Making Mathematical Practices Explicit in Discourse: Experienced and Beginning Instruction

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Abstract

Making mathematical practices explicit while still providing students authentic opportunities to engage in these practices can make instruction more equitable. Through examining whole class discussions in four inquiry-based mathematics classrooms, this study compares how experienced and beginning teachers make mathematical practices explicit in discourse and what this might reveal about the knowledge and skills entailed in doing so. The experienced teachers made mathematical practices explicit in all of their class discussions through reprising students' activity. They also make key learning practices (not specific to mathematics) explicit. In contrast the beginning teacher made learning practices explicit but did not make mathematical practices explicit in discourse. Reflections from the beginning teacher's interview suggests possible explanations for these differences, centering around mathematical knowledge for teaching mathematical practices, the demands of noticing of student engagement in mathematical practices, and the learning needs of students new to inquiry-based mathematics instruction.

Introduction

Mathematical practices, like constructing arguments, are central to learning and doing mathematics (Common Core State Standards, 2010). To learn mathematical practices, students need opportunities to jointly participate in them through classroom interactions (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2011). But simply providing these opportunities may not be sufficient to support all students in learning them (RAND, 2003) and to meet the access and equity principle in the NCTM *Principles to Action* (2014). In the spirit of Chazan and Ball's (1999) argument around "beyond being told not to tell", this paper takes up the tension between concerns about explicitness and issues of equity and access to learning mathematical practices. Through analyses of whole class discussions in four middle and high school mathematics classrooms in the first month of school, I investigated how teachers made mathematical practices explicit in classroom discourse in ways that navigated this tension. Through exploring the contrast between beginning and experienced instruction around making mathematical practices explicit, this paper examines some potential demands of making complex disciplinary practices explicit for students.

Background and theoretical perspective

This study is grounded in a conceptualization of mathematical practices as more than ways that individuals (students, mathematicians, mathematics users) think about and do mathematics; mathematical practices, such as constructing arguments, are culturally organized practices that emerge out of social interaction (Scribner & Cole, 1981; Moschkovich, 2013; Cobb et al., 2011). Drawing on this perspective, students learn, or "appropriate" (Moschkovich, 2004; Rogoff, 1991), mathematical practices through jointly participating in mathematical practices in social interaction. Appropriating a mathematical practice involves learning what's often implicit, such as the meaning of particular symbols, signs, or terms or the goals of engaging in particular practices (Moschkovich, 2004). But normative ways of participating in mathematical discourse may be novel to certain groups of students, raising concerns about equitable access if these ways of working are not made explicit (Ball et al., 2005; Boaler, 2002; Delpit, 1988; Ladson-Billings, 1995; Lubienski, 2000). In response to concerns that problem-centered instruction might exacerbate inequalities due to some students' inexperience with particular discourse practices, Boaler (2002) argued that equitable instruction must give students access to learning these central mathematical practices. Ball, Goffney, and Bass (2005) also argued that providing explicit access to learning mathematical practices is a matter of equity, if only some students are able to learn what might be implicit. Research in science education has demonstrated the power of making scientific inquiry explicit for students, particularly for those students who have previously be labeled low achieving (White & Frederiksen, 1998). The math education community has cited the need to better understand how the detailed practices of teaching might make disciplinary ways of working in mathematics explicit for students to promote more equitable access to rigorous mathematics (Boaler, 2002; Jackson & Cobb, 2010).

Although there are arguments for making mathematical practices explicit and accessible, "explicitly" teaching mathematical practices as the focus of instruction, as often happens with proof in high school geometry classrooms, could also be problematic for a number of reasons (Schoenfeld, 1988; 1989). First, explicit instruction in mathematical practices could lead to the separation of practices from content (e.g., teaching proof as a separate unit in a geometry class). This misrepresents the discipline of mathematics, as mathematical practices are ways in which mathematicians engage with, construct, verify, and communicate about content (Bass, 2011; RAND, 2003). This separation could lead students to see mathematical practices as something that they only need to do in a particular unit or type of task (Schonfeld, 1989). Another potential concern about making mathematical practices explicit is that teachers might turn mathematical practices into prescriptions, in the way that has often happened around problem-solving strategies. Consider the Common Core mathematical practice standard, *MP1 – Make sense of problems and persevere in solving them*. Sometimes this practice, which is complex and non-routine, becomes a prescription in mathematics instruction, such as when students are told to follow a process like underlining key words and numbers, writing them down, using the key words to determine the operation, writing the equation, and finding the answer (e.g., Darch, Carnine, & Gersten, 1984). This misrepresents and narrows the flexible, non-routine, and rich nature of the mathematical practice of making sense of problems. A third concern around teaching mathematical practices explicitly is that such instruction might result in the teachers doing the bulk of the work of engaging in mathematical practices. This would limit students' opportunities to participate in mathematical practices, a central opportunity for students to appropriate mathematical practices (Moschkovich, 2004; Rogoff, 1990).

