Teachers Beliefs as Portrayed in NCTM's *Principles to Actions*

NCTM's (2014) recent publication, *Principles to Actions* (*PtA*), sets forth an ambitious agenda for the mathematics education community. Within one year, this visionary document has become prominent in the field of mathematics education, and as such, the community must seek to develop a shared understanding of the ideas expressed within the document. *PtA* contains six guiding principles for school mathematics: effective teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. For each guiding principle, NCTM (2014) identified a list of unproductive and productive beliefs. Unproductive beliefs were described as those that might "compromise progress" toward achieving NCTM's vision for school mathematics. In contrast, the productive beliefs are those that NCTM posits as supportive of ambitious teaching, thus should be embraced by the mathematics education community.

For decades, beliefs have been a significant part of the discourse in the mathematics education community. The study of this construct has garnered a great deal of attention because teachers' instructional decisions are considered to be a product of their beliefs. Researchers (e.g., Cross, 2009; Cross Francis 2014; Pajares, 1992; Philipp, 2007) suggest that any initiative focused on instructional reform must attend to teachers' beliefs in order to be successful. As such, given the strength of the belief-action relationship, we sought to identify the nature of the beliefs of elementary teachers as a part of our ongoing professional development work with them.

The objective of this research is to investigate the extent to which elementary teachers' beliefs align with those expressed in the *PtA* publication and to understand how teachers interpret the statements made by NCTM. By understanding how teachers interpret these statements, we will be better prepared to support teachers to strive toward meeting NCTM's vision for mathematics teaching and learning.

Methodology

Participants

This mixed methods study draws on data collected from 55 elementary school teachers situated in three urban school districts. The participants were engaged in a multi-year professional development program designed to increase the teachers' mathematical knowledge for teaching, enhance their potential to teach using the Standards for Mathematical Practice, and teach using culturally-responsive methods. Most of the teachers in the study had recently completed one year of the professional development program. For the purposes of this paper, pseudonyms have been used when referring to individual teachers.

Data Sources

Data was collected from two sources – a survey designed using a five-point Likert scale and transcripts of interviews with the teachers targeting their interpretations of a selected set of items from the survey.

PtA Survey. All participants attended a summer institute designed to bridge the professional development activities between years one and two of the professional development program. During the summer institute, the participants were asked to respond to a survey instrument that included 34 beliefs statements that either appeared in *PtA* verbatim or were slightly adapted from the *PtA* publication. These belief statements were intentionally selected as items that might be considered controversial among elementary teachers of mathematics. Beliefs were selected from each of the six guiding principles identified in *PtA.* Participants indicated their level of agreement with each statement on a scale from 1 to 5, where 1 indicated they strongly disagreed with the statement and 5 indicated they strongly agreed with the statement. Percentages for each response and each item were tabulated.

The unproductive belief items were reverse scored so all items would correspond to a scale where 1 represents a participant belief that strongly contrasts those stated by NCTM and 5 represents a participant belief strongly aligned with NCTM's intent. Frequencies and percentages were calculated for each item, and average responses were calculated for each of the six guiding principles.

Interviews. Ten items were selected for additional analysis. These items were posed during individual interviews where a subset of 35 participants were asked to consider the ten statements, share to what extent they agreed or disagreed with each statement, and explain their reasoning. The audio-recorded interviews were transcribed, and each statement examined and analyzed using emergent coding techniques.

Results and Conclusions

Table 1 provides the results of the teachers' average agreement with the beliefs corresponding to each of *PtA*'s six guiding principles. In general, all of the averages fell within a fairly narrow range from 3.64 and 3.84 on a scale from 1 to 5, with 4 aligning with "*Agree*." The data indicates that the participants tend to have beliefs that are consistent to NCTM's positions as described in *PtA.*

Table 1. Average Teacher Response by Guiding Principle

Guiding Principle	**Number of Belief Statements**	**Average Response**
Teaching & Learning	4	3.76
Access & Equity	9	3.83
Curriculum	5	3.64
Tools & Technology	6	3.84
Assessment	4	3.82
Professionalism	6	3.67

