Practice-based Perspectives of Inquiry Teaching of High School Algebra

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Although there are suggested guidelines about teaching algebra from a reform perspective, what happens in the classroom depends on the teacher. This study investigated inquiry-based teaching of algebra based on the thinking and practice of three experienced exemplary high school mathematics teachers. Findings based on interviews and classroom observations identified three different perspectives of their inquiry approaches with central features specific to each that afforded different types of learning of the algebra concepts. Four central features of the teachers' thinking were also identified as bases of their sense-making of their inquiry-oriented approaches.

Introduction

The National Council of Teachers of Mathematics [NCTM] (NCTM, 2000) promotes algebra as fundamental to the basic education of all students from prekindergarten through grade 12. This suggests the importance of making algebra accessible to all students, which will depend on how it is taught, and thus, on the teacher. Traditional instructional approach of high-school algebra generally involves the teacher demonstrating two or three "worked examples," with emphasis on procedure and symbol manipulation, followed by practice exercises. This paper focuses on high-school mathematics teachers who deviated from this approach. It reports on a study of the teachers' inquiry-oriented approaches based on their thinking and practice. It addresses two research questions: What are the central features that characterize the teachers' inquiry-oriented approaches based on teaching systems of equations? What are central features in their thinking that allow them to make sense of their approaches?

Related literature and theoretical perspective

Algebra education has been an active field of research covering a range of issues in terms of the nature, teaching, and learning of algebra (e.g., Bednarz, Kieran, & Lee, 1996; Herscovics & Linchevski, 1994; Kaput, Carraher, & Blanton, 2008; Sleeman, 1986; Stacey & MacGregor,

1999; Stacey, Chick & Kendal, 2004). Concerns about students' inadequate understandings and preparation in algebra, algebra being difficult to learn, algebra curricula, and algebra instruction have been foci of this body of research. To address some of these concerns, many suggestions about meaningful instruction are offered in the literature (e.g., French, 2002; Swan, 2000). In general, reform perspectives of mathematics teaching (e.g., NCTM 1991, 2000, 2014) are promoted to make a difference. The NCTM's Principles to Actions (2014) place significant emphasis on the teaching and learning of mathematics highlighting the importance of "teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p.4). The eight Mathematics Teaching Practices identified to characterize this type of teaching embody key notions of inquiry teaching, which provides the theoretical perspective for this study.

Inquiry is considered to be effective for teaching both content and process skills. As a basis of learning, inquiry is well established in the literature dating back to Dewey (1933). The key components that define inquiry are: posing a question, investigating it, creating new knowledge, communicating the knowledge, reflecting on the knowledge in relation to the question posed, and considering new questions that could start a new cycle of inquiry. Inquiry-based teaching focuses on the learner and learning (e.g., Dewey, 1938). Thus, it provides opportunities for students to be actively engaged in the construction of mathematical knowledge with deep understanding; to make connections between prior, existing and new knowledge and experience; to work and learn collaboratively; and to take responsibility for their own learning. In this study, in considering teacher's inquiry-oriented approaches, the focus was on how the teachers made sense of it in their practice and thinking. The goal was not to investigate a particular view of inquiry, but to understand what the teachers' were able to do.

Research method

This study is part of a larger nationally funded research project with a focus on elementary and secondary school mathematics teachers' thinking and use of arithmetic and algebraic word problems in their teaching. The research method involved case studies (Stake, 1995) grounded in a naturalistic paradigm (Lincoln & Guba, 1985) focused on the experiences of the participants in a natural setting to understand their realities by identifying significant patterns/themes in their thinking and actions while participating in the educational activities.

The three experienced high school mathematics teachers considered here had 16 to 20 years of teaching experience. They were from different local public schools. They were considered in their school systems to be excellent mathematics teachers and had won provincial teaching awards. They became the focus of this study during data analysis of the larger project as noted below.