One potential way to negotiate this tension might be to make mathematical practices explicit or visible to the whole class as students participate in these practices in mathematical discourse. To develop a shared understanding of language, symbols, goals, and ways of engaging in mathematical practices (Moschkovich, 2004; Rogoff, 1990), instruction would need to promote joint-attention of the class to make what might be implicit explicit. In Goodwin's (1994) study of professional vision, he articulated a set of practices, such as *coding* and *highlighting*, which support those learning a profession to build professional vision, which consists of "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606). For example, *highlighting* involves "making specific phenomena in a complex perceptual field salient by marking them in some fashion" (p. 606). This suggests that the set of practices that foster professional vision might have purchase in the complex perceptual fields of classrooms. Lobato and colleagues (2013) have extended Goodwin's notion of professional vision to students' mathematical noticing, showing how the practices of *highlighting* or *renaming* particular phenomena influenced what students noticed mathematically with regards to content. Mann, Owens, and Ball (2013) also showed how the moves of *naming* and *highlighting* made mathematical practices visible to elementary students. Similarly, Selling (2014) developed a framework of moves that made mathematical practices explicit as they emerged in secondary math discussions.

Even when teachers are committed to teaching mathematical practices, there may be differences among teachers' levels of explicitness and what they make explicit in classroom discourse. Through comparing experienced and beginning instruction in four inquiry-based mathematics classrooms, this study explores (1) differences in how experienced and beginning teachers made mathematical practices explicit and (2) what such differences might reveal about the knowledge and skills involved in making mathematical practices explicit.

Methods

This study draws on data from larger project investigating how teachers cultivate opportunities for mathematical practices. I conducted parallel case studies (Yin, 2009) of four inquiry-based mathematics classrooms in the same secondary school, an urban charter school in an under-resourced district in California. More than 90% of students at the school were eligible for free lunch, and the student body was racially and ethnically diverse. I conducted case studies of four classes: a 6^{th} grade math class, a 9^{th} grade algebra class, and a 10^{th} grade geometry class, and a 10^{th} grade advanced algebra class. Three teachers taught the four cases study classes. Mr. Carrera[1] taught the 6^{th} grade math and 10^{th} grade advanced algebra classes. Ms. Larson taught 10^{th} grade geometry, and Ms. Lane taught 9^{th} grade algebra. I purposefully selected classrooms to find the most productive sample to address my questions (Marshall, 1996), using prior research on what supports engagement in mathematical practices. I selected teachers that (1) believed students should be active participants in mathematical activity (Boaler & Greeno, 2000); (2) worked to support collaborative inquiry (Staples, 2007); (3) implemented cognitively demanding tasks (Stein et al., 2000); (4) facilitated class discussions (Chapin et al., 2013); and (5) were committed to teaching mathematical practices. Mr. Carrera and Ms. Larson were both experienced (23 & 7 years respectively); Ms. Lane was a first-year teacher. Although the contrast between beginning and experienced instruction was not one of the original goals of the study when the teachers were selected, this later emerged as a theme in the findings.

The data include video-records of all whole class mathematics discussions from the four case study classes across the first month of school (n = 30). These discussions ranged from 5-45

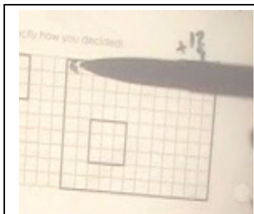
[1] All names are pseudonyms.

minutes. This study focused on instruction during the first month of school, since this time particularly relevant for understanding how teachers work to establish a classroom culture and norms for what it means to do and learn mathematics (Yackel & Cobb, 1996).

