COLLABORATIVE MODELING IN ELEMENTARY MATHEMATICS

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Abstract: This paper examined elementary students engaged in a modeling activity, and identified three shifts in thinking that helped students develop a viable model. We argue that encouraging students' collaboration, a teacher can successfully orchestrate a modeling activity that emphasizes the tension between the experienced world and the underlying mathematical idea.

Keywords:

Theoretical Perspectives

Modeling is an important mathematical practice, and educators have called for the integration of mathematical modeling into K–12 curricula (e.g., CCSSI, 2010). Although there is agreement that integrating modeling into the mathematics classroom could be productive, teaching mathematical modeling involves negotiating a complex set of priorities, processing mathematical content knowledge and its application to other fields, and attending to student needs, all of which can vary depending on the modeling problem, mathematical objectives, and pedagogical context (Blum & Borromeo-Ferri, 2009; Doerr, 2007). In this paper, we analyze a group of students collaboratively engaged in a modeling activity. We argue that by structuring a task to encourage collaboration among students a teacher can successfully orchestrate a modeling activity that emphasizes the tension between the experienced world and the underlying mathematics.

We borrow Doerr & Tripp's (1999) viewpoint of *a mathematically significant model*: it must "focus on the underlying structural characteristics of the experienced system that is being described or explained" (p. 232). To examine modeling in a classroom setting, we highlight learners' interaction with the experienced world and with mathematical representations. Moreover, because we will focus on students in a group, we emphasize students' sharing and refining ideas, which Doerr and Tripp argued are "essential characteristics of modeling activity" (p. 232).

Doerr and Tripp identified shifts in students' thinking that supported the development of mathematical models—the mathematical model in question was improved and students made a connection between the phenomena and the mathematical representation—in a college pre-calculus class. The shifts occurred at certain instances of *conjecturing* (when students make initial and/or follow-up interpretations to the situation), *questioning* (when students request information, challenge an interpretation), and *impasses to progress* (when the group could not reach an agreement with an idea), and *use of technology-based representations*. In this paper we analyze a case in an elementary mathematics classroom, to see whether similar shifts emerge and, if any, how those shifts help students develop a viable mathematical model.

Methods

Our research questions are:

1. What are the shifts in elementary students' thinking when they are engaged in a modeling activity?

2. How do these shifts occur during students' collaborative work and in what ways do these shifts support the development of a mathematical model?

The data is drawn from a professional development program for grades 5-9 mathematics teachers. Through the program, which included teachers engaging in three semester-long graduate courses, we have gathered an extensive collection of videos that document lessons taught by the teachers. These lessons exemplify classroom activities that draw from standard mathematical curriculum rather than radically reformed modeling activities.

When we selected the case, we first looked for elementary mathematics lessons that involved modeling activities. From the candidate lessons we picked one case that we thought best illustrates productive shifts while students were engaged in collaborative work.

We then carried out a qualitative analysis of the classroom videos and transcripts. Within the lesson we identified episodes during which productive shifts occurred. We identified shifts when the mathematical model in question was improved and students made a connection between the phenomena and the mathematical representation. Lastly, we examined the transcripts to identify at what instances shifts happened, and classified the instance following Doerr and Tripp's terms of *conjecturing*, *questioning*, and *impasses to progress*. We also paid attention to other instances that emerged in our work but were not in Doerr and Tripp's.

Here, we present an episode that illustrates shifts, with triggers and effects, to address our research questions.

Data Sources

The case we present involves a group of 5th grade students taught by Ms. J., one of our program teachers. Ms. J. used this lesson for her final course project. The students' task focused on the growth of two hypothetical children. One child, Tara, was 80 cm tall when she was 2 years old. She then grew at a steady rate from age 2 to 10. Students were asked to fill out data tables with Tara's heights from year to year and draw graphs. Ms. J. adopted this activity from Pearson's *Investigations* (2008).

Six students took part in the activity, and four of them were involved in the discussion pertinent to the episode we analyzed: Laura, Charlotte, Sharon, Hannah. They sat around a table, discussed and filled out their worksheets in loose pairs (each one partnered with the child next to her): Laura with Charlotte, and Sharon with Hannah. Ms. J. walked around and listened to their discussion, occasionally joining in and asking questions. When Ms. J. posed a question to the entire group, everyone responded to her.

