Examining Students' Academic Language Skills in Narrative Solutions

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Abstract

This study examines the use of academic language in formal written mathematical solutions by high school students (n=18). The author discusses the students' use of mathematical vocabulary, syntactic complexity, linking words, and how it relates to indicators associated with content understanding.

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Academic language is used by learners to understand and communicate in academic disciplines (Cummins, 1981; Solomon & Rhodes, 1995) often through conveying the understanding of content. As there is growing interest in the integration of socio-linguistic interactions in mathematics education, educators have come to recognize the importance of developing pedagogy for facilitating productive classroom discourse (Smith & Stein, 2011; Stein, 2007), where students are provided with opportunities to better understand the various aspects of language and mathematics. Efforts to train students in actively using academic language to learn mathematics are often influenced by the view that academic language facilitates communication within the practice of formal mathematics, and that learning mathematics provides a similar experience for students (Sfard, 2000).

Need for the study

There is a language we call mathematics, there is a language we call academic language, and then there is what we may call everyday language with regard to languages neither academic nor mathematical. Granted that language plays the essential role of representing, transmitting, and developing thinking and knowledge, all languages used by students in mathematics classrooms may be characterized as *academic,* since learning is occurring in an educational setting (Schleppegrell, 2010). Due to the common understanding that academic language is the language of the classroom, research of academic language is often reserved for those not proficient in English, as English is the primary language of U.S. classrooms (Snow & Uccelli, 2009). As such, it is not surprising that most research on academic language focuses on defining and developing the construct of pedagogies for English language learners.

Many mathematicians support the notion that mathematics is a language (Ford & Peat, 1988). Yet, if the language used by students and teachers to communicate in the classroom is the academic language of mathematics, how is it different from mathematical language? As the study further examines academic language by looking at different student skills of using specialized language in mathematical communication, we can discover a mechanism for language development that is closely associated with the language of mathematics – including symbols, notations, quantifiers, and others collectively called mathematical registers. This implies that all leaners of mathematics use and process mathematics as a second language. Therefore, language supports in classrooms do not apply exclusively to English language learners. Mathematics is a language to be mastered in addition to English, and that all students need language supports targeting mathematical registers while leveraging a common language in the class (e.g. English) for socialization (Schleppegrell, 2010).

In the study, college-bound mathematics learners (i.e., 12th graders) whose primary language in the classroom was English were given two mathematics problems and asked to construct narrative solutions responding to them. The study investigates how students use language – not necessarily academic language – to process and express mathematical ideas and see how their language skills relate to mathematical achievement. Our research question was: How do students' use of written language in mathematical solutions relate to indicators associated with content understanding?

Theoretical Perspectives

Symbolization and Narrative Writing in School Mathematics

The fundamental need in math is to be able to represent the relationship between a sign, number, or value. Certain ideas and concepts can only be clearly illustrated by the creation and use of symbols (Cai & Knuth, 2011). Symbolic notations in mathematics not only enhance understanding but also provide a universal manner in which math functions or illustrations of sequence can be shown (Gowers, Barrow-Green, & Leader, 2008). Measuring and representing math relationships through the use of symbols not only serve to simplify this process, but also give a clearer understanding of math concepts (Morgan, 1996, 2005).

The language of school mathematics involves both spoken and written forms. Students engage in dialogic interactions with peers about mathematics, studying mathematical texts in which symbolization—which can include numbers, operations, syntax, and conventional grammar of math relationships—is proposed and strictly enforced (Moschkovich, 2007). Mathematical texts can also be characterized as rhetorical, narrative, or argumentative (Pugalee, 2005), emphasizing the use of language in mathematics.

As it is, the language of mathematics has yet to become a part of students' colloquial language due to the specialized ways that mathematics registers with certain syntactic preferences (Schleppegrell, 2010). This is particularly true of written mathematics (McCarthy, 2008). Over the years mathematics has developed in the structural coherence and consistency of written expressions, especially in the collection of mathematical syntax with regard to elements, operators, and relations (Pimm, 1987; O'Halloran, 2000). Pimm (1987) elaborated the syntax of written mathematical forms, explaining that symbols are combined to represent mathematical relationships and ideas and that algebra involves a great deal of symbol manipulation following the grammar of symbolic expression.

Vocabulary

Vocabulary is a multi-layered concept involving subject-specific meanings that vary according to content, academic terms utilized across disciplines, and definitions applying to specific disciplines. There exist the specialized patterns of language used in academic subjects (Lemke, 1988). As students build their knowledge of mathematics, they learn the language of words that mathematicians or scientists use to talk about their worlds (Templeton, Bear, Invernizzi, & Johnston). While vocabulary may be taught in the context of a system of words and phrases with subject specific meanings (Cummins, 1981), it may also refer to general academic vocabulary used across disciplines (e.g., compare, analyze, evaluate).

