First grade students' uses of tables as they explore functional relations

Abstract

We present first graders' uses of function tables as they explored relationships between quantities, drawing from a design study investigating functional thinking. Findings suggest children used tables as organizational tools and they looked not just *at* but also *through* tables at functional relationships. We take this as evidence that children were using tables as tools to think about functions.

Theoretical Framework & Prior Studies

Our theoretical framework on representations as tools is grounded in a sociocultural perspective on the mediating role of tools on one's thinking (Vygotsky, 1978). In the field of mathematics education, Kaput (1991) has described how representations help us reorganize our ways of thinking. Kaput, Blanton, and Moreno (2008) have also described how when working with symbols, one can either look *at* them "as opaque objects in their own right" (p. 26) or *through* them "to what they might 'mean'" (p. 26). In our study, we will explore how children look *at* and *through* tables when exploring functions.

Some prior studies have explored children's use and understandings of tables among first to fifth graders (e.g., Brizuela & Alvarado, 2010; Brizuela & Lara-Roth, 2002; Martí, 2009; Schliemann, Carraher, & Brizuela, 2007). These studies indicate that children are able to successfully solve more complex (transformation-transformation-transformation) additive problems when using tables, but not when solving more simple (state-transformation-state) problems. They also indicate that specific ways of structuring tables (e.g., introducing labels *a*

priori or not) may encourage children to reason in different ways about similar problems. Students in Marti's (2009) work tended to create lists instead of leveraging the coordinating information of tables. He concluded that primary school children need to be introduced to tasks in which knowledge is expressed in different forms (such as tables) and that require different representations.

Methods

Considering tables as tools for thinking about functional relationships and the ways in which children looked *at* and *through* them, we considered the following questions:

RQ1: Looking *at* tables: *How did children organize the quantities in the problem as they constructed their tables?*

RQ2: Looking *through* tables: *In what ways did tables become tools for thinking about functional relationships?*

We report on individual interviews with four first grade students who participated in an 8-week classroom teaching experiment [CTE] that was part of a larger design-based research study (Cobb, et al., 2003) in six classrooms—two at each of grades K–2—from two demographically diverse elementary schools in the Northeast United States. The CTE consisted of two 40-minute lessons per week (16 lessons total). The first 4-week cycle focused on tasks involving functions of the form $y = mx$; the second 4-week cycle focused on tasks involving functions of the form $y = x + b$. This paper focuses on individual, semi-clinical interviews that we carried out at pre-,

mid-, and post-points during the CTE, interviewing the same subset of four students. Each interview lasted about 30 minutes and consisted of the student solving a task similar to those used in the CTE and describing his or her thinking in response to a common interview protocol generated by the research team.

Data Sources and Analysis

Our data were drawn from video recordings and written work produced in three interviews carried out with four students. These students were selected for interviews from one classroom of 18 first grade students through teacher nomination. Data analysis began with a line-by-line review of each of the interview transcripts, tagging moments when children had to make decisions regarding the quantities in the problem and how to represent them in the table (RQ1) and moments when tables were used as they were thinking about functional relationships (RQ2). Tagged episodes were then analyzed with regards both research questions. Once these descriptions were developed within individual interviews, we looked across interviews and identified common descriptions among the four students. From this analysis we identified three themes, one in relation to RQ1 (how children organized the quantities in the problem) and two in relation to RQ2 (whether or not the children were prompted to produce a table; and how children used the tables as they were thinking about the functional relationships they were exploring).

Results

Figure 1 provides an overview across the pre-, mid-, and post-interviews for each of the four students in the study reported here regarding the three themes identified above.

Looking *at* tables

How did children organize quantities?

Children were able to decide which were the two quantities involved in the problem and were able to label them: In only four of the 12 interviews did children need help identifying which were the quantities involved in the problem; in all other cases they did so independently: number of dogs and number of noses (pre-interview); height without the hat and height with the hat (mid-interview); and number of stops made and total number of train cars collected (post-interview).

Places dependent variable in the input column: There was only one instance in which one of the children (Eleanor in the pre-interview) placed the dependent variable (the number of noses) in the input column of the table.

Places independent variable in the input column: In the other 11 interviews, in two cases children needed help deciding which quantity to place in the input column of the table. In the other nine cases, they placed the independent variable in the left column of the table.

Collectively, these results indicate that as children were looking *at* tables, they were making important decisions regarding the quantities involved in the problem and how to use the table to organize these quantities to reflect the relationships between them.