A majority of the teachers held beliefs that were consistent with the NCTM position for 28 of the 34 beliefs statements. The six belief statements in which no majority existed appear in Table 2, as well as the percentages of participants who responded with each of the five response options (strongly disagree (SD), disagree (D), neutral (N), agree (A), or strongly agree (SA)). There was not a clear consensus among the teachers on comparisons of equity and equality, issues of tracking low-achieving students, placement of students in ability-based groups, or using pacing guides to ensure coverage and continuity. Additionally, the teachers had mixed feelings on two qualities of teachers, whether strong content knowledge is sufficient for effective teaching and whether highly effective teaching is an innate ability.

Table 2. Beliefs Statements without a Majority of Teachers Responding Consistent with NCTM's Position

Belief Statement	Guiding Principle	Productive or Unproductive?	SD	D	N	A	SA
Equity is the same as equality. All students need to receive the same	Access & Equity	Unproductive	7%	36%	16%	24%	16%

learning opportunities so that they can achieve the same academic outcomes.							
Tracking promotes students’ achievement by allowing students to be placed in “homogeneous” classes and groups where they can make the greatest learning gains.	Access & Equity	Unproductive	9%	35%	49%	7%	0%
The practice of isolating low-achieving students in low-level or slower-paced mathematics groups should be eliminated.	Access & Equity	Productive	2%	29%	25%	33%	11%
Implementation of a pacing guide ensures that teachers address all the required topics and guarantees continuity so that all students are studying the same topics on the same days.	Curriculum	Unproductive	5%	18%	42%	33%	2%
A deep understanding of mathematics content is sufficient for effective teaching.	Professionalism	Unproductive	9%	36%	22%	25%	7%
Highly effective teachers have an innate and natural ability to provide innovative instruction that results in high levels of student achievement.	Professionalism	Unproductive	2%	24%	29%	35%	11%

Results from Interview Analysis

The results we share below are based on the analyses, of seven of the additional items that were investigated during the analyses of teachers’ responses during interviews.

Teaching and Learning: The 'frustration' fine line

In regards to the unproductive belief statement "An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused," 60% of the teachers stated they agreed or strongly agreed, whereas 25% stated they disagreed on some level. When teachers were asked to explain why they agreed or disagreed with the statement, their awareness of students' emotions became visible. Specifically, teachers' statements reflected their awareness of the difficulty in managing students' engagement in productive struggle in ways that didn't lead to frustration. Several described their goal of minimizing students' "frustration", as the basis of their support for "guiding students step by step through problem solving." A continuum of a frustration scale could be developed based on the teachers' answers. Thus, a teacher's perception of how much frustration could be tolerated by students was the dividing "fine line" between those that agreed and disagreed with the statement.

On the disagree side of the argument, where the majority of teachers fell, teachers justified their decision by stating that ultimately frustration is viewed as a critical component to learning and thus should be valued over step by step teaching; therefore "step by step" would not be ideal for supporting the learning process, although in some cases step by step could come later in the learning process. Ms. Devine summarized this in her statement, "Because they have to be frustrated in order to learn." Ms. Willis expanded in her statement, "I think confusion is good; you remember confusion. You remember when you struggled, and you don't make those mistakes again." Many of the Common Core mathematical practices (albeit indirectly) were other common justifications across all teachers that disagreed, such as persevering, valuing mistakes and making them learning sites for everyone, recognizing and encouraging there is more than one way to solve a problem in mathematics, and how being frustration-free does not model real life.

Those few teachers who agreed with the statement saw frustration as a barrier to learning and step by step teaching as a means to that end. Ms. Morris summarized, "I agree (be)cause I like to (guide) my students step by step so they'll understand exactly what to do because if they miss a step, and they get it wrong, then they are frustrated, (and) well they will be confused." These teachers viewed the "step-by-step" process as important for learning, conjecturing that otherwise, in many cases, "certain" students would shut down or be defeated before even starting the problem. These two sides of the "frustration fine line" were toggled by teachers on the continuum, several of whom also suggested that step-by -step instruction was one way of differentiating instructions for students. These teachers, such as Ms. Vantlin, identified this as a strategy that was good for "some kids" but maybe "not all of the time." As Ms. Howder stated, "I don't really agree with that in all situations but I don't want to start them being frustrated to the point of where they don't want to try."