Academic vocabulary is mostly categorized into (1) key content words; (2) words and phrases that play a role as functional language such as in explaining a procedure and interpreting a mathematical representation; and (3) English morphology (Pierce & Fontaine, 2009). Subject-specific words also comprise a part of mathematics vocabulary. The school mathematics register is most readily identifiable in highly specialized vocabulary—including specific mathematics terms such as parallelogram, polygon, trigonometry, quadratic equation—that are rarely used in daily life and not usually confused with other words in meaning. On the other hand, words like right angle, factor, even/odd, or difference are used extensively in general language but also have precise meanings in mathematics (see Thompson & Rubenstein, 2000).

Syntactic Complexity

The writing process involves syntactic complexity with the use of complex linguistic devices (Alamargot & Chanquoy, 2001). Syntactic complexity is evident in writing in terms of variation and sophistication of sentence structures (Ortega 2003; Lu, 2010, 2011). A large number of different measures have been proposed for characterizing syntactic complexity in writing; Ortega (2003) categorized the measures into five different types (see Table 1). (1) The first type consists of three measures of production length at the clausal, sentential, and T-unit levels: mean length of clause (MLC), mean length of sentence (MLS), and mean length of T-unit (MLT). (2) The second type consists of a sentence complexity ratio (clauses per sentence, or C/S). (3) The third type consists of four ratios reflecting the amount of subordination: T-unit complexity ratio (clauses per T-unit, or C/T), a complex T-unit ratio (complex T-units per T-unit, or CT/T), a dependent clause ratio (dependent clauses per clause, or DC/C), and dependent clauses per T-unit (DC/T). (4) The fourth type consists of three ratios measuring the amount of coordination: coordinate phrases per clause (CP/C), coordinate phrases per T-unit (CP/T), and a sentence coordination ratio (T-units per sentence, or T/S). (5) The fifth type consists of three ratios considering the relationship between a particular syntactic structure and larger production unit: complex nominals per clause (CN/C), complex nominals per T-unit (CN/T), and verb phrases per T-unit (VP/T).

Table 1

The Fourteen Syntactic Complexity Measures (Lu, 2010, p.479)

Measure		Definition
Type 1: Length of production unit	Mean length of clause	# of words / # of clauses
	Mean length of sentence	# of words / # of sentences
	Mean length of T-unit	# of words / # of T-units
Type 2: *Sentence complexity*	Sentence complexity ratio	# of clauses / # of sentences
Type 3: *Subordination*	T-unit complexity ratio	# of clauses / # of T-units
	Complex T-unit ratio	# of complex T-units / # of T-units
	Dependent clause ratio	# of dependent clauses / # of

		clauses
	Dependent clauses per T-unit	# of dependent clauses / # of T-units
Type 4: *Coordination*	Coordinate phrases per clause	# of coordinate phrases / # of clauses
	Coordinate phrases per T-unit	# of coordinate phrases / # of T-units
	Sentence coordination ratio	# of T-units / # of sentences
Type 5: Particular structures	Complex nominals per clause	# of complex nominals / # of clauses
	Complex nominals per T-unit	# of complex nominals / # of T-units
	Verb phrases per T-unit	# of verb phrases / # of T-units

Linking Words (Logical Connectors)

Logical connectors are used to join or connect two ideas that have a particular relationship. These relationships can be sequential (time), adversative (opposition and/or unexpected result), conditional, or a reason and purpose. Sentence connectors serve a significant semantic function in written English. Though they have no meaning of their own and their usage is predetermined by the sentences in which they are used, they often determine the logical relationships of the sentences they connect. While one connector creates a set of suppositions about the truth or nature of the two sentences it connects, another connector can create a different set of suppositions about the same two sentences. This means the semantic interpretations of two sentences may vary depending on the sentence connector used in connecting them. In this study, the following logical connectors (Biber et al., 1999) were used as research targets.

Table 2

Classifications of Linking Words (Narita, Sato, & Sugiura, 2004, p.1172)

Semantic Category	Logical Connectors
Enumeration/Addition	First, next, in addition, similarly, also, furthermore, likewise, moreover, besides
Apposition	For example, for instance, that is (to Say), like
Result/Inference	Therefore, thus, then, as a result, to conclude, it follows, hence, of course, consequently, finally
Contrast/Concession	On the other hand, in contrast, however, yet, instead, nevertheless, still

Methods

Task

The design of the study included an untimed task of solving two algebra problems from a pool of NAEP mathematics problems, each requiring a constructed response using these mathematical reasoning or problem-solving strategies: (1) analyzing conjunction and disjunction of inequalities; and (2) explaining and generalizing a given number pattern. Students were asked, in isolation, to solve two algebra problems (see Appendix A) and present written-out solutions which were later graded. Researchers evaluated the solutions according to these following key components: syntactic complexity, use of vocabulary, use of linking words, and use of mathematical symbols. Lastly, the study analyzed how the elements of language in the solutions related to various performance indicators including the score for the solution, course grade, and teacher evaluation.

