Patterns of Variation and Productive Discourse: An International Comparison

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Abstract

This cross-cultural, comparison study was designed to explore how strategically adapting and implementing tasks can create different learning opportunities from a theory of variation perspective. By comparing reform-oriented, exemplary lessons developed through lesson study in the U.S. and China, this study aimed to explore how lessons can maximize student-learning opportunities by appropriately introducing patterns of variation and productive discourse. Using the theory of variation along with funneling and focusing patterns of questioning, we compared the two lessons with respect to these five Mathematics Teaching Practices: establishing mathematical goals to focus learning; facilitating meaningful mathematical discourse; posing purposeful questions; supporting productive struggle in learning mathematics; and eliciting and using evidence of student thinking. Although the two exemplary lessons were initially based on the same lesson plan and shared somewhat similar learning goals, the lessons created substantially different learning opportunities with respect to the patterns of variation co-constructed through purposeful discourse. Theoretically, this study demonstrates how the theory of variation can be used to analyze mathematics lessons in such a detailed and fine-grained way in order to gain a deep understanding of mathematics teaching. Practically, the vivid description of how each lesson enacted objects of learning through constructing dimensions of variation systematically provides catalysts for readers to reflect on the ways of improving their teaching within their culture. Comparing and contrasting the similarities and differences in these two lessons enhances reflection upon maximizing learning opportunities in mathematics classes.

Introduction

Classroom instruction is one of the key factors contributing to student achievement in mathematics. The success of Chinese students on the Programme for International Students Assessments (Organization for Economic Co-operation and Development, 2009, 2012) and the stark differences in culture and the beliefs about the teaching and learning of mathematics between China and the U.S. (Cai & Wang, 2009; Li, 2011) make for interesting comparisons between them. This study was designed to explore how strategically adapting and implementing tasks can create different learning opportunities from a theory of variation perspective (Marton & Pang, 2006; Marton & Tsui, 2004). By comparing reform-oriented strategies employed in lessons from the U.S. and China, this study aimed to explore the following question: how can lessons maximize student-learning opportunities by appropriately introducing patterns of variation and carefully guiding discourse?

Theoretical Framework

In this study, we adopted a specific framework known as the theory of variation to analyze each learning situation. According to Marton and Tsui (2004), learning is a process in which learners develop a certain capability or way of seeing and experiencing. In order to see something in a certain way, the learner must discern certain features of the object. More specifically, it is important for the learner to be able discern the patterns of what varies and what is invariant in a learning situation so that they can experience critical features of the object of learning. In this study, we focused on the patterns of variation co-constructed through the implementation of varying tasks. In regards to this process, Lo and Marton (2011) stated:

Contrast helps the learner to discern a particular phenomenon, concept, or aspect and

> differentiate it from other phenomena, concepts, or aspects. By paying attention to what remains unchanged in the background of varying appearances, the learner can generalize to an invariant principle or aspect. (p.11)

As patterns emerge from these varying tasks, lessons should engage students in meaningful discourse that focuses student learning towards the established mathematical goals (National Council of Teachers of Mathematics [NCTM], 2014). Discourse should provide students with opportunities to share ideas and clarify understandings, construct viable arguments regarding why and how things work, and learn to see things from other perspectives (NCTM, 2000). To enact such productive discourse, posing purposeful questions that elicit student thinking is crucial (NCTM, 2014). To identify productive discourses, patterns of funneling and focusing (Truxaw & Defrance, 2008; Wood, 1998) were explored.

Methodology

To explore ways of maximizing learning opportunities in mathematics classes, two similar lessons focusing on patterns within a monthly calendar developed in the U.S. and China were examined (see Huang, Prince, and Schmidt (2014) for a detailed description of the U.S. lesson). Both lessons were created through lesson study groups in which the teachers participated in iterative cycles of planning a lesson, observing teaching of the lesson, and debriefing using the data collected during the lesson. Both lesson study groups provided data including three lessons plans, videotaped lessons, post-lesson reflection and comments, and the demonstrating teacher's post-lesson reflection. Using the theory of variation and patterns of funneling and focusing, we compared the two lessons with respect to these five Mathematics Teaching Practices (NCTM, 2014): establishing mathematical goals to focus learning; facilitating meaningful mathematical discourse; posing purposeful questions; supporting productive struggle in learning mathematics;

and eliciting and using evidence of student thinking.

Results

Although the lessons were initially based on the same lesson plan and shared somewhat similar learning goals, the two exemplary lessons created substantially different learning opportunities with respect to the Mathematics Teaching Practices (NCTM, 2014). The following sections will describe these differences.

Establishing Mathematics Goals to Focus Learning

The U.S. lesson plan mainly focused on students understanding the *average identity* pattern in detail. The *average identity* pattern states that the sum of consecutive numbers in any linear grid on the calendar equals the middle number times the total number of the consecutive days when the total number of days is odd. For example, the sum of days 8, 9, 10, 11, 12 is 10x5 = 50. The focus of the lesson was on understanding when this pattern held and how to justify algebraically that it worked for various grid shapes and sizes. Alternatively, the Chinese lesson plan went further by also focusing on alternate conjectures for when the number of days is even (i.e., *exchange identity*). The *exchange identity* states that the sum of an even number of consecutive days equals half the number of days times the sum of the two middle numbers. For example, the sum of 2, 3, 4, 5 is 2(3+4) = 14. These differences in established goals led to drastically different learning opportunities for students.