All discussions were transcribed. The analysis of the discourse then proceeded in three phases. The first phase involved identifying student engagement in mathematical practices. To determine an appropriate unit of analysis, I drew on the conceptualization of mathematical practices as socially constructed practices (e.g., Moschkovich, 2004; 2013) that can be co-constructed by multiple actors in social interaction. I conceptualize a *mathematical practice episode*, defined as beginning with the first student move that represents engagement in a mathematical practice. Each episode typically consists of one or more talk-turns, although it can also involve non-verbal moves. An episode could be constructed by an individual, but more frequently an episode was co-constructed by multiple participants. The end of an episode is determined in a number of ways: (1) the teacher reflects verbally on the interaction, (2) the class begins to participate in a different mathematical practice, or (3) the talk shifts and the class does not return to that practice. Every discussion was segmented into episodes and coded for the mathematical practice(s) in which students were engaged.

The second analytic pass involved identifying teaching moves that supported student engagement in mathematical practices. For each episode, I identified the talk turns that *initiated*, *sustained*, and *reprised* student engagement in that episode. An *initiating* talk turn is defined as a move that facilitates the beginning of student participation in a mathematical practice. These types of moves are similar to prior research on talk moves in math discussions (e.g., Chapin et al., 2013). A *sustaining* talk move occurs within a practice episode. This type of move presses on or further elicits student participation in mathematical practices. This is similar to what Staples

(2007) describes as "scaffolding the production of student ideas" (p. 173). A *reprising* move is when the teacher explicitly reflects back on student participation in mathematical practices. This type of talk is similar to what Cobb et al. (1997) call reflective discourse in that what was just discussed becomes an explicit object of discussion. Figure 1 shows an example of a co-constructed practice episode and the moves that initiate, sustain, and reprise student activity. I use colors (red, blue, green) to differentiate between moves that initiate, sustain, and reprise.



	Teacher: And Leo, where did you count nine?	Initiating move
	[Leo draws a line in the left column of 9 squares.]	
	Teacher: So put a nine on the side there.	Sustaining move
	[Leo writes a 9 on the left side.]	
	Teacher: Exactly. So what you just did Leo is label the dimensions.	Reprising move

MP1 – Make sense of problems – connect representations
MP2 – Reason abstractly and quantitatively – attend to the meaning of quantities
MP6 – Attend to precision – labeling

Figure 1. Moves that initiate, sustain, and reprise engagement in math practices

The third analytic pass involved coding the identified teacher moves to look for patterns in how teachers made mathematical practices explicit. I coded all of these moves using a combination of emergent and a priori codes (Goodwin, 1994; Lobato et al., 2013). These eight codes, along with descriptions and examples, are shown in Table 1. A second trained researcher double-coded a subset of the moves (~15%), achieving inter-rater agreement of 94%.

Table 1. Reprising move framework (Selling, 2014)

Move	Description	Example(s)
Highlighting aspects of engagement in MPs (Goodwin 1994; Lobato et al. 2013)	This type of talk reflects back on what students just did mathematically in the discussion with respect to mathematical practices, but without necessarily naming the practice. This involves talk, often coordinated with non-verbal communication to promote joint attention, such as gesture or use of a laser pointer.	"I really liked that you had- are you guys looking? These 3 right here circled [points with the laser] and you also circled the 3 in Javier's calculation"
Naming the MP(s) in which students engaged	The teacher names student engagement in mathematical practices. Sometimes naming involves the use of conventional terms to describe practices; other times, the naming involved more locally meaningful terms.	"So you know-you started to code the picture"

Making evaluative statements about engagement in MPs	This code describes evaluative statements (positive, negative, or mixed) about student engagement in mathematical practices. This type of move almost always occurs together with one or more of the first two types of reprising moves.	"So you know-you started to code the picture which I really liked and then you used too many circles"
Explaining the goal(s) for engaging in an MP	This type of reprising move provides a goal/rationale or multiple rationales for why it would be important for students to engage in a particular practice.	"That kind of coding for your work helps me see what you're thinking"
Connecting students' engagement in MPs	This involves connecting different students engagement in particular practices. This type of move helps students see how different instances of student engagement in mathematical practices were the same or related.	"It's like when Graciela was labeling the pile pattern yesterday."
Expansively framing student engagement in MPs (Engle et al. 2012)	This type of move involves framing student engagement in mathematical practices as part of a larger conversation that extends across time, settings, and participants. This includes references to when students will need to engage in mathematical practices in future units, in future classes, and in other contexts. It can also reference other participants, such as teachers, employers, classmates.	"In college, the professors won't necessarily be around while you're doing your homework, so it's important that you can make sense of problems and figure things out by working with others"
Eliciting self-assessment with respect to MPs	This type of move involves asking students to assess how well they had understood other students' engagement in mathematical practices or how well they thought they would be able to do something similar.	"I want you to write down on a scale of 1 to 5 how well you understood what Adrianna, Graciela & Josh just did with pile patterns"
Referring to a teaching narrative about MPs	The teacher makes explicit reference to either noticing student improvement in mathematical practices or to having designed instruction particularly to help students work on particular mathematical practices.	"One of the reasons we're talking about this problem is that I want you to be thinking about when formulas make sense and when they don't"