Instructions for the task stated: “Solve the problem and provide a detailed and descriptive solution including symbols, expressions, equations, diagrams, words, complete sentences (preferred), and paragraphs that you believe best constitute a logical body supporting your solution and demonstrating your thinking and reasoning. Please assume that those who read your solution will have the same level of mathematical competency as yourself.” Prior to the assignment separate letters were sent home asking parents or guardians to not provide the students with help, informing them that all student solutions are for teacher research and no grades are given for the assignment.

Participants

Participants of the study were 12th grade students (n = 18) enrolled in a college algebra class in the spring of 2013, at a private high school in a major city of a southern state. Of the 21 students in the class three students chose to opt out of the study. There were ten Caucasian students, four Asian students, two Latino students, and two African American students in the study. All students stated that English was their primary language and the levels of their second language were described as elementary proficiency (n=7), limited working proficiency (n=9), and working proficiency (n=2). The languages included Spanish (n=10), French (n=3), German (n=2), Chinese (n=2) and Korean (n=1). Finally, the teacher was a Caucasian female in her 30’s with seven years of teaching experience and a master’s degree in mathematics education, currently working towards getting her doctoral degree in mathematics education.

Data Analysis

Other elements were the teacher’s descriptions of the participating students’ mathematical competency in response to our following request: “Please describe the student’s mathematical ability and level of content understanding as well as a rating of the student’s mathematical ability: 1(poor), 2(below average), 3(average), 4(good), 5(excellent), or 6(exceptional). Please do not consider student attitudes or work ethic in your assessment of mathematical competency." Additionally, the students' current course grades were used for analysis.

While examining the students' narrative solutions, two scorers scored the solutions against the scoring guide provided by NAEP with a scale of 0 to 4 points. When there was discrepancy in scoring, a third scorer reviewed and reached a consensus score. The elements of our analysis included syntactic complexity and the use of mathematical symbols, logical connectors, and mathematical symbols/notations. Syntactic complexity was measured by using a computational system called the L2 Syntactic Complexity Analyzer (Lu, 2010), for a composite score of syntactic complexity with scores for sub-components. This system was able to automatically analyze 14 different measures of syntactic complexity, covering (1) length of production units, (2) amounts of coordination, (3) amounts of subordination, and (4) degree of phrasal sophistication and overall sentence complexity. The following table illustrates two sample solutions with the scores (see Table 3) and the analysis of syntactic complexity (see Figure 1).

Table 3

Sample Solutions to the First Problem and the Consensus Scores

Label	Solutions	Score
Text #1	(10n + 5)^2 = 100n^2 + 100n + 25 = 100(n^2 + n) + 25 Two terms are multiples of 100. Third term is 25.	4 out of 4 point
Text #2	Any positive integer times itself will always have 25 at the end of the answer because if 15^2 = 225, 15 x 15 = 225 It ends in 5 so it will have 25 in the answer. Example 45 x 45 = 2025. 5x5 = 25 That's how you get 25 at the end of each answer.	1 out of 4 point

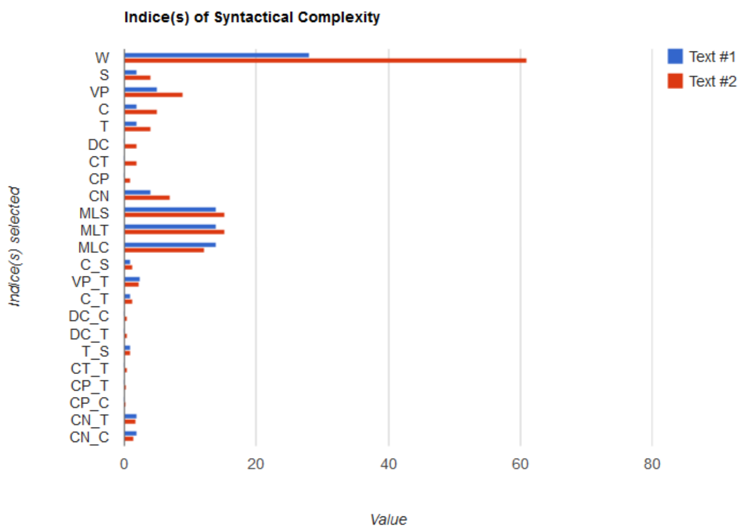

Figure 1. Graphical representations of two solutions on elements of their syntactic complexity

Vocabulary use was analyzed by counting academic vocabulary and special terms of mathematics. The usage of linking words was analyzed by frequency of the presence of appropriately connected points making a logical argument. Additionally, the use of mathematical symbols was evaluated by the frequency of mathematical syntax errors; for example, "$\sin\theta^2+\cos\theta^2 = 1$" was coded as an incorrect form, because the form "$\sin^2\theta+\cos^2\theta =1$" was more acceptable.

