Preparing Math Teachers for the Common Core:

Are they (we) ready for this?

Steve Williams
Husband to April and Father to Kaiden (9) and Bowen (3)
Runner
Professor of Mathematics
Coordinator of Secondary Mathematics Education
Secondary Math Teacher at Heart
Lock Haven University of PA
swillia6@lhup.edu

The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

- G. H. Hardy, 1940, *A Mathematician's Apology*

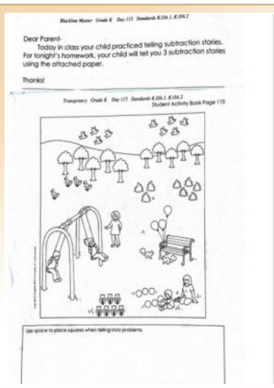
Standards or Curriculum?

The Social Media Problems!

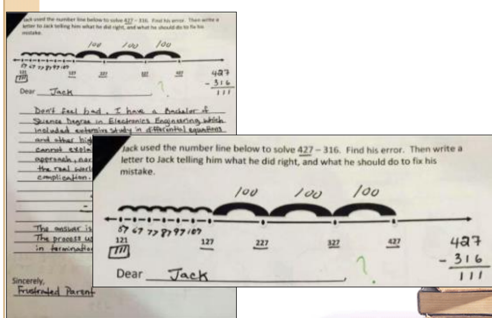
8 toys are in the chest and 6 toys are on the shelf. Which can be used to find how many toys in all?

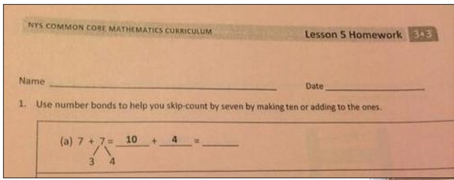
A. 8 - 6 = 2
B. 6 + 2 = 8
C. 8 + 4 = 12
D. 10 + 4 = 14

1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).


Dear Parent-
Today in class your child practiced telling subtraction stories. For tonight's homework, your child will tell you 3 subtraction stories using the attached paper.
Thanks!

Understand addition, and understand subtraction.
CCSS.MATH.CONTENT.K.OA.A.1
Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.



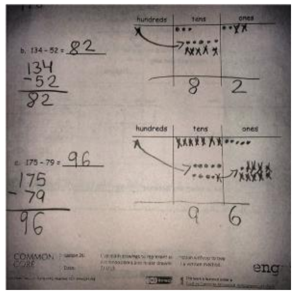

NYS COMMON CORE MATHEMATICS CURRICULUM

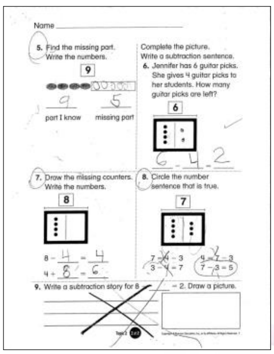
Lesson 5 Homework 3•3

Name ______________________ Date __________

1. Use number bonds to help you skip-count by seven by making ten or adding to the ones.

(a) 7 + 7 = 10 + 4 = ____





2.NBT.7

(**BEFORE** standardized testing begins)

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

But don't look now…

4.NBT.4

Fluently add and subtract multi-digit whole numbers using the STANDARD ALGORITHM!!!

The Check!

How carefully are we reading the standards?

6.RP.2

Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

$\frac{3}{4}$ isn't a unit rate...or is it?!?!

Don't abuse fractions! They can be looked at as two integers that each mean something or as one number itself.

7.G.4

Know the formulas for the area and circumference of a circle and use them to solve problems; **give an informal derivation of the relationship between circumference and area of a circle.**

How would you find the area of a circle if you were given the circumference?

Have you ever seen a formula that gives the area of a circle in terms of its circumference?

Three axioms for teaching

1. Know the content being presented.
2. Know more than the content being presented.
3. Teach from the overflow of knowledge.

4. Know the standards inside and out!

Goals for Presentation

- To examine the Common Core State Standards (both the content standards and the standards for mathematical practice) and use them to motivate participants to consider more conceptual, structured, and rigorous pedagogical strategies to teach secondary mathematics (it isn't always our students' fault for not learning).
- To motivate participants to consider other mathematical concepts where we have "short-changed" our students.
- To provide participants with some alternate ways of viewing certain math topics that are more conceptual than traditional ways of viewing them.
- To help participants begin to be able to see mathematical concepts and structures despite being blinded by the mindless procedures.

Ultimate Goal

To find ways to help our future teachers become competent teachers of mathematics in the Common Core Era.

(Consider this: We are about to ask our students to become teachers within a system in which they have not participated.)

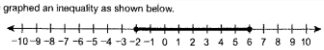
Where did this stuff come from?