Results

Analyses of the 30 class discussions revealed that all three teachers engaged in *initiating* and *sustaining* mathematical practice engagement in classroom discourse; however, comparing *reprising* moves showed noticeable differences between experienced and beginning practice. Mr. Carrera and Ms. Larson engaged in reprising mathematical practice engagement in ways that made them explicit; Ms. Lane did not. I first briefly illustrate how Mr. Carrera and Ms. Larson made mathematical practice explicit through reprising moves, as well as how they attended to more general learning practices. I then contrast their instruction with Ms. Lane's reprising moves, which focused exclusively on learning practices not specific to mathematics. Finally, I unpack a number of potential hypotheses for what could account for the differences.

Reprising mathematical practices (and learning practices): Mr. Carrera and Ms. Larson

Both experienced teachers made mathematical practices explicit in every discussion, using a variety of different reprising moves. For example, in a discussion about a t-table task (Figure 1) in Mr. Carrera's class, Daren suggested the rule "times three plus five" and showed how this would work for 10 and 35. Sofia suggested multiplying by four and subtracting five would also work. Daren disagreed:

Daren:	We need the same rule because two times four equals eight. But minus five equals three.
Mr. Carrera:	So you're saying that Sofia's plan B wouldn't work.
Daren:	Yeah, like it would only work for ten and 35 but not the others.
Mr. Carrera:	Excellent, Daren. You're talking about how to check your rule, which is what I was going to ask you about next.

Mr. Carrera reprised Daren's activity by *naming* it, making the practice of checking a rule explicit. Both teachers did more than name practices. In a discussion Ms. Larson facilitated about oblique triangles, Alejandra used color (Figure 2) to both connect representations (SMP1) and highlight composed structure (SMP7). Then Ms. Larson reprised: "Yeah, color code things. Color code shapes and stuff. Now it makes it way easier for me to see the red triangle on the left and the blue triangle on the right", first **naming** color-coding and then **explaining its goal.**

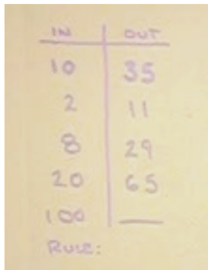


Figure 1. T-table task

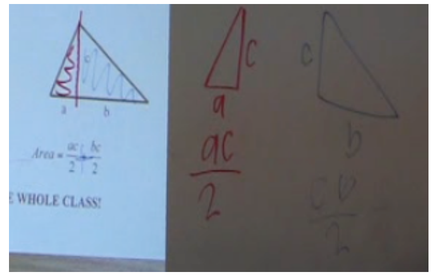


Figure 2. Alejandra's color-coding

In addition to using reprising moves that made mathematical practices explicit, both Ms. Larson and Mr. Carrera reprised student engagement in learning practices (Cohen & Ball, 2001) that were not necessarily specific to a math classroom such as taking risks and listening carefully to classmates. For example, in a 6th grade class discussion, a student named Juan shared his incorrect solution to an exponent problem. After extended discussion with other students presenting solutions, Juan went back up to the board and shared a correct solution to a follow-up problem. Mr. Carrera then engaged in a reprising move that reflected back on Juan's participation in the discussion:

> Very good. Exactly. Good. Perfect you guys. Juan, I think one of the hardest things to do is to go up there because you think you have an answer but it turns out you're not right and then go up there again, right afterwards, and show the class that you've learned it. I think that takes a ton of courage and I really appreciate both of those.

Here Mr. Carrera **highlights** a learning practice of taking a risk to go back up after a previously incorrect answer, and **makes an evaluative statement** regarding Juan's productive participation. This practice of risk-taking and being willing to make mistakes publically is not necessarily unique to mathematics; rather it is a productive student practice related to the social norms and classroom culture that support inquiry-based mathematics learning (Yackel & Cobb, 1996).

