Principles of Productive Disciplinary Engagement: Framework for Design

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This investigation explored the ways in which the four *principles of productive disciplinary engagement* (Engle & Conant, 2002*)* may be used as a tool for informing the design of the norms, structures, and classroom features that combine to form a learning environment that supports the CCSS-M. The study examined both the instructional practices employed by the teacher and the nature of student engagement. Using the principles of productive disciplinary engagement as a lens, transcriptions of classroom videotapes, mathematical tasks, student work, and student questionnaires were used to analyze the ways that the teacher and students participated in creating the environment. Evidence illustrates several elements of classroom design that encourage students to be active participants in the mathematics classroom. Results point to the interrelated nature of the four principles and student behaviors that occur when the social configurations are arranged so that students assume some of the roles typically associated with the teacher.

Keywords: productive disciplinary engagement, mathematical tasks, five practices for orchestrating discussion, participation pattern, teacher questions, noticing, teacher-as-partner

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) has led an effort to change mathematics teaching and learning for more than twenty-years. Among the sources fueling the need for change was recognition that in an increasingly technological society, mathematics plays a central role and students were not being adequately prepared for this responsibility. Our global society demands now that students are able to think, reason, and problem solve in addition to developing skills related to computational accuracy (Schoenfeld, 2013). Students are expected to understand mathematics not only as they master facts and procedures, but to see connections among multiple representations while building interpretive frameworks to make sense of their experiences (Engle, 2011).

The Common Core State Mathematics Standards (CCSS-M, 2010) provide an opportunity to reenergize the efforts of NCTM. Although these standards do not dictate curriculum nor pedagogy, the emphasis they place on student reasoning and communication challenges the traditional method of delivery, wherein teachers model procedures and students use the procedures in repetitive fashion (Lampert,1990; Ball, Goffney, & Bass, 2005). Supporting students in a way that encourages a belief in their own efficacy and a positive disposition toward mathematics, necessary for successful implementation of CCSS-M,, demands teacher reflection regarding the vision of good instruction and the related classroom culture that supports it (Hill, Rowan, & Ball, 2005).

When one considers classroom culture, teaching mathematics in a way that is consistent with the Common Core State Standards includes more than teaching mathematical content. The Standards for Mathematical Practices are an integral part of the Common Core

State Standards. The first three practices: make sense of problems and persevere in solving them; reason abstractly and quantitatively; and construct viable arguments and critique the reasoning of others, focus on making sense of problems and solutions through the process of logical explanation as well as through probing the understanding of others as students construct arguments, identify correspondences among approaches, and explore the truth of conjectures.

Creating Supportive Environments

If social relations and communication are considered to be essential elements of learning, then the environment that supports interaction must be carefully considered. Research has identified "design principles" or "principles of learning" that capture key theoretical ideas underlying innovative learning environments and provide guidance so that others can recreate them (Boaler & Staples, 2008; Tarr et. al., 2008; Kazemi & Stipek, 2001; Chapin & O'Connor, 2007; Goos, 2004; Hiebert & Wearne, 1993; Silver, Smith & Nelson, 1995). Of particular interest to the study being proposed herein is the Productive Disciplinary Engagement framework (Engle & Conant, 2002). Presented as a theory, the *principles of productive disciplinary engagement* were proposed in response to a challenge to the design-based research community that included a request for a consensus on a small set of common principles that research suggested were critical for supporting effective learning environments. The *principles of productive disciplinary engagement* were presented as a proposal to members of the research community as a set of principles that they likely shared. Thus, the goal of Engle and Conant (2002) was " to abstract principles that could apply across learning environments in ways that could inform both the design of a wide range of new learning environments as well as research about existing ones" (Engle, 2011).

Consistent with the sociocultural and situative perspective of learning (Greeno, 1989, 1991; Lave & Wenger, 1991; Rogoff, 1994), Engle & Conant anchored their framework on the goal of "explaining students' deep involvement in and progress on concepts and/or practices characteristic of the discipline they were learning about" (Engle & Conant, 2002, p. 400). As an organizer for thinking about instruction that supports *productive disciplinary engagement*, four principles were specified: authority, accountability, problematizing, and resources. These principles, described in the paragraphs that follow, provide the basis of the proposed study.