Data sources for the larger project included interviews, classroom observations and teaching artifacts (e.g., lesson plans). Two open-ended interviews (two to three hours each) prior to classroom observations explored the participants' thinking and experiences with word problems in three contexts: past experiences as students and teachers, current practice, and future practice. Interview questions were framed in both a cognitive context to allow the teachers to share *their* way of thinking by providing "theoretical responses" (e.g., explicit conceptions) and a phenomenological context to allow them to describe their teaching behaviors as lived experiences (i.e., stories of actual events that embodied implicit conceptions). Classroom observations focused on the teachers' actual instructional behaviors during their teaching of topics that included word problems. For the three participants, this included lessons involving algebraic topics related to word problems such as systems of equations, which formed the basis of this

study. For each these teachers, ten lessons (60 to 85 minutes each in grades 10 and 11) were observed during a school term and audio-recorded. Post-observation discussions focused on clarifying the teachers' thinking in relation to their actions.

Data analysis for the larger project involved the researcher and two research assistants working independently to thoroughly review the data to identify attributes of the teachers' thinking and teaching based on statements/actions that reflected their judgments/intentions/ expectations/values regarding word problems and teaching with word problems. Emerging themes were validated through an iterative process of identification and constant comparison. Inquiry-oriented teaching emerged as one of the themes in terms of the ways some teachers engaged students in teaching with word problems and is the basis of this paper with a focus on the three teachers who used this approach consistently throughout their teaching. This theme was further investigated by examining the data in relation to the two research questions: What are the central features that characterize the teachers' inquiry-oriented approaches based on teaching systems of equations? What are central features in their thinking that allow them to make sense of their approaches?

The focus on teaching systems of equations emerged from the data which consisted of information on this topic for the three teachers. This provided a basis for comparison of their approaches. In this stage of the analysis, first, key components of the inquiry process and structure were identified by analyzing the structure of the teachers' instructional approaches. This included: isolating the stages of the lessons on systems of equations; highlighting the different ways in which students were engaged in each stage; comparing the stages to identify patterns/cycles and prominent features in students' engagement; and assigning themes based on patterns emerging to represent the central features of the approaches. Second, the coded data

were examined to identify central features of the teachers' thinking related to their inquiry approaches. This was guided by theory of key characteristics of inquiry instruction, for example, learner-focused, question-driven, investigation, communication, reflection, and collaboration.

Findings

The following overview of the findings highlights the inquiry-oriented approaches of the each teacher and the central features of their thinking that supported the approaches. The three teachers used aspects of all three approaches in their overall practice, but the focus here was on what was dominant in teaching systems of equation. All names are acronyms.

Inquiry-oriented approaches

Each teacher's approach consisted of central features specific to it that suggested three possible approaches to engage students in inquiry in learning the algebra concepts involving systems of equations. Ardise's approach was oriented to a *problem-solving approach.* For example, she introduced the new concept (systems of linear equations) by presenting students with a word problem framed in a real-world context. She allowed the students to work in groups on the problem to try and solve it on their own in any way that made sense to them. She led a whole-class discussion during which students shared and reflected on their attempts at a solution and the gaps in their knowledge. She built on the gaps to extend their knowledge of the concept through discourse and further problem-solving activities.

Beth's approach was oriented to a *research approach.* The approach consisted of an inquiry cycle with features that parallel a conventional research method as in the following two examples from her lessons on the concept of systems of equations: (1) Prior to learning the concept, students were required to plan an investigation of real-world graphical applications of the concept (i.e., linear or non-linear graphs that intercept); gather and analyze information in

terms of visual representations and meaning; draw and reflect on conclusions; share, discuss, and apply outcome; and extend the investigation to interpret the graphical solution. (2) After understanding the nature and use of systems of equations, students, working in groups, were required to investigate solutions of systems of linear equations to understand the process. Each group was assigned one approach and was required to plan their investigation of it; gather and analyze information from studying the solved examples; draw conclusions; reflect on and justify their findings through their own examples and counter examples provided by teacher; share their findings in a way to convince their audience (whole class) of the efficiency of their approach; and develop examples to apply their approach.