Reprising learning practices: Ms. Lane

Analyses of Ms. Lane's discussion facilitation in the 9th grade algebra class showed that, like Mr. Carrera and Ms. Larson, she was able to initiate and sustain student engagement in mathematical practices. Additionally, Ms. Lane also engaged in reprising student activity in eight different reprising talk turns across her four class discussions, but the nature of her reprisals was qualitatively different; in particular, instead of reprising student engagement in mathematical practices, Ms. Lane reprised student engagement in learning practices (Cohen & Ball, 2001) that were not necessarily specific to a math classroom such as taking risks or asking questions. For example, at the end of the first class discussion in her 9th grade algebra class when Alejo presented his thinking on a number sequence task, Ms. Lane engaged in the following reprising move that reflected back on student activity during the discussion:

> I want to give some shout-outs. First, a shout-out goes to Alejo for taking a risk, even though he wasn't sure how to do it. I want to give a shout-out to Oscar for taking a risk and saying "What did you say? Slow down". A huge shout-out to Hector for taking a risk and saying "What did you say? That didn't make sense". Cause that's the way you're going to…That's the way you're going to take care of yourself and get an A in this class. When something doesn't make sense.

In this set of reprising moves, Ms. Lane **highlighted** what several students (Alejo, Oscar, Hector, Jorge) had said or done in the discussion, even using quotes from those students to reference what had happened more specifically. She also **named** the student activities as "taking a risk" and "listening". Additionally, Ms. Lane **explained the rationale** for why a particular aspect of student activity was productive when she said "That's the way you're going to take care of your self and get an A in this class. When something doesn't make sense". This statement also **expansively framed** this context and student activity by referencing the future when Ms. Lane referenced "how to get an A in this class". She continued to frame student activity expansively when she referenced Jorge's listening, "His eyes were actually up here and we want to make sure we keep doing that", which frames this student behavior as part of an on-going activity in this math class.

This aspect of Ms. Lane's instruction was consistent across the three class discussions she facilitated in this data set. For example, in the second whole class discussion, Ms. Lane engaged in a similar reprising move at the end Jamal's presentation on a test average problem and the class discussion around his ideas: "Wait, you guys. Jamal, thank you so much for taking a risk and coming up here even when you didn't do the problem. And thank you for the people who helped out [she points to the class] by following along. This will be on a future quiz." Notice that again, Ms. Lane refers to specific aspects of student activity such as taking a risk and listening attentively when others are presenting.

From these excerpts, it seems that Ms. Lane engaged in a number of the same types of reprising moves that were highlighted in Mr. Carrera's and Ms. Larson's instruction; however, what she reprised was qualitatively different. While Mr. Carrera and Ms. Larson reprised student engagement in *mathematical* practices, Ms. Lane reprised student engagement in productive *learning* practices (Cohen & Ball, 2001), such as risk taking, asking questions when one does not understand, and listening attentively to one's peers. These learning practices, while central for learning, are not necessarily specific to learning mathematics. Her reprisals of these learning practices make explicit aspects of the social norms for the classroom without necessarily attending to the sociomathematical norms (Yackel & Cobb, 1996). Ms. Lane's own reflections on student presentations in her classroom align with these observations:

> And all I'm doing is standing back and making sure I say things like "okay, Riccardo, do not start until you feel like you have the respect of everyone in your class" and making even a humorous joke like make sure if someone looks confused or looks like this and they making a confused face you call them out because we're taking care of each other and things like that. So I think there's a social aspect to the structure of my homework and the homework check as well as a mathematical purpose to it, and so far it's been pretty good.

Ms. Lane's reasoning about her role in these discussions of student presentation highlights her emphasis on the social aspect of these discussions, in addition to the mathematical purpose.

What might account for the differences in what the teachers made explicit in discourse?

The noticeable difference between the reprising moves in Ms. Lane's discussion facilitation and those in Mr. Carrera's and Ms. Larson's raise questions about why these differences might exist. Why did Ms. Lane reprise student engagement in learning practices while Mr. Carrera and Ms. Larson reprised student engagement in mathematical practices and also learning practices)? This is a particularly interesting question since Ms. Lane's discussion facilitation around student presentations looked very much like Mr. Carrera's in many respects, likely a relic of having spent her student teaching year in Mr. Carrera's classroom. A number of possible explanations exist to account from the differences in reprising moves.