The first principle, authority, reflects the idea that in order for students to become genuinely engaged in problems, they must have intellectual authority to do so. As learners are authorized to share their thinking, they become recognized as authors of the ideas and contributors to the ideas of others, leading to students becoming local authorities on a subject. In order to balance authority, the principle of accountability addresses the need for students to be accountable to explain their own thinking; making sense of their own thoughts in light of other people's ideas. As accountability increases, learners improve their ideas so they are ready to be challenged more thoroughly by peers, internal authorities, and finally external disciplinary authorities (Engle, 2011). The assumption is that as a learner is expected to explain the reason that his ideas make sense given the relevant idea of others, the process provides the social conditions that prompt the learner to revise his ideas for the better. Other people's ideas become resources for revising, refining, and better defending one's own.

A learning environment embodies the principle of problematizing to the extent that learners are encouraged to address problems that engender genuine uncertainty, are responsive to the learners' own commitments, and embody central aspects of the discipline. Problemetizing can be achieved by creating uncertainty regarding what to do, what to conclude, or how to justify what one is doing. Providing resources, the fourth principle, provides balance to problematizing. The provision of relevant resources that are necessary for the work may be provided insufficiently, resulting in learners being overwhelmed with the problem at hand. In contrast, if too many resources are provided, the problematic nature of the work may be reduced so that the potential for productive disciplinary engagement is lost (Engle, 2011).

Research since the original work that introduced the principles of productive disciplinary engagement has been extensive (Windschitl & Thompson, 2006; Gresalfi, Hand, & Hodge, 2006; National Research Council, 2008). Engle (2011) reviewed seventeen case studies that were explained using the principles of productive disciplinary engagement. The work to date suggests that the principles of productive disciplinary engagement appear to capture some consensus ideas within the research community related to a wide variety of respected educational innovations developed over the last twenty years (Forman, Engle, Venturini, & Ford, 2013). However, the work to date provides little guidance to teachers or teacher educators regarding ways to operationalize these ideas in the classroom. Articulating the knowledge and skills necessary for creating the kind of learning environment that implementation of the principles of productive disciplinary engagement demands has yet to be defined.

The Purpose of the Study

This investigation explored the ways in which the four *principles of productive disciplinary engagement* may be used as a tool for informing the design of the norms, structures, and classroom features that combine to form a learning environment. The study examined both the instructional practices employed by the teacher and the nature of student engagement in a seventh grade classroom over the course of one unit of study, following the implementation of intentional pedagogical practices aimed at implementing the four *principles of productive disciplinary engagement* during the initial half of the year. The guiding assumption is that for most students, the extent of their engagement in personal thought and the thinking of peers defines their learning. Further it is assumed that when all four of the *principles of productive disciplinary engagement* are realized together in the learning environment, *productive disciplinary engagement* has been achieved.

Learning to talk with peers regarding the discipline is critical, and depends on specific teacher practices to encourage this kind of behavior. Although Engle & Conant's work provided a synthesis of design features that were highlighted in individual research studies, this study adapts their framework as a practical tool for use by a classroom teacher in the design of the learning environment. Supporting teachers in a way that enables them to encourage student learning by creating environments that foster communication and mathematical reasoning, consistent with the CCSS-M, Mathematical Practices calls for a great deal of learning on the part of teachers. Transforming teachers' knowledge, beliefs, and habits of practice will require professional development that can lead to changes in the judgments and complex decisions that teachers make on a moment-by-moment basis. If opportunities to develop new levels of awareness and knowledge are to be provided, research that decomposes effective practices and positions them in a way that professional developers may present them to teachers will be crucial to the successful implementation of the consensus of ideas that research on this subject has captured (Grossman, Hammerness, & McDonald, 2009).

THE RESEARCH QUESTION

This study examined the instructional practices and the nature of student participation in a seventh grade mathematics classroom over the course of one instructional unit in the second half of a school year, following the implementation of intentional pedagogical practices aimed at implementing the *principals of productive disciplinary engagement* during the initial two quarters of the year.

