# Reasoning about Fractions: Using Number Lines to Understand Fraction Comparison

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## Session Overview

We will discuss:

- Relevant CCSS Standards and other recommendations
- Models, activities, and online resources to help students understand and reason about comparing fractions on the number line.



## Some of the CCSS "Big Ideas (Clusters) in Grades 3 – 5: Number and Operations—Fractions

1. Develop understanding of fractions as numbers (gr. 3)
2. Extend understanding of fraction equivalence and ordering (gr. 4)
3. Use equivalent fractions as a strategy to add and subtract fractions. (gr. 5)



## More about CCSS

- Greater emphasis on using the **number line model** to represent and act on fractions.



## Grade Three CCSS

- Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.A.2)



## Grade Three CCSS (cont.)

- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. (3.NF.A.2.A)

## Grade Three CCSS (cont.)

- Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (3.NF.A.2.B)



## Grade Three CCSS (cont.)

- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.A.3)



## Grade Three CCSS (cont.)

- Compare two fractions with the same numerator or the same denominator by reasoning about their size.
- Recognize that comparisons are valid only when the two fractions refer to the same whole.

(continued)



## Grade Three CCSS (cont.)

- Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (3.NF.A.3.D)



## Grade Four CCSS (cont.)

- Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½.

(continued)



## Grade Four CCSS (cont.)

- Recognize that comparisons are valid only when the two fractions refer to the same whole.
- Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (4.NF.A.2)

## Improving Fractions Instruction

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Developing Effective Fractions Instruction for Kindergarten through Eighth Grade: A Practice Guide (Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, Thompson, & Wray, 2010).



## Considerations

- Most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions
- Students who don't have mental images for fractions often resort to whole number strategies.

(Post et al., 1985; Cramer et al., 1997)

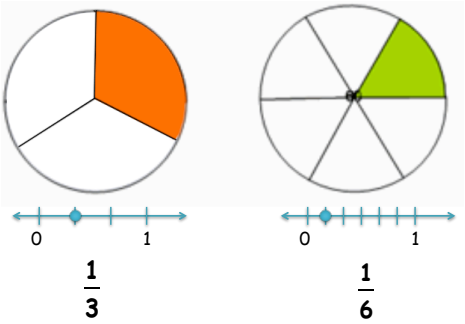


## Types of Models for Fractions

- Area/region
  - Fraction circles, pattern blocks, paper folding, geoboards, fraction bars, fraction strips/kits
- Set/discrete
  - Chips, counters, painted beans
- **Linear**
  - **Number lines, rulers**



## Comparing Fractions with a Model




$\frac{1}{3}$ $\frac{1}{6}$



## One Fifth-Grader's Understanding of Comparing Fractions

Circle the larger number or write "=" if they are equal in the pairs below:

1. $\frac{1}{6}$ $\frac{1}{3}$
2. $1$ $\frac{4}{3}$
3. $\frac{3}{6}$ $\frac{1}{2}$
4. $\frac{1}{7}$ $\frac{2}{7}$
5. $\frac{3}{10}$ $\frac{1}{2}$
6. $\frac{1}{2}$ $\frac{4}{6}$



## One Fifth-Grader's Understanding of Comparing Fractions

[video—no permission to share]

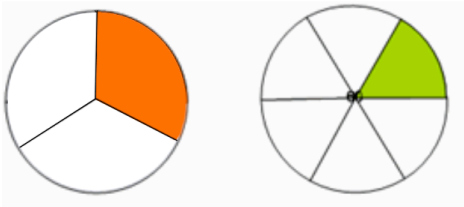
## Comparing 1/6 and 1/3:

- According to Ally, "1/3 is bigger, because if you change the digit down from 3, if it was 1/1 it would be equal to 1 and one's a whole number so it's bigger".
- What does she understand and what is she struggling to understand about comparing fractions?



## Think about the Language of Comparison

- Should we use "Bigger" or "Greater"? (or "Smaller" or "Less than"?)





## Ordering Fractions

$$\frac{5}{8}, \frac{3}{8}, \frac{6}{8}$$

Fractions with the same denominator have the same-sized pieces, so the numerators tell which fraction has more pieces (and is greater).



## Ordering Fractions

$$\frac{4}{8}, \frac{4}{5}, \frac{4}{6}$$

Fractions with the same numerator have the same number of pieces, and the denominators tell us which pieces are larger (and which fraction is greater).



## Ordering Fractions

$$\frac{3}{4}, \frac{2}{5}, \frac{1}{2}$$

Fractions close to a benchmark (such as ½ or 1) can be compared by finding their distance from the benchmark.



## Fractions Equivalent to One-half

$$\frac{2\frac{1}{2}}{5} = \frac{1}{2}$$

The denominator is twice the value of the numerator, so it's equal to 1/2

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## Ordering Fractions

$$\frac{7}{8}, \frac{3}{4}, \frac{2}{3}$$

Fractions close to one can be compared by finding their distance from one, for example, by focusing on the amount that's missing from the whole.



## Ordering Fractions

$$\frac{99}{100}, \frac{6}{7}, \frac{15}{16}$$

Fractions close to one can be compared by finding their distance from one, for example, by focusing on the amount that's missing from the whole.



## Ordering Fractions on a Number Line: The "Clothesline" Activity

- Task:
  - Order fraction tents using a clothesline to represent a number line and
  - mathematically justify the reasons for your ordering.
- Materials: fraction tents and clothesline (string, yarn, etc.)



## In A Third Grade Classroom



## "Clothesline" Fractions Activity

$$\frac{1}{2}, \frac{3}{4}, 1$$



## "Clothesline" Fractions Activity

$$1\frac{2}{3}, \frac{7}{4}$$

## "Clothesline" Fractions Activity

$$\frac{1}{3}, \frac{3}{4}, \frac{5}{8}$$



## "Clothesline" Fractions Activity

$$\frac{3}{5}, \frac{4}{9}, \frac{3}{4}$$



## "Clothesline" Fractions Activity

$$\frac{1}{8}, \frac{7}{8}, \frac{11}{12}$$



## "Clothesline" Fractions Activity

$$\frac{1}{4}, \frac{3}{13}, \frac{6}{27}$$



## In A Third Grade Classroom



## Free Online Fraction Resources

ConceptuaMath
www.conceptuamath.com

Resources→ Tool Library →
"Try the Tools"

## Strengthen Students' Fraction Reasoning by Helping Them:

- Develop understanding of fractions as numbers.
- Understand fraction concepts, order, and equivalence,
- Use number lines as a central representational tool (but not as the first model students use for fractions) in teaching fraction concepts from the early grades onward.
- Make "Why?","How do you know?","Can you explain?" classroom mantras.



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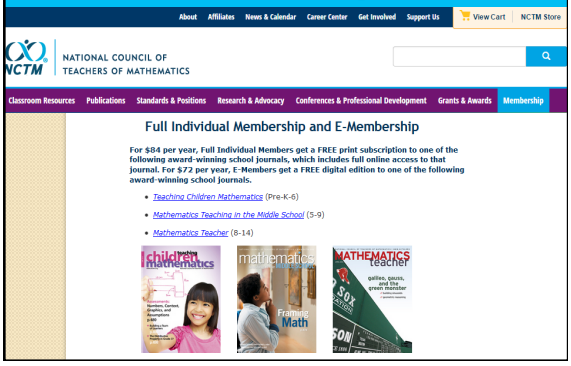


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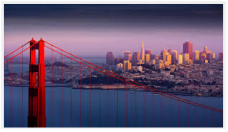

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