Pedagogies for Enacting Secondary Instructional and Mathematical Practice

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Abstract
Policymakers and politicians are raising questions about the quality of teacher education program outcomes and how to prove or improve these outcomes. For all students to be college and career ready teacher candidates (TCs) need instructional skills upon entering the profession that enable them to enact ambitious teaching in pursuit of ambitious mathematics with students. Our research and development design aims for teachers to gain improved skill with *high leverage* instructional practice through a process of investigation and supported enactment or what has been termed in the literature, pedagogies of practice. This paper examines TCs' instructional tool use as they participate in pedagogies of practice to support secondary mathematics students. In addition, we examine the mathematical resources secondary mathematics students employ to solve a contextualized rate problem. Using a lens of mediated action, our coding of TCs' written analyses, video of their enactments, and tools provided within pedagogies of practice we were able to coordinate TCs' development of skilled use of instructional tools and their communication about these instructional tools. TCs initial engagement in pedagogies of practice afforded the use of a limited number of instructional tools – eliciting students' contributions and discursive practices – to varying degrees these tools were used consistently and repeatedly across TCs' enactments. Our findings offer insights on how pedagogies of practice support candidates' development of instructional skill while offering secondary students' opportunities to engage in authentic disciplinary practices.

Pedagogies for Enacting Secondary Instructional and Mathematical Practice

Policymakers and politicians are raising questions about the quality of teacher education program outcomes and how to prove or improve these outcomes (Grossman, 2010). Studies of teacher preparation highlight the lack of consensus in preparation components and the weak content preparation of teachers (Tatto, Schwille, Senk, et al., 2012). Other researchers have documented the meager impact on novice teachers' preparation to meet the needs of classrooms (Gainsburg, 2012; Valencia, Martin, Place, Grossman, 2009). One imperative to stem the rising tide of public debate is the need for teacher education to develop evidenced based designs that build novices' capacity with skillful instruction.

Simultaneous to the debate on teacher preparation programs' worth, U.S. schools are experiencing high levels of teacher attrition. Statistics on teacher retention suggest U.S. classroom are often populated by a stream of novices (first to fifth year teachers) who are in the process of building skill and at the same time have the responsibility of educating a majority of U.S. students (Darling-Hammond, Wei, & Johnson, 2009; Ingersoll, 2001; Kumashiro, 2010). Because of this, a second imperative for teacher education is to prepare novices for the first few years of teaching so that they have deployable instructional skills (NCATE, 2010) that support each learner to understand mathematics and to use mathematics in authentic tasks to solve problems (National Governors Association Center for Best Practices and Councils of Chief State School Officers, 2010).

For all students to be college and career ready teacher candidates (TCs) need instructional skills upon entering the profession that enable them to enact ambitious teaching in pursuit of ambitious mathematics with students (Lampert, et al., 2013). This means that TCs need to leverage instructional resources, such as mathematical problems, lesson designs, and pedagogical strategies or practices similar to those called for by the recent NCTM publication *Principles to Action* (Leinwand, Brahier, Huinker, et al., 2014). These instructional skills afford students opportunities to pose mathematical questions and build arguments, examine the structure of processes to leverage procedures, and bring to bear multiple representations to gain traction on problems. In other words, TCs' capacities with these high leverage instructional practices that advance productive trajectories of student participation have the potential to engage students in key practices of the discipline.

For TCs to gain skill with the ambitious teaching envisioned by NCTM and others they need more than the opportunities to investigate teaching during teacher preparation, they need to enact the complexity of teaching in supported and strategic ways (Grossman, Compton, Igra, Ronfeldt, Shahan, Williamson, 2009). Our research and development design aims for teachers to gain improved skill with *high leverage* instructional practice through a process of investigation and supported enactment (Ball, Sleep, Boerst, & Bass, 2009; Kazemi, Lampert, & Franke, 2009). Drawing on recommendations from McDonald and colleagues, we have designed *secondary mathematics pedagogies of practice* that support TCs developing skill with instructional practice (McDonald, Kazemi, & Kavanagh, 2013). Our research investigates how TCs used tools and resources to participate in the work of teaching with secondary mathematics students. We define tools as the instructional activity and instructional routines practices and strategies TCs had available. Our attention to resources included students' interactions and the mathematical strategies and practices employed. The questions examined in this paper are:

1) What tools and resources do teacher candidates use as they engage in secondary mathematics pedagogies of practice?

2) What tools and resources do secondary students use as they engage with teacher candidates in mathematics classroom activities?

Relevant Literature and Conceptual Framing

Evidence based teacher education designs need to build TCs' capacity with skillful instruction. Recent teacher education research has shown that in order to develop instructional skill TCs need supported opportunities to engage in the work of teaching using instructional tools integral to ambitious goals for student learning. Pedagogies that support TCs to investigate, enact, and reflect on teaching are currently being employed in teacher education. However, the tools for developing TC's knowledge and skills to enact ambitious instruction and the outcomes from employing these tools are still in need of investigation (Lampert, et al., 2013).

TCs' engagement in supported, authentic teaching experiences and analysis of that teaching are central features of *pedagogies of investigation and enactment* (McDonald, Kazemi, & Kavanagh, 2013). We situate our work within a growing movement to integrate pedagogies of investigation and enactment into teacher education (Grossman & MacDonald, 2008; Grossman Compton, Igra, Ronfeldt, Shahan & Williamson, 2009; Lampert, 2010). We frame this work via two constructs; *pedagogies of investigation and enactment* that inform methods for supporting teachers in developing instructional skill (McDonald, Kazemi, & Kavanagh, 2013), and *mediated action* from sociocultural theory that informs how teachers and students learn to use (or appropriate) tools (instructional and mathematical) in teaching interactions (Rowe & Bachman, 2012; Wertsch, 1991, 1994). The coordination of these frameworks allows us to consider the ways that TCs' skill with instructional tools (such as, routines, practices and strategies) develops within interactions with students in classroom settings.