The study examined the following research question:

- In what ways are the principles of productive disciplinary engagement: 1) evident in the instructional practices implemented by the teacher and 2) enacted by the students?

METHOD

Participants and Context of the Classroom Study

This study focused on the students and teacher in one seventh- grade mathematics classroom in a suburban school. Each of the lessons that together comprise one mathematics unit from the seventh grade *Connected Mathematics* unit focused on probability was captured. The unit of study included fifteen lessons and took place over a four-week period.

The teacher who also served as the researcher and the students in the seventh period, Pre-Algebra class were the subject of this investigation. The nineteen students in the class were Caucasian and represented a range of socio-economic conditions that included approximately 10% who qualified for free and reduced lunches. Four students had Individualized Education Plans (IEPs). The teacher/researcher was a doctoral candidate in Mathematics Education.

DATA SOURCES AND ANALYSIS

The examination of an instructional environment required a description of an amalgam of pedagogical features and student behaviors. Answering the research questions required a close examination of the classroom so as to determine the ways that students and the teacher interacted to construct the environment. There were several data sources that were used: verbatim transcripts of video recorded lessons, mathematical tasks, and student work that was produced as a result of the completion of assessments or was used during whole group instruction. In addition, a student questionnaire served to triangulate the data gathered by the teacher/researcher.

Video of Instruction

Video records of daily, seventh-period, classroom lessons, transcribed verbatim, in their entirety, were the primary source of data for this study. The first phase of the data coding included highlighting every instance of the principles of productive disciplinary engagement that were apparent in each of the video transcriptions using operational definitions defined prior to the start of the investigation. Review of the transcript and modifications to the operational definition occurred iteratively until each transcript had been reviewed exhaustively.

Mathematical Tasks and Student Work

Another source of data that was used in the study included the tasks posed to students. Because the tasks chosen contributed to the establishment of problematizing within the instructional environment, they were considered carefully. The tasks were selected to: 1) meet the criteria of a task of high cognitive demand, 2) engender genuine uncertainty within students, and 3) reflect mathematics that is part of the CCSS-M content standards for seventh grade.

Each task that was assigned to students was collected for later analysis. Student work was not coded, but was collected and used to provide clarity related to whole class discussions and assessments. The tasks posed, that contribute to the establishment of problematizing within the instructional environment, were coded using the Math Task Analysis Guide (Stein, Smith, Henningsen, Silver, 2000).

Student questionnaire

A student questionnaire that sought to gather student perceptions related to the classroom instructional practices and the classroom environment was given to students at the conclusion of the data collection process.

RESULTS

The analysis of the verbatim transcriptions of the fifteen classes that were part of this study indicates that *productive disciplinary engagement* was accomplished. Every lesson includes examples of students assuming authority through atypical participation patterns. In addition, students were held accountable by peers as well as the teacher, and problematizing was accomplished using a high percentage of tasks of high- cognitive demand. Resources were strategically selected to balance the challenge of each task. One exemplar is offered in the following paragraphs to illustrate the reflexive relationship among the *principles of productive disciplinary engagement.*

In the segment shown in Figure 1, the typical IRE (Mehan, 1979) participant structure was replaced by students assuming consecutive turns of talk. In this seventh lesson of the study, the teacher has selected Dominique to come to the document projector to describe her tree diagram, representing the following problem. "Suppose that you spin the pointer of a spinner at the right (having 2 colors) once and roll the number cube. The numbers on the cube are 1,2,3,4,5,6. Make a tree diagram of the possible outcomes resulting from a spin of the spinner and a roll of the six-sided number cube." (Lappan, et.al, 2014). The diagram that Dominique presented is shown below in Figure 2. Her presentation follows class time wherein students worked in small groups toward completion of the task. In addition, the transcript in Figure 1 follows several minutes of whole class discussion regarding the number of outcomes represented by her tree diagram. Dominique has expressed uncertainty regarding the number of outcomes possible. Several classmates have made the point that there are twelve outcomes.

