dimensional teaching

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See how one seventh-grade teacher engages students by melding NCTM's Process Standards, CCSSM's Standards for Mathematical Practice, and varied contexts.

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Mathematics, like many other forms of knowledge, is situated in a cultural context (Lave and Wenger 1991). Too often, learners experience school mathematics as something detached from any meaningful context and unfamiliar, leaving many with the sense that such knowledge is inaccessible (Ellis 2003). However, students bring with them informal knowledge of mathematics through everyday interaction with their cultural and social backgrounds.

Multidimensional mathematics teaching, as we will discuss in this article, encourages the use of context in mathematics teaching and learning to help teachers better recognize and build upon the cultural and social resources students bring to the classroom. Such efforts can impact students' performance and can help students make sense of decisions about their choice of problem-solving procedures (Boaler 1998). Multidimensional teaching also represents a way to address equity concerns in school mathematics by supporting—

- an understanding of the cultural and social resources that students bring to the mathematics classroom;
- the development of knowledge within those cultural frameworks; and
- an understanding of mathematics within varying cultural frameworks (Secada 2000).

For example, consider the way in which a seventh-grade teacher, Ms. Canady, used the local newspaper as a resource when presenting the following contextual problem involving integer arithmetic:

> In Friday night's football game, the Spartans lost 11 yards on first down then gained 5 yards on second down. What was the total yardage for these two plays?

Donovan, Shonda, and Cheri worked together on this problem. Donovan drew a picture that resembled a football field, showing that the team started on the 20 yard line and ended on the 14 yard line. He stated that the answer was 14. Shonda looked at Donovan's pictures, and the conversation continued:

Shonda: Yeah, they lost 6 yards.
Canady: Suppose the Spartans started on the 50?
Donovan: 44.
Shonda: Minus 6.

During the whole-class discussion, Marcel, a student working in a different group, drew an image similar to Donovan's with two exceptions: The starting point was 0 and the ending point was –6. Donovan made a connection from Marcel's strategy by stating to his tablemates that "it does not matter where they start; they lose 6 yards."

After Canady's contextual problem set the stage, she introduced the use of a number line to teach addition and subtraction of integers. The context, coupled with sharing strategies, gave students opportunities to make connections with different representations of integers. It also supported the problem's transition from a concrete, real-world situation to the abstract representation of a mathematical relationship.

Given that standards call for students to learn mathematics as connected and meaningful and to develop positive dispositions, teachers must recognize and build on various factors:

- Students' cultural and social experiences
- The use of context that can positively impact students' performance on mathematical tasks
- Mastery of mathematical procedures
- The ability to productively apply mathematics to solving problems
- Attitudes toward the subject (Boaler 1998)

We focus on identifying points of intersection among the Process Standards in *Principles and Standards for School Mathematics* (NCTM 2000), the Standards for Mathematical Practice in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), and multidimensional mathematics teaching. These intersections reflect an awareness of both learners' diversity and the definition of mathematical proficiency called for by *Principles and Standards* and CCSSM.

MULTIDIMENSIONAL MATHEMATICS TEACHING

In contrast to a narrow focus on abstract reasoning and rote procedural learning, multidimensional mathematics teaching supports a broader range of learning preferences and outcomes. Based on their community practices and social-cultural norms, many students have a preference for holistic, contextual, and relational learning (Malloy and Malloy 1998). *Holistic learning* refers to starting with a big concept at the macro level before delving in to focus on more discrete and specific parts of the whole. *Contextual learning* refers to using meaningful activity during the learning process including activating kinesthetic, affective, and visual modalities, thus making the use of manipulatives, visual models, and graphical ways of organizing ideas important (Shade 1997). *Relational learning* refers to valuing communication and connections among people and ideas during sense making. This form supports movement, creativity, rhythm, and engagement with patterns (Shade 1997).

To support a broader, more diverse range of students in learning mathematics as conceptual, connected, and meaningful, teachers must provide opportunities for learners to engage with big ideas, contextualize learning, exercise divergent thinking, relate mathematics to their own interests, activate multiple modalities, and learn through informal and formal class discussions (Berry 2003; Boaler 2000; Malloy and Malloy 1998).

Emphasizing these strategies is not meant to imply the elimination of a focus on proficiency with facts and procedures, opportunities to work individually, or engage in abstract reasoning; these elements are clearly critical aspects of mathematical knowledge. The point is that once students are engaged in reasoning and sense making via holistic, contextual, and relational means, they will more readily see the need for specific factual knowledge. They will also be able to transition more successfully to analytical reasoning about abstract mathematical ideas. Thus, multidimensional mathematics teaching must strive for a balance of holistic and specific learning, contextual and abstract learning, and individual and relational learning.

Multidimensional mathematics teaching requires that teachers know and understand the multiple ways that students might interact with teachers'—

a. pedagogical content knowledge;
b. beliefs about mathematics teaching and learning; and
c. understandings about students' social and cultural contexts (Lampert 2004; Moschkovich 2002).