Mathematics and science teacher education researchers are advancing models of teacher education within which TCs are supported to develop skilled instruction through *investigating* the work of teaching and learning and *enacting* the work of teaching with peers and students (Kazemi, Lampert, & Franke, 2009; Windschitl, Thompson, Braaten & Stroupe, 2012). To engage in investigations and enactments of teaching aimed at improvement of instruction requires supported learning opportunities to intervene directly on TCs' interactions with students (Grossman et al., 2009). Further, TCs need opportunities to decompose lessons to expose their disciplinary and pedagogical structures in ways that allow them to analyze lesson enactments in order to unpack the highly complex and relational learning opportunities made available in classroom interactions. We posit that teacher education can no longer be satisfied with just developing knowledge of teaching, additionally it must support TCs development of skilled practice and in particular the use of high leverage instructional practices so that TCs enter classrooms equipped with skills to support all learners.

Pedagogies of practice require the identification of high leverage teaching practices that are at the core of these pedagogies (Ball & Forzani, 2011). Recent research in mathematics and science education has attended to this by developing a set of instructional practices that are high leverage for supporting ambitious learning goals for all students (Forzani, 2014; Grossman, Hammerness, & McDonald, 2009; Kloser, 2014 see https://cset.stanford.edu/research/core-practices); Lampert, 2010; McDonald, Kazemi, & Kavanagh, 2013). High leverage instructional practices are routines and strategies that are accessible to novice and more experienced teachers occurring with high frequency in teaching and positively impacting student learning of key content while providing equitable access to all students (Ball, Sleep, Boerst, & Bass, 2009). Our attention to high leverage practices are situated within routines and activity structures (i.e., IAs) that place at their center equitable access, such as launching tasks, building mathematical explanations, and examining errors (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Leinhardt & Steele, 2005). Both routines and practices are enacted within particular contexts using a variety of questioning and student participation strategies.

One routine central in this study is building explanations using student contributions. Leinhardt and Steele's (2005) analysis of teaching suggest that this instructional routine is central to supporting students' mathematical learning. Similarly, Stein, Engle, Hughes and Smith (2008) offer that orchestrating discussion can be accomplished through a series of practices – anticipating, monitoring, selecting, sequencing and connecting students' contributions. These authors' research provides insights on an essential routine for ambitious mathematics instruction. We have termed this routine "building explanations." Using pedagogies of practice we have constructed learning opportunities to support TCs' developing skill and knowledge to orchestrate building explanations via eliciting students' contributions and questioning that leverages key connecting across contributions. Researchers who have examined teachers' instructional skill for building explanations suggest advancing a mathematical point is a complex endeavor (Sleep, 2012) and the highly relational work that calls teachers to elicit contributions, interpret contributions, and construct a response that moves a discussion toward a mathematical aim remains of the most essential, yet allusive routines for TCs (Ghousseini, 2009). We hypothesize that in order for TCs to develop skill with instructional routines, such as building explanations, they need repeated opportunities to investigate the routine, engage in the routine, and analyze instruction. It is through supported participation in the work of teaching, pedagogies of practice, that TCs may develop ambitious instructional skill.

In order to understand how these pedagogies of practice impact TCs' developing instruction we call on *mediated action*. We posit that through pedagogies of investigation and supported enactment high leverage practices are instructional tools (material, linguistic, and conceptual) that mediate the interactive work of teaching (Wertsch, 1991, 1994). From the perspective of mediated action, within enactments TCs make use of tools inside contexts toward goals; and within investigations TCs further specify and appropriate instructional tools. Though engagement in both investigations and enactments TCs transform these tools of communication and interaction into individual tools for thinking and doing.

Next we elaborate the classroom-embedded, teacher education design which is the setting for the study reported here. After detailing the project, we elaborate on the IA participants enacted with secondary mathematics students and analyzed.

Math Practice Cycle

The Math Practice Cycle (MPC) is a classroom-embedded, teacher education model supporting the development of teachers' skilled instructional practice, via investigation and enactment, while leveraging ambitious learning goals for K-12 students. The data for this study are drawn from a multi-year research project aimed at investigating teachers' development using pedagogies of practice and the participation of K-12 students. MPC research examines teachers developing skill through cycles of investigation, rehearsals, and analyses using artifacts and video records of instruction.

The MPC places at its core a set of high leverage practices (see Table 1) adapted from the Learning In, From and For Teaching Practice (LTP) project (Lampert, et al., 2013). This set of high leverage practices serves as a set of instructional tools that TCs gain skill with prior to certification. By using an adaptation of a similar set of high leverage practices we have opportunities to examine TCs' appropriation of tools (routines, practices, and strategies) and investigate design considerations supporting TCs' enactments.

Table 1: Set of High Leverage Practices Adapted from LTP (Lampert et al, 2013)

1. Teaching toward a clear learning goal
2. Representing student reasoning verbally and visually
3. Constructing and organizing public records
4. Eliciting and responding to student contributions
5. Orienting students to one another and to the discipline
6. Making sense of students' participation to inform instruction
7. Positioning students as competent
8. Developing and maintaining a productive learning environment

The MPC model consists of four components that are repeated across an academic year (see Figure 1). To begin the cycle, teacher educators collaborate with one or two host secondary mathematics teachers from a collaborating school. In concert with the host teacher(s), teacher educators select a day for the classroom rehearsal, identify a learning goal that will be appropriate for the host teacher's(s') students, and construct initial designs of an IA. The mathematical learning goal is the anchor for designing an IA around which pedagogical leaning goals for teachers are identified. The IA contains a math problem(s) and specifies the segments of the lesson, including a lesson trajectory, specific prompts, and supports for teacher and student interaction. The aim is for TCs to successfully support students' learning central mathematical ideas and provide opportunities to be ambitiously responsive to students' contributions.