Figure 1. Students' use of tables during the three interviews in the CTE. Y=presence; N= absence; WH=with help; E=Eleanor; N=Nolan; Ro=Robert; Re=Rebecca.

	PRE-INTERVIEW $y = x$ How many noses do any number of dogs have?				MID-INTERVIEW $y = x + 1$ What is the height of any person who wears a 1-foot hat?				POST-INTERVIEW $y = 2x$ and $y = 2x+1$ A train car picks up 2 cars at every station (with and without counting the train engine).			
	E	N	Ro	Re	E	N	Ro	Re	E	N	Ro	Re
QUANTITIES IN THE PROBLEM AND HOW TO REPRESENT THEM IN THE TABLE												
decides which are the 2 quantities and labels them	Y	Y	Y	Y	Y	WH	Y	WH	Y	WH	WH	Y
places dependent variable in the input column	Y	N	N	N	N	N	N	N	N	N	N	N
places independent variable in the input column	N	WH	Y	Y	Y	Y	Y	Y	Y	Y	WH	Y
TABLES AS TOOLS FOR THINKING ABOUT FUNCTIONAL RELATIONSHIPS												
WERE CHILDREN PROMPTED TO PRODUCE A TABLE?												
prompted for table	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N
asked to organize information and sets up table with no further prompting	N	N	N	N	N	N	N	N	Y	Y	Y	Y
HOW WERE TABLES USED?												
constructs a generalization	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y
"matches" with other pairs of values	Y	Y	Y	N	N	N	N	N	N	N	N	Y

Looking *through* tables

Were children prompted to produce a table?

Our data suggest that in the pre and mid-interviews children did not produce a data table until prompted by the interviewer. For example, in Eleanor's pre-interview, the interviewer said to her, "So, we're gonna [sic] keep track of the information that we're talking about." Eleanor constructed a list, and the interviewer prompted her by suggesting a table and saying, "And, an even better way to keep track, and I don't know if you've seen this before…". However, by the post-interview children produced tables as soon as they were asked to "organize the

information," indicating that they had been appropriated as a representational (and organizational) tool.

How did children use tables?

Use the table to construct a generalization: In the 12 interviews analyzed, there was only a single case (Eleanor in the mid-interview) when a child did not refer to the information collected in the table to construct a generalization. For example, in the mid-interview, the interviewer asked Robert how he would use the table to help a friend figure out what a person's height is without the hat if they measure seven feet with the hat, and Robert stated, "Well…the number that is with the hat (pointing at the right hand column in the table) is one more than … without the hat (pointing at the left hand column in the table)."

Use the table to match the relationship with other pairs of values: The four students referred to "matching" pairs of corresponding values. In the pre-interview, Eleanor, Nolan, and Robert talked about "matching" the number of dogs and the number of noses, by which they meant that they always needed to make sure that the numbers in the left hand column matched the numbers in the right hand column. In the post-interview, Rebecca also talked about "matching." When asked how she knew to represent the relationship between the number of stops (represented by Rebecca as R) and the number of train cars (represented by Rebecca as V) as $R + R = V$, she said, "Because the rest [of the table] it says one plus one and you get the one from the other side, and I want it to match [indicating that she wanted the form of the generalized equation to match the equations relating specific pairs of co-varying values]."

Children's use of tables to construct a generalization and to match the relationship with other pairs of values indicates that they were looking *through* the tables at the functional relationship represented to make inferences and draw conclusions.

Overall, our findings suggest that children in our study were able to both look *at* and *through* tables. Importantly for functional thinking, when looking *at* tables children in our study mostly focused on how to identify and organize the quantities; when looking *through* tables the structure of tables helped to highlight for children the underlying relationships (by "matching") and scaffolded their generalizations. Our findings also suggest that a short 8-week CTE can support children's appropriation of tables as a mathematical tool and their use of tables as tools to organize information.

Significance of the Research

Given the central focus of algebra throughout the mathematics curriculum (e.g., Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000, 2006; National Mathematics Advisory Panel Report, 2008) as well as the current focus on mathematical practices that include modeling, looking for and making use of structure, and looking for and expressing regularity in repeated reasoning (see CCSSI, 2010), it is important to understand the ways in which children use tables, an ubiquitous algebraic representational tool. Understanding their ways of using tables can help us leverage their abilities to their fullest.

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