	TEACHER	STUDENT	COMMENTARY
	Teacher: Well, how are you going to help her make sure she knows there aren't 24? Just talk to her.		The teacher places the responsibility for Dominique's understanding on the class.
		Jacob: The spinner is first. **Alex : They're the same thing.** **Lauren: It's just the same thing twice, the numbers are in a different order.**	Students assume responsibility for explaining their views to her.
	Teacher: On the first one, Dominique, you have spinner , 1 or 2, right? So you have 1 or 2 and the cube, 1 thru 6 for each one, right? So you have 12 outcomes. The only difference with the second one is you rolled the dice first. ..and you had 1 thru 6..then you had spinners on each dice roll. But it's the same 12 outcomes. It's just written in a different order. The first time you did the spinning, then you rolled the dice. The second time you rolled		Teacher assumes authority and summarizes the discussion so far.

	the dice then did the spinner. I just wanted to be sure you didn't think it was two separate things. They're both right, but you only need one of them. It doesn't matter what order you do them. You guys all see that? You do.		
		Alex: Why? I don't understand. Like they're two separate things.	Expresses uncertainty.
	Teacher: This one has both the spinner and the dice in it. This includes both what happens with the spinner and what happens with the dice, right? The 1-6, these are the dice (pointing), right?		Teacher assumes authority and tries to clarify for Alex using Dominique's diagram.
		Alex: Yea, I see that . I understand that.	.
		Madison: She just did the dice backwards from the spinner.	She assumes authority and restates the logic behind each representation.
	Teacher: Right.		Interjects authority.
		Madison: If she'd just pick one it would be the same thing as the other one.	Continues to provide explanation.
	Teacher, What's your question, Alex?		Teacher tries to determine the point of partial understanding for Alex.
		Alex: Why are they linked?	
	Teacher: They're not linked. They're two separate things. The way she has them written, she wrote them two different ways, but they represent the same		Draws attention to the resource provided by Dominique to address Alex's question.

	number of outcomes. This has 12 and this has 12 (pointing). It's the same 12 outcomes written in two different ways. You don't see it.		
		Lyla: Ok, it's like she said before. It's like if she started with the spinner first then went on to the dice or the second one she started with the dice then went onto the spinner.	Student assumes authority and restates the logic without encouragement from the teacher.
		Ute: For the spinner, if you land on one you can get any number 1 thru 6 but then the second one is say it's the other way around it lands on 1 it says spinner could land on either 1 or 2. They're the same exact thing, they're just (inaudible) different . (several students finish his sentence)	Student summarizes the discussion, assuming authority with no prompting.

Figure 1- Topically Related Segment Regarding Dominique's Tree Diagram-Day 7

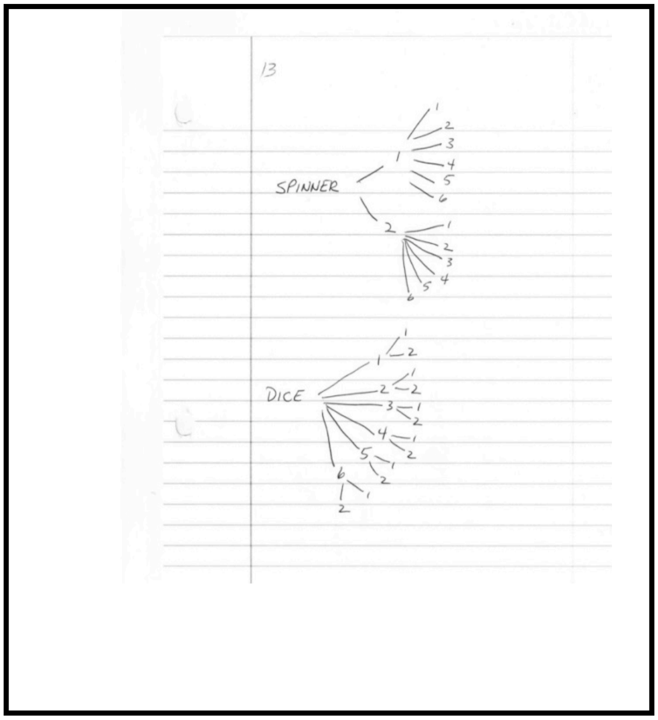

Figure 2- Dominique's tree diagram

Throughout the segment, the non-traditional role of the teacher and students is apparent. The teacher neither speaks every other turn, nor is she the sole authority. The turn-taking pattern itself indicates a change in power, with students consuming more of the talk turns.

