graph, they can mark off the top of each column and then stack themselves "on top of" the prior results after the next trial.

Preparation time for the lesson involves consideration of time and space constraints in order to determine and arrange for a site for the random walk. The Flip Trip has been conducted in school hallways, a parking lot, a gymnasium, and could also be held on a sports field. Markers to show students how far to step are useful. (Tiles on a floor can serve as a built-in number line.)

Preparing for the lesson primarily involves thinking about the possible mathematical questions students may propose based on the experiment. Teachers should also consider which student questions to focus on in order to reach the initial lesson goals.

Two 40-minute periods are recommended to conduct the Flip Trip and allow students to propose their own questions, debate the questions that arise, and draw their own conclusions. Closure on some of the important goals of the lesson may come on Day 1 if travel time to the walk site is short.

In conducting the lesson for the first time, anticipate logistical difficulties with students moving around in a hallway or open space. Inform other teachers who may be affected by the movement of the students to and from your chosen location, or with your use of a hallway as a teaching space.

Have fun! Middle school students enjoy the chance to move around, and are curious about probability. Listen to their ideas in order to help them make connections between their initial thoughts and their discoveries upon seeing the random walk conducted by multiple people at once. Assess your students' prior knowledge of probability to determine how quickly you might move towards extensions of the activity.

Let’s take a Flip Trip!

If you flip heads, you will step forward 1 unit on the number line.
If you flip tails, you will take a step back 1 unit.

In the table below, record your flips and the location on the number line you will arrive at with each flip.

	Flip 1	Flip 2	Flip 3	Flip 4	Flip 5	Flip 6
Location	Below, record your location on the number line after each flip.					
0						

Noticings and Wonderings

Analyzing our Trip