Cindy's approach was oriented to a *dialogic-discourse approach.* In this approach, the students' inquiry of the concept was driven by student-student and student-teacher dialogue during mathematical discourse. The approach consisted of a discourse-inquiry cycle initiated by students' and teacher's questions or conjectures during discourse or a predetermined inquiry task presented by the teacher. Students then investigated or reflected on examples of the concept to identify what they noticed; verified what they noticed; made and investigated conjectures; and discussed and reflected on their findings. For example, in one of Cindy's lesson on systems of equations, the cycle was initiated with students' example of a system consisting of linear function and a quadratic function. They investigated, reflected on and discussed what they noticed about these two functions. The discussion resulted in a new discourse cycle based on students' noticing and questioning: "why does the quadratic not cross the x-axis?" Although this was not Cindy's intent for the lesson, she allowed the students to pursue an investigation of it to arrive at a generalization to answer their question.

Central features of the teachers' thinking

The three inquiry-oriented approaches were supported by central features of the teachers' thinking involving: algebra concept, task, inquiry, and peer interactions. The teachers posed tasks that reflected how they made sense of the algebra concept and what students were to learn about and through it. Ardise emphasized the concept as *strategy*, Beth as *useful tool* and Cindy as *pattern and connection*. This allowed them to make sense of the tasks they posed for students' inquiry as *situated strategies* (Ardise), *meaningful experiences* (Beth) and *mathematical structures* (Cindy). They also engaged students in ways influenced by their thinking about inquiry and peer interactions. For inquiry, Ardise wanted students to *play* with the problems, Beth wanted them to *investigate*, and Cindy wanted them to *reflect*. Supporting students' autonomy and initiatives were important to them to facilitate these processes as were their sense-making of peer interaction as *source of information* (Ardise), as *collaboration* (Beth), and as *mirrors* (Cindy).

Conclusions and Implications

Inquiry-based teaching could be a challenge for high school mathematics teachers who are accustomed to teacher-centered classrooms because it requires teaching differently from how they were taught and different skills from the traditional classroom. This study provides examples of teachers who were able to adopt an inquiry-oriented approach to their teaching. While these approaches vary based on the teachers' sense-making, they have at their core key features of inquiry-based teaching. They have a surface structure that is consistent with theory, i.e., a cycle of: begin with a question, investigate, discuss, and reflect. The other aspect of the three approaches consistent with theory involves an emphasis on learning and not teaching, e.g., support for students' autonomy and initiative and students' engagement in interactions with peers and the teacher; an emphasis on students' experience (prior and current) in learning; and allowing

students to inquire into self (what they know or not know, can do or can't do) to determine direction of learning.

These three inquiry-oriented approaches afforded different types of leaning (knowledge construction) of the concepts. For example, Ardise's students had opportunities to create knowledge of different strategies to solve problems involving the concept and develop problem-solving thinking and procedural understanding in relation to the methods to solve systems of equations. Beth's students had opportunities to create knowledge of real-world sources, meaning and usefulness of the concepts, to develop problem posing, problem solving, collaborative and inquiry skills, to develop algebraic thinking and conceptual and procedural understanding in relation to structural and applied meanings of the concepts, and to develop an inquiry disposition. Cindy's students had opportunities to create knowledge of the mathematical structures and meanings of the concepts, to develop algebraic thinking and conceptual and procedural understanding in relation to structural meanings of the concepts, to become critical thinkers, and to develop collaborative skills, an inquiry disposition, and reflective thinking.

This study contributes to our understanding of the nature of and relationship between teachers' thinking and practice which is important to teacher education and professional development. It offers examples of specific features of inquiry-oriented teaching approaches and teachers' thinking that reflects their sense-making of these approaches for a central area of school mathematics. As NCTM (2014) noted, teachers need "to be skilled at using instructional practices that are effective in developing mathematics learning for all students" (p.4). This study offers examples of inquiry teaching and teachers' thinking that are oriented towards such practices. However, the nature of inquiry-oriented approaches as presented here is based on the practice of three teachers, so it is likely limited in scope. Further research of other classrooms is needed to

develop a deeper understanding of these and other ways teachers' thinking can shape inquiry in practice.

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