Given that Ms. Lane was a first year teacher, it seems likely that her mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) was less well developed than both Ms. Larson and Mr. Carrera, who were both experienced teachers. Recognizing and naming student engagement in mathematical practices likely rests on a deep understanding of the content domain under discussion, the mathematical practices themselves and how they might emerge in that content, and knowledge of students and mathematical practices. Although mathematical practices are not yet explicitly named in Ball and colleagues' (2008) well-established framework for mathematical knowledge for teaching, it seems likely that teachers might need to know common student misconceptions and challenges around particular mathematical practices (Lessig, 2011), such as the common student misconception that empirical arguments, often described as example-based arguments, are sufficient to prove something is true In other words, it is possible that Ms. Lane did not yet have the mathematical knowledge for teaching that would have allowed her to recognize and reprise student engagement in mathematical practices. This would align with studies of teacher noticing, which have shown that attending to and interpreting students' mathematical thinking is a challenging part of developing professional vision that can be learned over time (Jacobs, Lamb, & Philipp, 2010). In my interview with Ms. Lane in the fall, she lent some evidence to this possible explanation when she responded my question about what she had noticed about student engagement in mathematical practices in the earliest days of school. Ms. Lane reflected: "Yeah, I don't think that I had this toolkit or the mentality as a first-year teacher to be really be thinking about that very important question in the first week of school, and I think that's a really important question". In this statement, Ms. Lane herself identified her lack of a "toolkit" or "mentality" as a first year teacher to be able to make such observations about her students and mathematical practices.

Another possible explanation for Ms. Lane's focus on learning practices might be that she identified different needs in her case study class of 36 9th grade algebra students, all of who were new to the school and six of whom had IEPs. When I asked Ms. Lane about what she thought was happening in her classroom to help students learn to engage in mathematical practices, her response included the following observations:

> I've noticed that students really are so afraid of being wrong or dumb here. You read about that and you hear about that in grad school. But I have students here, we do a lot of reflection and quick writes who have said I don't want to go up to the board because "I'm afraid of being dumb", "I'm afraid of what students will think of me", and it's a very real thing. And I think that that is step one, getting rid of that before you learn math because I certainly felt that way when I was younger too. It was like if you're constantly thinking what do I look like, what do I look like, your mind is not even open to understanding what's going on. So I think that that is kind of – anything to set that environment I think is helping students learn math. And then maybe next year maybe I'll start thinking about what kind of numbers should I use so students will learn this math concept better, all that stuff. I'm sure there's a whole other can of worms that I'm not even going to look at right now about that.

In this excerpt, we learn that Ms. Lane had clearly identified her students' discomfort and fear around sharing ideas publically and possibly being wrong, something she identified with through connected it to her own learning experiences in math. Then her statement "anything to set that environment I think is helping students learn math" suggests that she chose to focus on establishing a productive learning environment to meet this identified need. Her reflection about how she'll start thinking more deeply about the mathematical part next year also connects back to previous potential explanation around MKT (Ball et al., 2008).

It seems likely that both of these explanations (and possibly others relating to being a first year teacher) might have contributed to the differences between Ms. Lane's and the other two teachers' reprisals of student activity. However, despite these differences, Ms. Lane's instruction was remarkably skilled for a first year teacher. Like the other two teachers, she was able to initiate and sustain student engagement in mathematical practices in the first few weeks of school with students who were little experiences in such ways of working mathematically. She also noticed and made explicit productive learning practices (Cohen & Ball, 2001) that are central for doing the work of learning. In response to a question about whether she had observed any changes in student engagement in mathematical practices over the first two months of school, Ms. Lane responded, "I think I've seen the biggest changes and the biggest strides in student perseverance and students' willingness to try problems and that's really exciting as a teacher I think". This reflection celebrates a big accomplishment for a first year teacher to be helping students learn to persevere and take risks over a short time period. It also connects closely to her focus on productive learning practices such as taking a risk.

Implications

This study begins to unpack the knowledge and skill needed to make mathematical practices explicit in discourse (Ball et al., 2005). The case of Ms. Lane suggests that teachers may need to have well-developed mathematical knowledge for teaching specific to mathematical practices (Ball et al., 2008) to be able to recognize and describe engagement in mathematical practices at different proficiency levels. Additionally, what teachers notice (e.g., Sherin et al.,

2011) in a discussion would likely also influence what they were able to make explicit with respect to mathematical practices. Finally, Ms. Lane's focus on reprising learning practices that were not specific to mathematics suggests that this productive aspect of her instruction might provide a bridge towards making mathematical practices explicit in her own instruction, and potentially points to a productive avenue for others' learning. Future research will need to continue exploring potential demands of making mathematical practices explicit, as well as examining how both new and practicing teachers might develop this aspect of their instruction to meet the *Principles to Action'*s call for equitable access to high levels of mathematics learning.

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