The study provides a table of mathematical performance with scores of the solutions, course grades, and teacher ratings (both numeric and descriptive). The course grades were the current course grade and the course grade of previous school year. Examples and counter-examples of each element of academic language, in the students' narrative solutions as well as the teacher's descriptions of students' mathematical abilities and content understanding, were shared and revised for consistent coding (Gibbs, 2007) and categorizing in various consensus meetings. Although there were some instances of vague wording in the teacher's evaluative statements, the teacher later provided alternative synonyms that provided clearer meanings for interpretation.

Key Results

Our preliminary findings indicate that solutions with high scores demonstrated more linking words than solutions with low scores. Most solutions with high scores demonstrated a moderate level of vocabulary use and syntactic complexity. In fact, syntactic complexity generally had little impact on the mathematical soundness of solutions as indicated by scores, as some students had positive teacher evaluations and high course grades demonstrating high syntactic complexity. Although the narrative solutions with rich vocabulary did not correlate much with scores of high content understanding, teacher descriptions, and current course grades, some students with positive teacher descriptions and high course grades did use rich vocabulary.

Most noticeable was that all of the students ($n = 6$) who used multiple linking words and did well on the problems were identified as successful mathematics students (e.g., “great thinkers or doers of mathematics”) by the teacher, with their course grades in the high range of 88 to 98 (the course grade average of the class was 81.2%, ranging from 61 to 98). Most students who demonstrated high syntactic complexity were described positively by the teacher. For example, such students with course grades ranging from 81 to 88 were described as “hard working, deserving good grades, thoughtful, attentive.”

Ultimately, there was no consistent pattern found between grades, teacher evaluations, solution scores, and the level of syntax errors. Other noticeable findings included a clear gap among students with regard to level of vocabulary use.

Reflections

The use of linking words appears most related to content understanding: those who were strong in the use of linking words received positive teacher descriptions about their math ability and performed well on the tasks. Those who were successful at creating solutions, however, were not necessarily more skilled in their use of vocabulary and writing with high syntactic complexity than those who were less successful. With a limited number of participants, the findings are not generalizable. Therefore, a more rigorous statistical analysis is undergoing consideration, especially with regard to variant levels and components of syntactic complexity. Nonetheless, the findings seem to lend credence to the view that mathematical thinking and reasoning has little to do with the syntactic complexity of written English, including the use of vocabulary.

On a different note, it is interesting that the teachers' perception of high performing students (i.e., their ratings and descriptions) seem to relate to the level of students' facilitation of (academic) language, as demonstrated by their high syntactic complexity and use of vocabulary.

The essential goal of school mathematics is to develop problem-solving skills and build foundational skills and interest in advanced mathematics. Thus, as we ask mathematics teachers to incorporate academic language in the teaching of mathematics, we may need to be clearer about the integration of language as a significant tool in socio-linguistic interactions. We may also need to reconsider the teacher's responsibility to provide language support in mathematics classrooms, rather than language instruction. In the case of graduate students in mathematics for whom adept skills in speaking and writing in mathematics are essential to their professional careers, understanding the use of academic language is perhaps the most urgent aspect of mathematics education. Future research is certainly necessary regarding the process and nature of mathematical learning (i.e., secondary mathematics, college mathematics, abstract mathematics, etc.), for a greater degree of certainty in identifying the uses of language that facilitate content understanding.

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Appendix A.

Problem	Difficulty
1. Question A: If x is a real number, what are all values of x for which x > -3 and x < 5? Question B: If x is a real number, what are all values of x for which x > -3 or x <5? Barbara said that the answers to the two questions above are different. Dave said that the answers to the two questions above are the same. Which student is correct? Explain why this student is correct. You may use words, symbols, or graphs in your explanation.	Hard (14.69% Correct)
2. This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work. $15^2 = 225$ $25^2 = 625$ $35^2 = 1225$ The examples above suggest the following statement. When a positive integer that ends in the digit 5 is squared, the resulting integer ends in 25. Explain why this statement is always true. (Hint: $(10\,n + 5)^2 = ?$)	Hard (2.33% Correct)

Source. U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress Mathematics Assessment. (NAEP, 1992, 2013)