Facilitate Meaningful Mathematical Discourse

The U.S. students were more engaged in the process of constructing dimensions of variation, namely necessary conditions of learning. The U.S. students had more opportunities to express their thoughts, clarify their ideas, justify their observations, and learn from peers. In contrast, in the Chinese lesson, students were led as quickly as possible to get the answers that

the teacher expected. There was little opportunity for students to express their ideas, justify their observations, and critique each other. Although the mathematical goals in the Chinese lesson raised the task's level of cognitive demand, the implementation of the tasks provided students with very different levels of productive discourse.

Elicit and Use Evidence of Student Thinking

When exploring the *average identity* pattern, students need to experience two fundamental dimensions of variation with respect to the grid: shape and size. The U.S. lesson focused on exploring the *average identity* through creating appropriate dimensions of variation in sequence. However, the Chinese lesson included exploration of two patterns (i.e., *average* and *exchange* identities) simultaneously, which can make characteristics of the patterns difficult to discern according to the theory of variation. In contrast, the U.S. lesson provided students with a clear opportunity to discern the features of the average identity separately rather than simultaneously. Moreover, the U.S. lesson provided opportunities for groups to present their proofs to the class. The Chinese lesson did not allow much time for students to share their ideas.

Supporting Productive Struggle in Learning Mathematics

The U.S. lesson contained open-ended questions that encouraged students to make as many of their own observations as possible, which laid the foundation for further exploration. In addition, students in the U.S. lesson were given time to grapple with the mathematics, justify their conjectures, and critique the reasoning of others. In contrast, the Chinese lesson set higher expectations by providing more and higher-level tasks than the U.S. lesson did (e.g., 11 high level tasks in the Chinese class compared to 3 high level tasks in the U.S. class). In particular, the Chinese class contained various grids created by students that were more diverse and challenging. Even though students were provided some time to explore the patterns on their own,

a majority of the discussions were teacher-led.

Pose Purposeful Questions

When examining the discourses surrounding the creation of dimensions of variation, we found that students in the Chinese lesson were led to the intended goals as quickly as possible by adopting a funneling pattern of questioning. In contrast, the U.S. lesson tended to facilitate students' discussion and justification of their conjectures as much as possible by using a focusing pattern of questioning. Although both focusing and funneling patterns of discourse (Wood, 1998) can be appropriate, a focusing pattern allows student thinking to lead the discussion. In fact, NCTM (2014) recommended that asking questions can help teachers establish what students know, make connections among mathematical ideas, and reveal student reasoning.

Conclusions and Implications

In general, it is important to design and carry out a sequence of carefully selected tasks that allow students to experience critical features, one at a time. The Chinese lesson contained more of a funneling pattern of questioning, narrowing the space for students to express their thinking and justify their solutions, resulting in lowering of cognitive level of the implemented tasks. This raises an issue of how to implement complex tasks focusing on broader learning goals at a higher level in a large size class such as those found in China (45 students, 45 minute duration). This finding illuminates some of the cultural differences between the two countries.

The U.S. lesson demonstrated many elements of effective mathematics teaching practices (NCTM, 2014). The teacher achieved clear learning goals through deliberately sequencing and implementing high cognitive tasks, and carefully orchestrating students' work, within a focusing interaction environment. This finding implies that that reform-oriented teaching can be implemented in classrooms if teachers carefully plan and deliberately implement a lesson with

the support from peers and more knowledgeable experts.

From a theory of variation perspective, this study shows that the two exemplary lessons developed based on the same lesson plan provided students with substantially different learning opportunities. The findings offer both theoretical and practical implications. Theoretically, this study demonstrates how the theory of variation could be applied to analyze mathematics lessons in such a detailed and fine-grained way in order to gain a deep understanding of mathematics teaching. Practically, the vivid description of how the teachers enacted objects of learning through constructing dimensions of variation systematically provides catalysts for readers to reflect on the ways of improving their teaching within their culture. Comparing and contrasting the similarities and differences in these two lessons enhances reflection upon maximizing learning opportunities in mathematics classes. Although this exploratory study provides some insight into improving mathematics teaching, the nature of this study does not allow the findings to be generalized to the features of mathematics teaching in each of the relevant counties.

References

Cai, J., & Wang, T. (2009). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education, 13*, 265-287.

Huang, R., Prince, K., & Schmidt, T. (2014). Exploration of patterns in a calendar. *Mathematics Teacher*, *108*, 337-342.

Marton, F., & Pang, M. F. (2006). On some necessary conditions of learning. *The Journal of the Learning Science*, *15*, 193–220.

Marton, F., & Tsui, A. B. M. (2004). *Classroom discourse and the space of learning.* Mahwah, NJ: Erlbaum.

National Council of Teacher of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (2014). *Principles to action: Ensuring mathematical success for all.* Reston, VA: Author.

Organization for Economic Co-operation and Development. (2009). *PISA 2009 results: What students know and can do: student performance in reading, mathematics and science*. Paris: Author.

Organization for Economic Co-operation and Development. (2012). *PISA 2012 results in focus: What 15-year-olds know and what they can do with what they know*. Paris: Author.

Truxaw, M. P., & Defrance, T. (2008). Mapping mathematics classroom discourse and its implication for models of teaching. *Journal for Research in Mathematics Education, 39*, 489-525.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. B. Bussi, & A. Sierpinska (Eds.), *Language and communication in mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.