Critiquing the reasoning of peers is an indicator of student authority, and is apparent in this segment. As students developed a sense of agency, they assumed some of the roles that are traditionally held by the teacher including the evaluation of ideas. In Figure 1(Day 7), all of the bolded turns indicate student turns wherein the primary function of the talk was assessment of the information. Seven students demonstrated intellectual courage and engaged in Dominique's thinking. Jacob, Alex, and Lauren all make a contribution to the discussion noting the difference in the order represented. Lauren, in particular points to the idea that the representations reflect the same events. In making this evaluation, she has necessarily immersed herself in Dominique's diagram, truly attempting to analyze her thinking, Alex's question, and making a connection between them and the diagram. Later, Madison attempts to restate the logic presented earlier; assuming authority without prompting from either the teacher or students. Likewise, Lyla assumes authority and restates the logic without prompting. Finally, Ute assumes authority by explaining and summarizing the discussion. All of these students provide examples of student agency. In addition, they are holding one another accountable to the community for mathematical sense-making.

Common to the examples related to critiquing the reasoning of peers in the study were students having the time and agency to talk to each other about an idea or representation. Extended examples of students critiquing peer reasoning occurred during the wrap up of this task and others while students presented solutions. The topically related segment that addresses Dominique's tree diagram begins about thirty-two minutes into the class; after small groups have had the opportunity to discuss the task. Time in small groups offered an opportunity for each students to consider his own solution before engaging in the thinking of the class.

The selection of the solutions to be presented to the class was an essential element in encouraging other students to critique peer thinking (Stein, Engle, Smith, & Hughes, 2008). The tree diagram that Dominique designed had attracted the attention of the teacher as she monitored student progress in small groups. Dominique's tree diagram offered the opportunity for the teacher to formatively assess the capacity of students to understand the tree diagram model. By selecting Dominique to present her idea, the teacher encouraged class discussion and challenged students to explain their views of the representations.

The tree diagram served as a resource that helped Dominique make her thinking public and afforded her classmates the opportunity to explore mathematical meaning as they investigated and discussed the possible significance of the representation. In addition, the students served as resources for each other in this segment. For example, Lyla assumes authority and restates the logic behind each representation. In so doing, she offers the other students an idea to consider; a resource for further consideration. The tree diagram resource balanced the challenge associated with the task. As students engaged with the visual representation that the tree diagram provided, they were afforded access to the challenge of the task. The resource offered each student a thinking tool as well as a common public thinking tool as Dominique shared her tree diagram.

Finally, problematizing was prominent in this segment. The task itself presented students with the opportunity to air uncertainty and to make public the construction of meaning using the tree diagram. Students persevered in problem solving to arrive at a solution that was accepted by their peers, and were positioned as decision makers, resulting in the authentic need for classroom discourse. The task selection, combined with careful teacher monitoring of student thinking as they worked in small groups, allowed for a robust discussion focused on the tree diagram that Dominique had produced. Both the resource and the task itself offered students something worthy of a discussion. The discussion allowed students to hold one another accountable and enabled students to assume authority, traditionally held by the teacher. The segment provides one example of the presence of all four principles synchronously. I argue that I could make this point for every lesson in the study.

The integral nature of the task to the principle of problematizing is apparent in the segment discussed. Because a well-chosen task provides some degree of uncertainty, students necessarily were engaged in talking about their mathematical thinking. The strategic choice of representations for class discussion provided an opportunity to both air uncertainty and to make public the construction of meaning among representations. Students necessarily persevered in problem solving in order to arrive at solutions that were accepted by their peers (CCSS-M, 2010).