- Seven years of teaching all levels of secondary mathematics
- Seventeen additional years of working with preservice teachers and talking to them about potential conceptual deficiencies or misunderstandings
- Twenty-four years of trying to closely examine as many mathematical concepts in as much detail as possible
- A sincere interest to have my students (and myself) develop that Profound Understanding of Fundamental Mathematics by looking at concepts in different or non-traditional ways

Three of my favorite problems to get us thinking outside of just procedures

My First "Favorite" Problem

David correctly graphed an inequality as shown below.



The inequality David graphed was written in the form $7 \le$ __?__ ≤ 9.

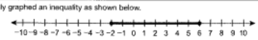
B. What is an expression that could be put in place of the question mark so that the inequality would have the same solution set as shown in the graph?

$7 \le$ ________ ≤ 9

Commentary

It's not about teaching your students to solve this type of problem. It's about teaching them to think about structure and use known tools to solve it.

David correctly graphed an inequality as shown below.



The inequality David graphed was written in the form $7 \le$ __?__ ≤ 9.

B. What is an expression that could be put in place of the question mark so that the inequality would have the same solution set as shown in the graph?

$7 \le$ ________ ≤ 9

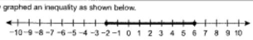
An Algebraic Perspective

$$7 \le ax + b \le 9$$

$$\frac{7-b}{a} = -2 \quad and \quad \frac{9-b}{a} = 6$$

$$\begin{cases} 2a - b = -7 \\ 6a + b = 9 \end{cases} \Rightarrow a = \tfrac{1}{4};\ b = \tfrac{15}{2} \Rightarrow 7 \le \boxed{\tfrac{1}{4}x + \tfrac{15}{2}} \le 9$$

David correctly graphed an inequality as shown below.



The inequality David graphed was written in the form $7 \le$ __?__ ≤ 9.

B. What is an expression that could be put in place of the question mark so that the inequality would have the same solution set as shown in the graph?

$7 \le$ ________ ≤ 9

A Transformation Perspective, 1

The inequality interval has been shrunk and moved.

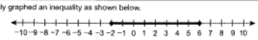
It has been shrunk by $\frac{1}{4}$ (from 8 to 2).

So: $\tfrac{1}{4}(-2) = -\frac{1}{2}$ *and* $\tfrac{1}{4}(6) = \frac{3}{2}$

Now move: $-\tfrac{1}{2} + m = 7$ and $\tfrac{3}{2} + m = 9 \Rightarrow m = 7.5$

Therefore, $7 \le \tfrac{1}{4}x + \tfrac{15}{2} \le 9$ will simplify to $-2 \le x \le 6$.

David correctly graphed an inequality as shown below.



The inequality David graphed was written in the form $7 \le$ __?__ ≤ 9.

B. What is an expression that could be put in place of the question mark so that the inequality would have the same solution set as shown in the graph?

$7 \le$ ________ ≤ 9

A Transformation Perspective, 2

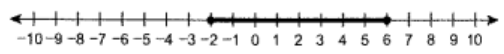
The inequality interval has been shrunk and moved.

It has been shrunk by $\frac{1}{4}$ (from 8 to 2).

move first: $\tfrac{1}{4}(-2+m) = 7$ and $\tfrac{1}{4}(6+m) = 9 \Rightarrow m = 30$

Thus, $7 \le \frac{1}{4}(x+30) \le 9$ will simplify to $-2 \le x \le 6$.

David correctly graphed an inequality as shown below.



The inequality David graphed was written in the form 7 ≤ __?__ ≤ 9.

B. What is an expression that could be put in place of the question mark so that the inequality would have the same solution set as shown in the graph?

7 ≤ ________ ≤ 9

A Combination Perspective

The inequality interval has been shrunk and moved.

It has been shrunk by a and moved by b.

move first: $a(-2+b)=7$ and $a(6+b)=9 \Rightarrow \begin{cases} -2a+ab=7 \\ 6a+ab=9 \end{cases}$

Thus, $b=\frac{1}{4}$ and $b=30$, so $7 \le \frac{1}{4}(x+30) \le 9$ will simplify to $-2 \le x \le 6$.

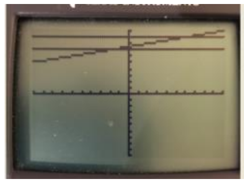
The pedagogical implications here are huge and should fundamentally change the way that most teachers teach the concept of linear functions.

What one student saw this semester:

-2 needed to get mapped to 7 and 6 needed to get mapped to 9.