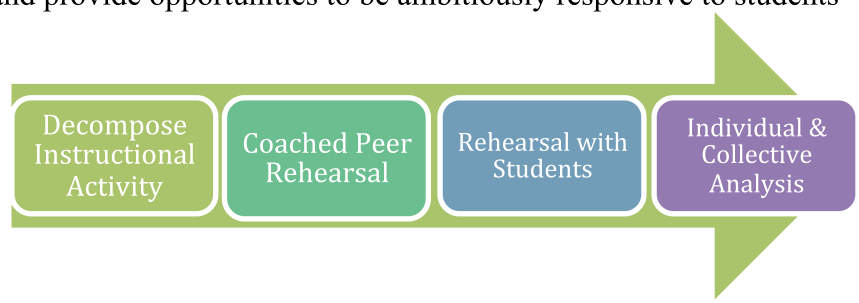

Figure 1. MPC Cycles of investigation and enactment.

During the investigation phase of the cycle TCs engage in the mathematics and pedagogy contained in the IA. The teacher educators engage TCs in tasks to reveal the mathematical knowledge teachers need to inform instruction (Ball, Thames, Phelps, 2009) and how the structure of the activity builds toward a mathematical learning goal. This phase takes many forms, including working on mathematics tasks that highlight key ideas, such as underlying meanings of mathematical concepts and procedures while exploring multiple representations of ideas, namely to build *specialized content knowledge* for teaching (Ball, Thames & Phelps, 2008; Elliott, Kazemi, Lesseig, et al., 2009; Susuka, et al., 2009). Further, TCs may experience the IA as a "student," with the IA being taught by a teacher educator. Through discussion, TCs collectively review the segments in the IA, unpack their corresponding pedagogical goals, and anticipate student responses to key activities.

Activities investigating the mathematics and the lesson structure are followed by rehearsals to build skill with segments of the IA (Lampert, et al., 2013). The collective group of TCs both rehearse the IA and act as participants within the instructional sequence. The goals of this work are to consider in real-time the IA's pedagogical and mathematical aims, explore the relational, interactive aspects of instruction – content, students, teacher, context, and develop TCs' confidence with new instructional tools, such as discourse strategies, eliciting students' contributions, or using protocols for student sharing (Chappin, O'Connor, Canavan-Anderson, 2003; Lampert et al., 2013; Schleppegrell, 2007). During the third phase TCs rehearse the IA with a group of students in a secondary mathematics class. Rehearsals with students are video-recorded and teacher educators circulate among TCs' rehearsals to confer with TCs as requested.

The final phase of the cycle is comprised of individual and collective analyses. TCs are provided structured analysis questions to examine video records of the student rehearsal. This is followed by an end of cycle video-club discussion using clips aligned to the student learning goals and the pedagogical learning goals embedded in the IA (Sherin & van Es, 2009). The current study reports on an investigation of eight TCs' individual written analysis of their video from the first cycle of four cycles in the 2013-2014 academic year. TCs worked with sixth-grade students on a proportional reasoning task in the first cycle. Below we describe the IA that was used in the student rehearsal and analyzed for this study.

Instructional activity: Going over a problem using equivalent ratios

As detailed in the IA document for TCs, the *Going Over a Problem* IA aimed for classroom students to understand ratios as the relationship between two quantities that can be iterated and/or partitioned and if one quantity is scaled, then the second quantity must be scaled by the same factor to maintain a proportional relationship. The eight TCs enacted two lessons in sequence over two days using the context of selling yogurt or cheese in a shop. The first day, four of the TCs launched the yogurt task ("Ten ounces of frozen yogurt costs $5.40. Chris wants to buy ___ ounces of yogurt. What will he have to pay $___?") developing students' explanations using a table to represent equivalent ratios. TCs recorded students' processes by which a ratio was iterated or partitioned by developing an inscription on the table similar to that shown in figure two. The second day, the second four TCs launched a similar task in the context of a cheese shop eliciting equivalent ratios and methods for determining the ratio followed by introducing an error (i.e., a ratio that was not equivalent) for student consideration.

Of interest in the task was that the launch of the lesson that allowed students to make sense of the situation, share initial ideas, and build a common understanding prior to students being given specific numbers for the task. We hypothesized that students may have unequal resources for working on the task and asking all students to engage in exploring the situation allowed for equitable access to the task (Jackson, Shahan, Gibbons & Cobb, 2012). The IA built in structured partner interactions and attention to TCs using strategies for furthering mathematical discourse, such as teacher revoicing, asking a student to revoice, or asking students to agree or disagree with student contributions. The IA built in these structures by reminding TCs within the IA document to use these strategies. TCs also rehearsed with their peers using these strategies prior to enacting the lessons with students.

Following the launch of the task, TCs supported students working on the task eliciting solution and placing them on a table. After a number of solutions were recorded, TCs probed for students' methods, and recorded on a table the mathematical computation that was utilized by students to arrive at a new ratio (see Figure 2).

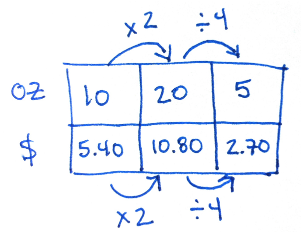

Figure 2. Example of a table of recorded solutions and methods.

By eliciting student methods and recording them TCs were prompted to probe on *how* students knew a method worked, in addition to that it worked (Chazen & Sandow, 2011). Within the routine of building an explanation (see Figure 3), TCs were supported, via the IA document, to construct a representation that allowed for comparisons of methods and discussion of the central math ideas.