Results of the student questionnaire point to the awareness that students developed regarding the types of tasks they were provided, and the expectation to persevere in problem solving. Question 5 asks students to circle a response ranging from *strongly disagree, disagree, agree, or strongly agree* related to the phase, "Encourages students to stop working when the work gets hard." Thirteen students voted *strongly disagree,* 4 voted *disagree,* 1 voted *agree,* and 1 voted *strongly agree.* Likewise, regarding the phrase, "Gives us work in class that is challenging", the overwhelming majority recognized that tasks were not routine; 11 voted *agree* and 5 voted *disagree (Not all students answered the question.)* Offering tasks that were of low cognitive-demand, but simply too difficult for students might result in a similar student response. However, I contend that because I have demonstrated that the majority of tasks were of high cognitive demand; that was not the case. Question 8, "Wants us to become better thinkers, not just memorize things" also points to the selection of tasks. Twelve students *strongly agreed* and 7 *agreed*. Students apparently recognized that thinking was valued and that struggling to complete the task was acceptable.

The results indicate that a high percentage of tasks utilized in this unit of study fall within the categories designated as high cognitive demand: either *procedures with connections* or *doing mathematics*. Specifically 9% of tasks were of low cognitive demand, and 91 % of tasks were classified as tasks of high cognitive demand. Within those tasks of high cognitive demand, 52%

were considered as *procedures with connections* and 39% were classified as *doing mathematics*. All three tasks having low cognitive demand were assigned for homework as procedural practice. Several of the tasks that were classified as *doing mathematics* were from the Connected Mathematics curriculum (Lappan et.al, 2014), while others were integrated from released NAEP items or items from the QUASAR study (Silver, Smith & Nelson, 1995).

CONCLUSIONS AND FUTURE RESEARCH

From the standpoint of a practitioner, the principles of productive disciplinary engagement may be very useful as a design tool. Results of this study indicate that it is possible to use the *principles of productive disciplinary engagement* as a tool for the design of a learning environment. By using these principles during the planning, enactment, and reflection of lessons, participation patterns were influenced. The instructional practices employed by the teacher aimed at supporting a culture where student thinking was valued and utilized as a tool for productive work were apparent in the results. The use of Accountable Talk (Chapin & O'Connor, 2007) and the five practices for orchestrating productive discussion (Stein, Engle, Smith, &Hughes, 2008) as well as the careful consideration of questions (Boaler & Brodie, 2004) encouraged students to engage in robust discussions. The selection of mathematical tasks of high cognitive demand provided features of mathematics for students to notice and critique in collective discussion.

It will not be enough for teachers to learn the instructional tools related to enacting the principles of productive disciplinary engagement described in this document. There are several teacher attitudes that will impact the implementation of the four principles. First, implementing these instructional practices demands flexibility on the part of the teacher. The common practice of teachers using guided notes and exact lesson plans from the year prior are indicative of a view that planning is a static process and that students learn the same material, the same way, at the same time. Planning the implementation of tasks is useful from year to year, but the plan for the entire lesson is much less predictable. I argue that fully embracing the principles of productive disciplinary engagement demands that teachers view planning as a dynamic process, and allow for some flexibility related to timing, so that teaching is responsive to students.

This study provides one glimpse of a classroom in which the principles of productive disciplinary engagement were evident. The decomposition of some of the supporting teacher and student behaviors may provide information for studies of larger size that might further this work.

Creating a learning environment that supports the students in productive disciplinary engagement is a practical challenge for teachers and teacher educators. Although the application of these design principles have been investigated in educational environments, including the one in this study, the articulation of the way to create the environment has not been articulated. This study helps to define the teacher and student behaviors that are evident once the environment is created, however, it doesn't address its creation at all. It leaves the reader wondering what happened from the first day of school until the study commenced. Did students come already knowing how to participate in the way that is described in this study? The way to develop the environment can be inferred, in part, from the study, but much more work is needed to decompose the teacher practices in a way that might allow teachers and teacher educators to apply these principles in a variety of educational settings with teachers of varying background and experience.

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