Results

We identified three major shifts where students moved back and forth and built connections from mathematics to experienced world. The first occurred when they transitioned from reading the task to carrying out their first mathematical description of the problem. Within Doerr and Tripp's framework, we view this as an instance of *conjecturing*. The students constructed their first, tentative model—a linear function increasing by the same amount every year between when Tara was born to when she was 10 years old. This led them to a first answer for Tara's height—400 cm at 10 years old. During this phase Ms. J. held back, allowing the students to arrive at this tentative model, which she knew would lead them to an impasse.

Laura	Tara was 80 cm when she was 2 years old.
Charlotte	So that means she was about 40 cm when she was 1 year old.
Laura	So do you want to add... 40?
Charlotte	So we should add 40 every time it grows a year.
Laura	Why 40?
Charlotte	Because that's 80 split in half.

Sharon	80 cm when she was 2 years old so I divided that by 2, which gave me 40 and then I was thinking maybe she grew like 40 cm each year then that would be like 400 cm, but I think that's way off.
Hannah	Well 400 cm....

The second shift occurred when they transitioned from focusing on the mathematics to focusing on the real world situation, right after students constructed the first model and got the answer of 400 cm for Tara. This shift involved the teacher, Ms. J., and was prompted by her *questioning*: challenging students' answer. In Doerr and Tripp's study (1999), the students posed questions to their peers, whereas in our study the teacher asked questions to students. Looking at and thinking about real life examples, students realize that a 10-year-old who is 400 cm tall is not reasonable.

Ms. J.	Tara grew at a steady rate until she was 10 years old, right? So about how tall would you expect a 10-year-old to be?
Charlotte	(Stands up) This tall.
Ms. J.	About how many cm do you think that is?
Charlotte	600 cm maybe
Ms. J.	You think 600 cm?

Ms. J.	I'm not sure what you have for final rates for Tara, but it looks like everyone has about 400 cm, right?
Whole group	Yeah.
Ms. J.	Ok, so I want to tell you that this (meter stick) is about 100 cm right here.

Laura	Ok, that's too much.
Ms. J.	And this room is about 2 1/2 of these (meter sticks). So this room is about 250 cm.
Laura	Ok, then that's not going to work.
Ms. J.	So a 10-year-old is probably not going to be 400cm. Why did you decide to add what you added each time?

The third shift occurred when the students re-focused on the underlying mathematical idea. Students found the problem in their model: they had applied the linear growth to the wrong domain. We classified this as an *impasse*: students arrived at an un-resolvable contradiction between the assumptions of their mathematical model and the real-world situation. What we call *impasses* here means an un-resolvable contradiction within a particular model, not students' disagreement with an interpretation, as is in Doerr and Tripp's study (1999).

Hannah	That's what Sharon and I were going to do, and Sharon was like, 'Oh, you can do 80 divided by 2'. But then it doesn't say she grew at a steady rate before she was 2. It doesn't say she grew at a steady rate. So she might have been really slow or really fast. And she was growing before, so I don't know if we can do that.

Students, from this point on, focused on the correct domain and on a steady rate of growth starting only when Tara is 2 years old and developed a sound model for Tara's growth

Conclusions

In this case, we identified three shifts in thinking when elementary students are engaged in a modeling activity. Working collaboratively, students shift their attention back and forth between to real-world situations and to the underlying mathematical ideas. The shifts are cultivated by students' *conjecturing*, by teacher's *questioning,* and by students' pondering upon an apparent *impasse*. The teacher, holding back or jumping in depending on students' discussion, orchestrated the modeling process successfully to help students develop a viable mathematical model for the situation in task.

References

Blum, W., & Borromeo-Ferri, R. (2009). Mathematical Modelling: Can It Be Taught And Learnt? *Journal of Mathematical Modeling and Application*, *1*(1), 45–58.

Council, N. R. (2012). *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas.* Washington, DC: The National Academies Press.

Doerr, H. M. (2007). WHAT KNOWLEDGE DO TEACHERS NEED FOR TEACHING MATHEMATICS THROUGH APPLICATIONS AND MODELLING? In *Modeling and Applications in Mathematics Education* (pp. 69–78).

Doerr, H. M., & Tripp, J. S. (1999). Understanding How Students Develop Mathematical Models. *Mathematical Thinking and Learning*, *1*(3), 231–254.

National Governors Association Center for Best Practices Council of Chief State School Officers. (2010). *Common Core State Standards*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Pearson/Scott Foresman, TERC (Firm), & Pearson Education, Inc. (2008). Investigations in number, data, and space. Pearson Scott Foresman.