STEP 4: Building an Explanation

Elicit answers for Table Ask for an amount and cost to add to table. Take a couple different pairs if lots of hands. If not ask someone to offer just an amount and then have students think about how to find cost. Take a moment to figure out the cost for each amount.	Answers will vary. Typically, some students will iterate equivalent ratios, some may use halving strategies (partitioning). A few students may combine two ratios to produce a third equivalent ratio. Some students may offer that one-ounce costs $0.54.
Elicit solution methods Ask students to share a method for solving the problem. [Record methods using arrows and the factors used.] Probe on how they knew their method would work. Ask other students for a thumb up or down to show if they think the solution method is correct or incorrect. Ask students if they have any other solution methods. Ask students for a thumb up or down to show if they think the solution method is correct or incorrect.	Here you are eliciting ideas about doubling amounts results in doubling costs, multiplying by ten in ounces means need to multiply by ten in cost.

Figure 3. IA document for teacher candidates' rehearsal.

Figure three shows a portion of the IA that was central to making students' mathematical reasoning public. This was the fourth step in the lesson, a whole group discussion in which students initially share solutions, without reasoning, as a means of verifying that there are numerous solutions to the situation. Further, eliciting solutions, without reasoning, allowed the TC to select a solution on which to probe reasoning. For example, the IA offers that students may double (iterate) or halve (partition) a ratio. These notes anticipate students' reasoning and provide TCs with support for interpreting solutions.

Methodology

To answer our first research question we coordinated the examination of eight secondary mathematics TCs' written analyses of student rehearsals using the Going Over a Problem IA with the video of their enactment. We also examined the affordances and drawbacks of the instructional tools made available in the IA. Recall our first question was: What tools and resources do teacher candidates use as they engage in secondary mathematics pedagogies of practice? For our second research question, we examined the video enactments for the student contributions during "step 4" of the IA. We selected step 4 as a site of focus because it is where students made their reasoning public and TCs were called to interpret and record this reasoning. Recall our second question was: What tools and resources do secondary students use as they engage with teacher candidates in mathematics classroom activities? Data were drawn from the first cycle of investigation and enactment as a part of a mathematics methods course. Examining data from this first cycle allowed us to consider the initial appropriation of instructional tools that TCs gleaned from their methods course and the ways that students employed mathematical tools participating in the IA.

Five of the TCs (all names are pseudonyms), Hunter, Mason, Lewis, Steve, and Hank, had completed a mathematics undergraduate degree and were enrolled in a master's of science and teacher certification program. One TC, Brooke, had completed the requirements for an undergraduate degree in mathematics and was completing coursework for teacher certification. Finally, two other TCs, Pam and Stewart, were working on undergraduate degrees outside of mathematics that would allow them to garner teacher certification in K-8 mathematics. Table two shows participating TCs program specifics and certification levels.

Table 2: Participating Teacher Candidates

Name	**Program**	**Certification**
Hunter	MS and certification	Grades 6-12
Mason	MS and certification	Grades 6-12
Lewis	MS and certification	Grades 6-12
Steve	MS and certification	Grades 6-12
Hank	MS and certification	Grades 6-12
Brooke	Math Undergraduate & certification	Grades 6-12
Pam	Other undergraduate & certification	Grades K-8
Stewart	Other undergraduate & certification	Grades K-8

Analysis

Our analyses of data entailed coding the TCs' written analyses of student rehearsals coordinated with the video of rehearsal. Further we analyzed the IA tool and the affordances and constraints of the tool. We also coded video of "step four" of the IA were secondary math students shared reasoning for four TCs who enacted the IA the first day. The aim of our video analyses, using a lens of mediated action where participation involves coordinating the mediational means of the sociocultural setting and the unique use of tools and resources within that context (Wertsch, 1994), was to identify the entailments of instructional practice, students' disciplinary work, and tool mediated participation.

Qualitative analysis of eight TCs' written analyses resulted in identifying 155 idea units across the data set. These idea units were distinguished by chunking the narrative by paragraph, which followed participants' attention to a topic sentence and elaborations on a topic. When more than one topic was linked within a paragraph the paragraph was broken into multiple idea units. Together two researchers coded a sample of the data to consider the robustness of the criteria for identifying idea units and our coding scheme.

A coding scheme was developed based on adaptions from Sherin and van Es (2009) and Santagata and Angelici (2010) who consider how teachers and TCs notice classroom interactions via video records. Our lens of mediated action has us examining TCs' communication about tool use to trace the ways social tools become individual tools for thinking and doing. We found the analytic stances of both research perspectives as commensurate and thus sensible to adapt coding schemes.

Our coding identified within each idea unit the: (i) *topic*; (ii) *level of evidence*; and (iii) *analytic stance of the TC*. Codes for topic included two possibilities; a focus on instructional work or student contributions. The level of evidence coded the presence of evidence and the nature of evidence with respect to the instructional system (Cohen, Raudenbush, & Ball, 2003; Lampert, 2001). That is, did TCs support claims with evidence and if so what kind of evidence did they bring to bear – attention to student, teacher, content, or context. A code of low evidence was applied to idea units that connected to no or one component of the instructional system. A code of high evidence was applied to idea units that connected to two or more components of the instructional system within the evidence provided. The stance coding included the following: *describe* – retelling events, *interpret* – making claims about events, and *critique* – positive or negative evaluation of events. Idea units could be coded with multiple stances (e.g., critique and interpret or critique and describe). Idea units coded interpret meant that the TC moved beyond just describing an event, thus no idea units were coded with describe and interpret.

Two researchers coded the entire data set, after coding together a sample. Coding resulted in a 92% agreement with disagreements rectified through discussion. All data were entered into an excel spreadsheet with associated coding. This allowed us to calculate the frequency and percentages per total idea units of each code and combinations of codes. From this inventorying of coding we constructed tables examining frequencies of data within and across codes.