Table 1 This grid connects NCTM's Process Standards, CCSSM's Standards for Mathematical Practice, and characteristics of multidimensional teaching.

NCTM's Process Standards	CCSSM's Standards for Mathematical Practice	Characteristics of Multidimensional Teaching
Problem Solving	Make sense of problems and persevere in solving them	Holistic Contextual
Reasoning and Proof	Reason abstractly and quantitatively Look for and express regularity in repeated reasoning	Relational
Communication	Construct viable arguments and critique the reasoning of others Attend to precision	Relational
Connections	Look for and make use of structure	Holistic Relational
Representations	Model with mathematics Use appropriate tools strategically	Holistic Contextual

In addition, to create opportunities for students to experience mathematics learning as relational teachers must know and understand learners' identities, histories, experiences, and cultural contexts (Boaler 1998; Gutstein 2003) and consider how to use these to connect students meaningfully with mathematics.

Teachers who situate mathematics teaching and learning as multidimensional are providing contextual opportunities for students to exchange strategies and share mathematical ideas. NCTM's Process Standards and CCSSM's Standards for Mathematical Practice provide a rich framework for understanding the learning preferences mentioned above and are aligned with multidimensional mathematics teaching and learning.

PROCESS STANDARDS AND STANDARDS FOR MATHEMATICAL PRACTICE

CCSSM describes eight Standards for Mathematical Practice across grades K–grade 12 that constitute habits of mind that mathematics teachers at all levels must seek to develop in their students (CCSSI 2010). Significant overlap exists among the ideas expressed in those eight standards and in the Process Standards.

Table 1 and the discussion below represent one set of connections among the Process Standards, the Standards for Mathematical Practice, and the characteristics of multidimensional teaching. Practice 1 (*make sense of problems and persevere in solving them*), connects with relational learning through the use of multiple problem-solving strategies, variation, and creativity. Both holistic and analytical learning preferences are supported by Standards 2 and 6 (*reason abstractly and quantitatively* and *attend to precision*) by using divergent and convergent thinking, employing concept organizers, and attending to connections among mathematical ideas. Standard 3 (*construct viable arguments and critique the reasoning of others*) overlaps with the importance of creating a relational learning environment in which students share and critique one another's reasoning.

In addition, Standards 4 and 5 (*model with mathematics* and *use appropriate tools strategically*) intersect with contextual learning through incorporating cultural and social contexts to model and make sense of mathematics. Modeling with mathematics requires knowing and doing mathematics in ways beyond simple paper-and-pencil computation such as understanding appropriate uses of mathematics within given contexts. Standards 7 and 8 (*look for and make use of structure* and *look for and express regularity in repeated reasoning*) connect with relational and holistic learning by giving students opportunities to explore regularities in patterns that can then lead to abstractions and opportunities to reason and understand how big mathematical ideas are constructed and connected.

MS. CANADY'S CLASSROOM

To better understand how these interconnected ideas might come to life in mathematics class, we offer a glimpse into the classroom of Ms. Canady, who has fourteen years of experience at Spartan Middle School (SMS) where she teaches seventh-grade mathematics and language arts. SMS, in a rural county in a southeastern state, has 514 students; 50 percent are black, 48 percent are white, and 2 percent come from other ethnic backgrounds. Fifty-six percent of the students receive free or reduced

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An Activity to Identify Standards

While reading the description of the lesson activities, use **table 1** as a guide to identify which of the Process Standards, Standards for Mathematical Practice, and characteristics of multidimensional teaching you see being enacted. You might choose one column from the table at a time and place check marks next to the items you notice; reread these descriptions to do the same for the other columns in **table 1**. If you are reading this article with a group of colleagues, assign a different column to different readers and compare notes afterward.

lunch. All three of Canady's seventh-grade mathematics classes were part of a larger research study from which the data for this article were generated. Of the 51 students from these classes, 33 were black and 18 were white.

Observations of Canady's classroom showed that she had a strong rapport with all students. She demanded students' respect and expected them to work hard by making sure they were productively engaged; she also connected with the students by asking questions about their families, discussing their hobbies, and discussing larger community issues. Canady can be described as being a "warm demander" (Bondy and Ross 2008), a teacher who knows the culture of students, has strong relationships with students, and commands that everyone within the classroom will be respected and follow classroom norms. Canady describes her pedagogy in this way:

> Knowing the backgrounds, family issues, and struggles of many students helped me to develop empathy and understanding and to want to reach my students. . . . This community has gone through some tough times. It is my job to make sure my kids can be successful. . . . Over the years, I've become creative in my teaching. My classroom has changed dramatically over the past years . . . from a class with me as the instructor to . . . me as the facilitator. Students really enjoy this environment and typically do well.

Canady began a unit on addition and subtraction of integers by working on contextual problems she created using the students' locale. On day 1 of this unit, students worked on contextual word problems that included a drought, temperature, football, and debt and income relationships. The context of drought was particularly relevant because the students' area was experiencing severe drought conditions at the time. The football task was also appropriate because high school football in that particular county is revered. Canady explained that she started with a real-world problem because she wanted students to have a context when manipulative use and abstraction were combined later in the unit.