So he found the equation of the line between these two points:

$$y=\frac{1}{4}x+\frac{15}{2}$$



Thinking like this allowed us to write any degree function that would fit between the horizontal lines $y=7$ and $y=9$, transforming the form

$$y=a(x+p)^n+q$$

The point here is to give deep thought to the underlying concepts rather than only considering procedures to common problem types.

CCSS Content

Creating Equations* A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

CCSS Mathematical Practices

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

CCSS Mathematical Practices

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might

My Second "Favorite" Problem

Circle the fractions from the following:

$$3,\ -2,\ \tfrac{2}{3},\ \sqrt{3},\ -4.6,\ 5.\overline{3},\ -\tfrac{5}{2},\ \pi,\ -\tfrac{2}{3},\ \tfrac{25}{5},\ 0$$

Concentrate on these!

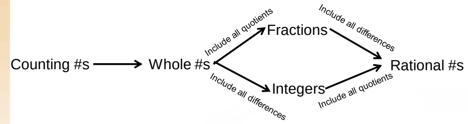
A "first" definition of a fraction: A part of a whole

The CCSS definition of a fraction:

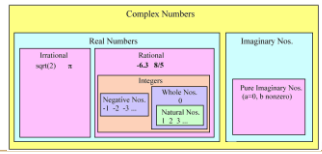
Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

The Developmental Progression of Fractions in the Curriculum



To a high school teacher or a college professor, the diagram above makes no sense.



There is no set called "fractions"

Why is this important?

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

My Third "Favorite" Problem

Write an equation that has the following solutions:

$$x = 8 \ \text{or} \ x = -2$$

$$x - 8 = 0 \ \text{or} \ x + 2 = 0 \quad \text{(what are the factors?)}$$

$$(x-8)(x+2) = 0$$

$$x^2 - 8x + 2x - 16 = 0$$

$$\boxed{x^2 - 6x - 16 = 0}$$

But what about $|x-3| = 5$?

$$x - 3 = 5 \ \text{or} \ x - 3 = -5$$

$$x = 8 \ \text{or} \ x = -2$$

How (and why) do we think about this problem?

CCSS 7.NS.1b/c

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

$3+5=8$ means that 8 is located $|5|=5$ units from 3 in the right direction.

$3+(-5)=-2$ means that -2 is located $|-5|=5$ units from 3 in the left direction.

$-3+(-5)=-8$ means that -8 is 5 units from -3 in the left direction.

$-3+5=2$ means that 2 is 5 units from -3 in the right direction.

What does this have to do with…

Write an equation that has the following solutions:

$$x = 8 \ \text{or} \ x = -2$$

If we talked like this, maybe we could take $x = 8$ and $x = -2$ and create an absolute value equation instead of only a quadratic one.

Questions to ask: What number is the same distance away from 8 and -2; and what is this distance?

Answers: Half way between, or (8 + (-2))/2 = 3 and the distance is (8 – (-2))/2 = 5.

Therefore, $|x-3| = 5$

What numbers are 5 units away from 3?

Problem

Consider this: $2\sqrt{51x}=10\sqrt{51}$. What is x?

Most teachers would immediately go to a procedure.
But why not look at structure first?

Since $10=2\bullet 5$, what number would go under the radical sign?

25

$\boxed{x=25}$

Problem

Consider this: $\sqrt{x}=14i\sqrt{3}$. What is x?

Now you have another way of looking at this that might lead to a procedure.

$$14i\sqrt{3}=\sqrt{(14)^2(i)^2(3)}=\sqrt{-588}$$

$\boxed{x=-588}$

In the Common Core Era, three things have impressed me the most

(which I really like and I think should transform how we train teachers.).

1. The Mathematical Practices, especially "looking for and making use of structure" (this is a lost art) and "looking for and expressing regularity in repeated reasoning."
2. The "story" of our subject—rather than a hodgepodge of disconnected "things to be able to do"—could return.
3. The mathematical rigor that should make its way back into our classrooms.

What is my point???

1. Our teachers are not ready for this.
2. We (people who train teachers) are not ready for this.
3. There are many concepts that preservice (and many inservice) math teachers do not know well enough to teach beyond a surface or procedural level. (I have a lot of these on file!)

Conclusions

- Secondary mathematics majors (and many inservice teachers) usually have not been provided the opportunities to closely examine many of the basic concepts that they take for granted and will one day have to teach.
- The traditional curriculum to prepare students to teach secondary mathematics feeds into this "gap" in teachers' knowledge. There is usually no course that students take in which to discuss these potential deficiencies.
- We, as their teachers, should admit some of the responsibility in helping to cultivate these deficiencies.
- The Common Core State Standards will require a deeper understanding of mathematics than most teachers currently possess.

Contact Information

Steve Williams
Professor of Mathematics/
Coordinator of Secondary
Mathematics Education
Lock Haven University of PA
swillia6@lhup.edu