Using our inventorying of coding, a secondary analysis examined how TCs referenced instructional tools within idea units. To carry out this analysis we sorted idea units coded within high confluence areas and created an emic-coding scheme, summarizing the nature of their contributions with the clusters of ideas units. This coding was conducted by two researchers separately and then shared to identify themes within high confluence areas.

Analysis of the instructional activity as a tool consisted of inventorying the routines, practices and strategies documented in the artifact. Here we looked at the specificity of instructional tools, the aim of the tools with respect to social and We also examined the mathematical aims of each routine and how practices specified or lacked specificity in terms of the mathematical aim.

For the analysis of video of the first IA's step four we identified the time when TCs elicited solutions (a marker of the beginning of step four). We then used Studiocode© (Studiocode Business Group, 2012), a qualitative and quantitative video analysis software package, that allowed us to segment the video into the sequence of student contributions. Each unique contribution became its own unit. Within each unit, we transcribed each student and teacher contribution. Finally we developed a set of descriptive codes, built on the instructional and mathematical practices teachers and students respectively employed. For example, instructional practices inventoried were: elicit solution, elicit method, solicit revoicing, solicit student agreement or disagreement with a method, teacher revoice, explain mathematical ideas verbally or explain mathematical ideas verbally *and* visually, questions eliciting how, and questions eliciting mathematical processes. Students' mathematical processes inventoried included: iterating, partitioning, adding of two equivalent ratios, offering evidence, and offering justification. Codes within and across enactments were clustered to consider themes. These themes are reported in the following section.

Results

Our analysis suggested that for first research question TCs initial engagement in pedagogies of practice afforded the use of a limited number of instructional tools – eliciting students' contributions and discursive practices – to varying degrees these tools were used consistently and repeatedly across TCs' enactments. Further, our analyses of the IA showed that we provided supports for these tools within the pedagogical structure of the lesson. Our analysis of the video of four TCs' enactments of the first IA showed that TCs indeed elicited numerous student contributions, recorded the contributions in varying ways, and attempted to connect contributions toward two aims – clarifying a procedure and justifying the procedure. The coding of TCs' written analyses uncovered further the ways that TCs communicated about instructional tools and coordinated the tools to issues of student access and key mathematical ideas. TCs also were frank about the challenges of deploying the instructional tools called for in the IA and the complexity of relational work of building an explanation aimed at the lesson goal.

For our second research question, our analysis of TCs' video-records uncovered that students deployed a number of mathematical methods to solve the task. Across the enactments, students offered numerous pieces of evidence to illustrate methods. What was more challenging was constructing a justification for a process rather than offer additional evidence. TCs' questioning varied in terms of supporting, or pressing, for justifications with only one of four TCs engaging students in this type of mathematical practice.

Analysis of the IA

The IA asked TCs to support students solving two contextualized ratio tasks, create a series of equivalent ratios with students devising their own solution methods, and collaborate with a partner to develop solutions and share methods. The goal of the task was to support reasoning about ratios as the relationship between two quantities and the coordination of two quantities to maintain equivalence by iterating and/or partitioning However, the IA did not specify what students' reasoning might sound like when justifying *why* iterating, partitioning or other valid methods worked. Nor did it unpack what it means to coordinate two quantities to maintain equivalence.

The IA also embedded instructional practices involved in launching and facilitating building an explanation among students. The IA asked TCs to use revoicing, soliciting students' evaluations of methods, elicit solutions, and elicit methods. The IA also asked TCs to question how students knew a method worked. Through the use of discourse moves and questioning guided by a specified mathematical learning goal, TCs were prompted to press on students' ideas in order to support students grappling with key mathematical relationships. The IA engaged students' participation with peers and asked candidates to record multiple solutions using a mathematical representation. The IA specified a structure for recording students' methods (see figure 2), but didn't unpack how verbal methods may get transposed into a written inscription to support all students reasoning and to advance the mathematical point. Further, the linguistic and mathematical demands of supporting students building a mathematical explanation were not explored in ways that anticipated these demands nor documented them within the IA (Chazen & Sandow, 2011; Schleppegrell, 2007).

Analysis of four TC enactments from video

The video analysis suggested that TCs' participation began to coordinate the instructional tools available in the IA – discourse moves, eliciting students' contributions, and recording contributions. Recall, we analyzed four TCs enactments of the first day's IA to be able to compare how TCs used the tools and resources available in the IA. Because the IAs differed in terms of mathematical resources available to students (e.g., an error is introduced in day two's IA) our video analysis was limited to the four TCs who worked on the same IA[1]. All four TCs successfully and repeatedly elicited students' contributions – both solutions and methods. Two of the four TCs employed discourse moves (student revoicing, soliciting agreement or disagreement) in the whole group discussion. A third was observed asking multiple students to offer insights on the same question. All four TCs revoiced students' contributions as he or she recorded methods. Given this was the first enactment with students we took this as evidence that TCs were developing awareness of discourse moves and initiating minimal use of discourse moves in the moment.

Typical in the four enactments was a participation structure placing the teacher in the center of the mathematical discourse, revoicing students' contributions, and often adding on to contributions to elaborate on mathematical procedures, summarize mathematical ideas, or in some cases correct short-cuts taken by students. For example, all TCs' enactments evidenced the use of verbally and visually explaining mathematical ideas. Two TCs' enactments took up clarifying students' contributions on multiplication or division by powers of 10 when students said, " I added zeros" or "I moved the decimal point." Here TCs reminded students they were multiplying or dividing by a power of 10. All TCs elaborated on mathematical ideas to draw students' attention to the idea that they were completing the same operation on each quantity to construct a second ratio. Less often TCs completed the elaboration with a rationale or justification of the import of this mathematical procedure. Further, only one TC, Hank, posed a question that asked why students would preform the same operation on quantities after solutions and methods had been explored.