In addition to context, the unit incorporated critical thinking and the use of hands-on and virtual manipulatives. On day 2, students were introduced to the idea of zero pairs (e.g., –2 and 2 combine to make 0) and worked with two-color counters to concretely represent this concept. Students solved words problems using both two-color counters and number lines to compare and contrast the two representations. For example, students were presented with this prompt:

> An architectural drawing of the county office building shows the elevation of the basement floor to be 12 feet below the surface. The elevation of the roof is 28 feet. What is the total distance from the roof to the basement floor?

Later during day 2's lesson, students worked on decontextualized problems, such as –8 + 3, also modeled with the two-color chips and number line.

On day 3, students represented and solved problems using two-color chips after accessing the color chip addition applet found on the National Library of Virtual Manipulatives website (www.nlvm.org). After initial work using the applet, students were told they would develop rules for adding integers and generate problems. Students could use number lines and two-color counters to develop their rules. The excerpt below is from a group of students discussing possible rules.

Amber: When you have a negative and positive, you subtract; if the negative is higher, then the answer is negative; if the positive is higher, then the answer is positive.

Angelic: OK, put it on the list. Here is mine; if there are two negatives, add them together, and the answer is negative.

The group: Alright, yeah, OK.

Shawon: If both are positive, then the answer is positive.

Tony: When adding opposites, you come out with zero.

During whole-class discussion, Tony asked, "Suppose you have a problem with two positives and one negative?" When Canady asked him to give an example, Tony replied, "7 + 6 + –10." Many students in the class either drew a number line or used two-color counters to explore this problem. After some discussion, they concluded that the answer would be a positive integer. After the discussion, students realized that there was not a rule to fit the case Tony posed. They realized they needed to simplify pairs of addends using the rules they derived until all addends are used up.

On day 4 of the unit, students were asked to develop song lyrics that communicated the rules for adding integers they developed the previous day. They worked in pairs or groups of three to four to develop lyrics that met several requirements. Students were to describe situations that involved the addition of—

a. two positive integers;
b. two negative integers;
c. one positive and one negative integer; and
d. opposite integers.

Most of the students wrote or adapted lyrics set to popular rhythm and blues, country, or hip-hop songs. Additionally, although not required, some students wanted to work out dance routines or movements for their songs and were allowed. The movements they developed complemented not only the songs but also the mathematics. For example, one group modeled a human number line, and their movements mirrored the rules as one move up or down the number line. In addition, students were asked to use the newspaper to search for stories involving integer use. The newspaper activity took the headline news activity on day 1 and turned it around; students were now looking for articles to share with their teacher and classmates.

Day 5 consisted of student performances of their songs, an analysis of some of the lyrics, and a review of the unit. Although the performances were entertaining, Canady kept them short because the goals were for students to understand the rules for adding integers, develop a context for knowing why they worked, and make sense of them for themselves. The analysis of lyrics consisted of the teacher writing down lyrics that expressed similar rules and asking students to compare and contrast their meanings. Students enjoyed discussing word choice and language use in relation to mathematics. They agreed that an important consideration when writing lyrics involved how a word would be interpreted by the listener.

MULTIDIMENSIONAL MATHEMATICS IN ACTION

We read the description of Canady's lessons on integer addition and reflected on which of the elements in **table 1** were seen throughout this five-day unit. We noted many instances of students engaged in conceptual, connected, meaningful learning through the Process Standards and the Standards for Mathematical Practice. Canady's use of strategies and activities strongly correlated with the characteristics of multidimensional mathematics teaching described earlier. The knowledge required for this sort of teaching is substantial.

In addition to mathematics content, teachers need to know and be able to teach problem-solving skills, represent mathematical concepts in multiple ways, connect mathematical concepts within mathematics and to other subject areas, and analyze students' thinking about mathematics. Effective mathematics teachers of diverse learners also know the cultural and social resources that students bring to the classroom and how to incorporate these resources into their teaching.

We have presented a snapshot of a classroom in a rural community. Although this specific context has limited generalizability, we cannot discount the fact that Canady's multidimensional pedagogy is far reaching. She understood and incorporated meaningful contexts into her lessons to make the mathematics engaging for her students, allowed them opportunities to communicate with and about mathematics, and planned for and supported their transition from concrete and contextual knowledge to abstract and generalizable mathematical understandings.

Taken as a whole, this multidimensional approach combined with Canady's "warm demanding" disposition motivated students to do mathematics. As the descriptions of students' activities throughout the integer unit reveal, such an approach aligns well with the Standards for Mathematical Practice. Indeed, when not approached as merely the delivery of decontextualized facts and skills, mathematics teaching can engage diverse learners in discussions of relevant social and cultural matters, promote students' creative thinking

and sense making, and strengthen students' ability to communicate their ideas clearly. These are goals worthy of striving for in all mathematics classrooms.

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