> H: Why are you doing the same thing to each of them?
> S1: if you multiple by 10 your adding a zero so if you subtract off one your dividing.
> H: Umm, yes I understand that. I am wondering, so you doing the same operation for the yogurt and the money. Right, you are multiplying by ten. Why are you doing the same in both places?
> S: ohh
> H: yeah [calls another student]
> S2: It's kind of like fractions. If you want to get an equivalent you have to multiple each number by the same thing.
> H: You used the word equivalent. So you have to get this fraction [points to a solution]. Is this a fraction?
> S2: No, yeah, using fractions, it would be...
> S3: ... well it can't actually be [a fraction] because 5.40 is a fraction.
> S2: yeah
> H: But you said equivalent. I really liked that word. We are creating an equivalent ratio [pointing to board]. This is equivalent 10 : 5.40 and 100 : 54 this is equivalent, and this is equivalent 1:54.

Hank's question potentially asked students to provide a justification for constructing equivalent ratios. The first student responded by providing an example as evidence for the process. Hank

[1] Future video-analysis will include day two's IA and compare across the two sets of enactments. Due to limited time and resources our analysis here is our initial work to develop a coding scheme to conduct further analyses of TCs' enactments.

rephrased the question and asked another student to contribute. This student drew upon a similarity between the situation and equivalent fractions. Hank picked up on the term equivalent and clarified if the two quantities form a fraction or a ratio. Hank provided silent, but positive agreement, when one student affirmed that the two quantities don't form a fraction because one value, 5.40, can be expressed as a fraction. From here Hank picked up again on the term, equivalent, and used this as an opportunity to elaborate on the fact that they have constructed a series of equivalent ratios. Hank posed a key question and refined the question when a student responded with evidence rather than a justification. However, rather than using other instructional tools, such as discourse moves or another question, Hank offered a claim that student had constructed a series of equivalent ratios. We saw this interchange of ideas as evidence that Hank needed support to anticipate a response to his questions. Further, the IA needed to offer TCs ways to distinguish responses to key questions that prompted for justification, rather than description of process. Hank's eliciting of ideas and pressing on why a process works was promoted in the IA. However, the challenges embedded in this work were not supported.

Within the four TCs' enactments we saw evidence of eliciting ideas in which TCs were faced with negotiating the verbal contribution into a written inscription of mathematical symbols, operations, and reasoning. This proved to be challenging. Evidence of this challenge was noted by the variety of representations constructed across the TCs' enactments, lack of consistency of representations across student contributions within a TC's enactment, and the discursive interplay of students offering evidence, teacher revoicing, teacher recording. For example, only one of the four TCs, Stewart, consistently used the table structure to record student contributions. One TC, Hank, recorded quantities on a table and recorded mathematical operations to find equivalent ratios in separate statements, inconsistently noting units, and using different methods to record across contributions. The remaining two TCs (did not visually connect the ratio being constructed with the student's method. We saw the disconnect between ratio and method and the varying ways of recording methods within an enactment as potentially problematic for advancing students' capacity to generalize across methods and to build a justification for why a procedure worked. We took the inconsistent use of representations as evidence that TCs were unsure of how to organize the records across contributions.

Further, the interplay of listening to contributions and recording contributions proved complex for TCs. Here we saw TCs use a variety of strategies to listen, record, and represent the mathematical ideas. Two TCs were observed recording students' contributions changing students' contributions or filling in information in order to record the ideas asserting that this must be what the student was doing. Another TC was careful to check with students when he recorded a statement to assure students understood the notation he was using. All four TCs moved between listening to verbal contributions and either rephrasing or asking a student to repeat a contribution in order to record a method. The interplay of listening, recording, verifying (or not verifying) contributions, and organizing the real estate available resulted in the visual display of information to vary across solutions, to be spread across spaces without visual cues of how ideas unfolded, and to often disconnect solution and method.

We took these findings as evidence that we underspecified within the IA the role of the recording structure and the linguistic and mathematical demands of building explanations. The tools – the IA, routines, practices and strategies – available to TCs to represent students' verbal contributions were woefully inadequate. We had not made explicit with TCs could organize real estate for displaying multiple methods. We provided a structure, but did not specify the import of its use prior to the enactment. We also did not anticipate and prepare for the complexity TCs faced listening, interpreting, and constructing an inscription in the moment. TCs' performances drew on the tools and resources they had available in the moment. Our analysis suggested that our pedagogy of practice underserved and under-supported TCs building instructional knowledge and skill with these practices.

Analysis of students' use of mathematical practices showed that across enactments students contributed evidence of iterating, partitioning, and adding of consistent quantities across equivalent ratios to find a third equivalent ratio. Students were able to provide mathematical valid evidence for their various methods. However, few were asked to provide justifications for their claims and if solicited justification was based on a motto of "what ever you do to one quantity you have to do to the other quantity". Unpacking why this was mathematically important and to what end was only explored in one TC – student interaction, as detailed above. We saw this as evidence that the goal of the IA, and the work to decompose the mathematical and pedagogical structures of the IA, did not support moving most TCs past a procedural goal for the enactment.

In summary, our analysis of video-records of four TCs' enactments showed us that TCs used key instructional tools – eliciting and discourse moves – to advance the instructional aims. However, the IA, as a tool, underspecified and under supported the complex interactive work of ambitious teaching for the TCs. Because we were interested in how our pedagogies of practices supported TCs' development of *knowledge* and skill we turned to our coding of TCs' written analyses to gain insights on TCs' thinking about their enactments.

Next we highlight the findings from our coding of eight TCs' written analyses of their enactments. We first describe the quantitative data generated by our coding to identify areas of high frequency that point to what were topics of interest for TCs. As we highlight the findings from this analysis we will note how the themes that emerged coincide with the findings from our video analyses. In our discussion section we will elaborate on how the coordination of our findings address our research questions.

Findings from 8 TCs' written analysis

Below we describe the findings from our coding. From our quantitative analysis there were two sets of idea units, coded with the most common stance, representing a majority of the data. These two sets of idea units were associated with a stance of describe or interpret. From our secondary qualitative analysis themes emerged. These themes suggested that TCs were connecting tools (routines, practices and strategies) with goals for these using these tools. Further, our analysis uncovered challenges TCs faced using tools in the complex system of instruction.

Tabulated below (Table 3) are coding results of TCs' written analyses. The first four numeric columns show the number of idea units coded with each combination of topic (instructional work or student contribution), level of evidence (high or low), and stance (critique, describe, interpret). The last column shows the percentage of the idea units that were coded with each stance. The two cells highlighted in **bold** indicate confluences of codes that we examine further.

Table 3: Topic, level of evidence, and stance of teacher candidates' video analysis

	Instructional Work	*Student Contributions*	*Percentage of idea units*

Stance	*High Evidence*	*Low Evidence*	*High Evidence*	*Low Evidence*	
Cri	6	17	2	1	19%
Des	5	26	3	5	**28%**
Int	*20*	27	*20*	6	**53%**

A large percentage of idea units were coded with the stance of interpret (53%), across topics, and levels of evidence. We found this intriguing given this was TCs' first analysis and this meant that they were moving beyond describing events to interpret the work using varying levels of evidence. TCs' use of evidence ranged from low to high meaning that they brought to bear either minimal evidence with minimal attention to the instructional system or they cited evidence and used more than one aspect of the instructional system (students, teacher, content, or context). We examine these idea units further in the qualitative analysis.

Second confluence of coding we found focused on describe across topics and levels of evidence (approximately 28% of idea units). We were not surprised by TCs' focus on describing events since they were asked to describe episodes selected after they transcribed them. We suppose that the low level of evidence cited in these idea units reflects that TCs' analyses contained transcriptions of events so that further description may have seemed redundant. We examine these idea units further coordinated with idea units coded with interpret in the qualitative analysis.

Qualitative Findings

In this section we highlight our examination of three sets of idea units that appeared with high frequency in our quantitative analysis; units coded with describe or interpret and idea units coded with perspective. Our secondary qualitative analysis revealed that TCs linked instructional tools to purposes, specified the mathematics in student contributions, and enumerated challenges they faced using instructional tools. We highlight our findings with illustrative quotes from TCs analyses attributing each quote to TC's pseudonym and an idea unit.

Stance of describe and interpret highlights TCs coordinating instructional tools and student contributions to goals. The theme of connecting instructional tools to goals emerged from the subset of idea units coded with describe and interpret within the topic of instructional work and student contributions and both levels of evidence. TCs' interpretations of instructional work had them specify particular instructional practices and strategies and connect them to a set of goals serviced in the enactment of the IA. Two common goals articulated in a variety a ways by TCs were: (i) providing the collective of students' access to other's contributions and (ii) underscoring the importance of key mathematical ideas.

Across idea units participants considered how instructional tools could make a student's contribution available to the class and leverage the IA's student learning goals. Pam offered how strategies she employed provided access for students when she stated,

> ...[A]s the second student was revoicing he took it more step by step, and I wanted to show on the board what he was saying so I said, 'Let me slow you down for a minute so I can write this as you say it.' I think others were able to follow it the second time as it was said slower and shown visually on the board (Pam-1B1).

Pam described how she made use of student revoicing and recording a student's contribution to ensure that all students had the opportunity to hear and make sense of a quickly shared and complicated solution method. Hunter ‘s contribution also illustrates how TCs used instructional tools to provide students access to ideas. He wrote,

> Having the student walk us through his process moved the class towards the mathematical point because he was able to explain that he multiplied both the amount of cheese and the cost of the cheese by the same number to get a new ratio. This was exemplified by the revoice by the other student because it showed that other students were following the first student’s explanation (Hunter-1B).

We saw Hunter attending to how a particular mathematical idea was introduced by one student and then taken up by other students in the class. Mason’s narrative also illustrated this goal as he noted how he elicited an anticipated strategy to consider the non-example or error introduced in the IA.

> The students gave me the idea of adding two of the previous number sets provided to disprove the non-example… In eliciting student strategies, I had an idea that this would come up and was sure other students were able to revoice what that student did (Mason-1M).

By eliciting an anticipated method to reason about the introduced error and using student revoicing, Mason considered how the instructional tools he deployed provided other students access to a method. We saw TCs’ attention to instruction tools of eliciting contributions and discourse moves supporting our video-analysis of the four TC’s enactments. We too saw them use these instructional tools. Our coding of TCs’ written analyses helps us understand what role they saw these tools playing. We were encouraged that they saw the import of the tools even if our coding of the video-analysis uncovered complexities that were challenging during the enactments.

In addition to framing instructional tools as useful for providing all students access to a solution method, the TCs also framed instructional practices and strategies as useful for underscoring the importance of key mathematical ideas. In Brooke’s analysis she commented, “Although I don’t think [my] question was ideal... it did lead to a generalization of why we have to coordinate both values or how that action is justified” (Brooke-1F). In this idea unit Brooke attended to how her question led students to generalize a key idea about ratio. Although Brooke’s enactment was not one of the video-records we coded, her comment highlights the work we observed in Hank’s enactment detailed above. Brooke’s consideration of a student contribution also showed how she needed to attend to questioning to reveal a students’ idea that could then be a part of the discussion. She wrote,

> I actually had to ask a lot of questions to make sense of student responses ... because I had to know what this student meant by doing the same thing or equivalence before I could try to facilitate a discussion about his idea. ...This was an instance where I had to ask more questions to really make sense of a student’s understanding. He knew we were doing the same thing to each quantity and knew something further about that relationship, but didn’t know how to articulate it. (Brooke, 1H)

We saw Brooke connecting students’ contributions to the goal of the lesson. Her move to elicit a student’s reasoning and press on that reasoning with the intention of facilitating a discussion with the group showed how she was holding both a need to understand students’ contributions and use instructional tools with a goal of advancing the mathematics.

Within these idea units, TCs not only saw how instructional tools leveraged key mathematical ideas, they also lamented on when their use of instructional tools did not highlight key mathematical ideas. Stewart's contributions within the set of idea units illustrated both success and where he saw room for improvement. He wrote, "I made sure to write separate models of how students created their specific answer... I believe that this gave students a clearer model" (Stewart-2B). He went on to say,

> I could have used discourse moves that will further the mathematical understanding of students... I quickly moved through it. I did not give students time to think about the strategy... we could have discussed how they relate to each other. I could have asked if students agree with the interpretation... 'Why or why not?' (Stewart – 3C1).

Stewart detailed how he enacted the practice of recording students' contributions and how the representation allowed students access to key mathematical ideas. He also shared spaces where he could have employed other instructional tools such as discourse strategies, asking students to compare solutions, and providing justification to advance mathematical ideas in the IA. Lewis also discussed how he used particular instructional tools, but also saw a need to improve when he wrote,

> While I was able to elicit a response from the student and respond to it by representing it on the board, I felt there was a one-on-one connection between the student and I that caused the rest of the group to be less engaged. I needed to back up and bring the rest of the group into the conversation (Lewis 1D).

We saw Lewis recognizing that although he used a variety of instructional tools his engagement with one student was not fully supporting all students in discussion, a goal he saw as important. He went on to say,

> I could have engaged them [the students] part way through [saying], 'Greg said that we need to multiply the top by 2. Why do you think he did that and what do you think he would do next?' (Lewis, 1D).

Here Lewis offered a replay of what he might have done to support the full group engaging in a discussion of a method. Although Lewis was not a part of the analysis of the four TCs enactments, Lewis's remarks corroborate our coding that suggested that limited use of discourse moves and a participation structure that placed the teacher in the center of the mathematical work.

Our coding in this confluence of idea units allowed us to see how TCs coordinated instructional tools, such as practices and strategies, with the purposes for using those tools, either making contributions from one student accessible to the group or highlighting key mathematical ideas. It also highlighted the challenges TCs suggested they faced when employing instructional tools to advance student learning. In the following section we discuss these findings in terms of our research aim and our framing of pedagogies of practice. Here we consider how TCs' communication about tool-mediated activity (pedagogies of investigation and supported enactment) provided insights on how TCs were appropriating instructional tools and the challenges they faced.

Discussion and Conclusions

The aim of this paper was to determine what tools and resources TCs used to engage in pedagogies of practice. Further, we examined the mathematical resources secondary mathematics students employed with TCs when TCs enacted instructional tools supported via pedagogies of practice. Based on coding of TCs' written analyses and the video of their enactments we were able to coordinate TCs' development of skilled use of instructional tools and their knowledge about these instructional tools. Further, our analysis of TCs' enactments allowed us to consider the ways students engaged in the mathematical ideas afforded by an IA. This study offers insights on TCs' initial participation in pedagogies of practice, their skill with new ambitious instructional tools and how they orchestrate the complex, relational work of building an explanation eliciting student contributions.

Our data show that TCs were able to elicit students' contributions with relative fluency. Our data analyses verifies findings from other studies that examine candidates' skill with facilitating discussion, namely that TCs' skill with interpreting students' reasoning varied in complexity, elaboration, and consideration of key mathematical ideas central to the learning goal of the lesson (Ghousseini, 2009). Further, candidates were challenged to leverage discourse moves in ways that supported interpreting student contributions that would advance a productive mathematical discussion aimed at the learning goal. Our coding of TCs' written analyses revealed that they saw the affordances of the instructional tools they were developing skill with and they acknowledged the challenges of enacting them. Our analysis of the IA suggests that our tools were underspecified to support the complex interactive work that TCs were engaged in. We also acknowledge that developing skill with these instructional tools takes practice. This was TCs' first cycle in the teacher preparation program and our hypothesis was that they need repeated opportunities to engage with the routines, practices and strategies being advanced via our pedagogy of practice. To be clear, we did not anticipate TCs to be masterful with the ambitious instructional tools. Our research showed that the tools provided TCs opportunities to build new skills and uncovered the challenges TCs face when engaged in the complex and relational work ambitious teaching. More specifically, this study uncovers some of the demands within the routine of building explanations that TCs and students faced. This will advance how subsequent tools and supports can be constructed to aid in the incremental improvement of instruction.

In our analysis of students' participation and tool mediated action, a theme that emerged was that students used the task in ways that opened up a variety of mathematical explanations, examined the structure of ratios and processes for maintaining equivalence and used everyday and academic language when pressed to share reasoning. They also leveraged social resources to some extent by sharing reasoning and to a much lesser extent to advance an argument or compare reasoning. Generally, students offered ideas to TCs for arbitration.

Candidate's initial experiences in pedagogies of practice established a discourse of practice that allowed for investigating the complexity of ambitious teaching. Our findings offer insights on how pedagogies of practice support candidates' development of instructional skill while offering secondary students' opportunities to engage in authentic disciplinary practices. Future research needs to coordinate TCs' and students' participation across multiple IA as TCs work with middle and high school mathematics students to examine the generativity of secondary pedagogies of practice for developing skilled instruction. A question that remains unanswered by this research is how secondary pedagogies of practice prepare TCs to navigate the sociocultural settings of schools and their capacity to enact ambitious instruction within student teaching and early career.

In the interactive paper session participants will have opportunities to examine key routines with the IA highlighted in this paper. We will also share two video episodes within the small group to consider ways of advancing TCs instructional skill using tools for research and development. Questions guiding discussion include: What learning opportunities and limitations can you identify in secondary pedagogies and practices? What research tools need developing to scale examination of pedagogies of practices within and across institutions?

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