There were also a few teachers who were indecisive – they agreed with parts of the item but disagreed with other parts. As Ms. Tooley summarized,

> I kind of agree and disagree, because sometimes you have to let them work on the problem themselves, make mistakes then ask them why do you think you made this mistake. And I do agree that I like to work through step by step, but I probably would want them to kind of fill it out on their own first and see how much they can do on their own.

In sum, teacher tolerance for student frustration, and what benefits and drawbacks such experiences bring for students are viewed and weighed by teachers differently, hence they guide beliefs that act as a filter for certain pedagogical decisions in the classroom and a lens through which teachers understand this belief being promoted within NCTM's *Principles to Actions* publication.

Access & Equity: All have the 'chutzpah' to persevere

The following unproductive belief from the Access and Equity principle resulted in the greatest amount of consensus among teachers in the study: "Only high-achieving or gifted students can reason about, make sense of, and persevere in solving challenging mathematical problems." The survey results showed that 98% of the teachers disagreed or strongly disagreed with this statement, and only one teacher (2%) chose undecided. This was the only item on the survey in which none of the 55 participants posited a stance oppositional (i.e., in agreement with the statement) to the views expressed by NCTM (2014).

During the interviews, many of the teachers suggested the need for all learners to persevere during the act of problem solving (or in some cases, participants referenced 'average' learners instead of 'all' learners). For example, Ms. Alvarez stated,

> You don't have to be gifted or high achieving to have the 'chutzpah' to keep going. Sorry, to keep going on working on these math problems, you don't back down from a challenge just because you know it's going to be hard. You have to step up to it and do it.

Teachers with the viewpoint that all students can learn to reason productively and persevere often referenced past students who they used as a counterexample to support their claim.

> Ms. Hughes: No, I disagree, I had a little girl this year that she was able to really persevere, her explanations sometimes maybe weren't as good, but she was able to see things that maybe you didn't expect that she would see them that way.

> Ms. Knowles: I've seen some slow learners come out with some, make some sense of some problem solving, and I'm like 'wow how did they?' It depends on the thinking, it depends on how they're thinking.

Although several teachers mentioned similar experiences of observing students they perceived as low-ability with strong reasoning skills, a smaller number provided descriptions of some of the difficulties that they had encountered with strong students engaged in problem solving.

> Ms. Ziggler: I totally disagree because it seems like the high achievers can't reason. They can tell you 'I have the answer.' When you ask how they got the answer they don't have a clue.

> Mr. Ford: I find that the high-achieving students tend to rush through those problems. And they get them wrong, because they're not thinking about them. They're like, 'Oh, I know how to do this. I'm just going right to the solving the problem.' And they miss a couple of steps, whereas the kid who's struggling and has to really think about it, they're thinking what each of the number's meaning is, what it stands for, and thinking about how to get to that end, but looking at each step along the way.

> Ms. Vantlin: No, it's amazing how some that are not so high level will persevere longer than the high level and have the patience and the willing to work even harder to get it, just because. And they're not considered high level, so again it depends on the student.

Like Ms. Vantlin, some participants offered an alternative explanation by focusing on the support provided by teachers rather than qualities of the students:

> Ms. Morris: I disagree with that. Because they could be a low achieving student, but if they have help and the problem can be broken down for them, then maybe they can get the answer. So I disagree that only high ability students can solve challenging math problems.

> Ms. Kerry: I think anybody can make sense of math, as long as it's, maybe you have to modify it so it's at different levels for students.

These participants suggest that the teacher was central to students developing their reasoning skills such as Ms. Morris's suggestion that teachers can break down a problem.

Curriculum: Changing all the time

A majority of the teachers (81%) disagreed with the statement "Mathematics is a static, unchanging field." A similar statement from the Tools and Technology section (i.e., "School mathematics is static. What students need to know about mathematics is unchanged (or maybe even threatened) by the presence of technology.) garnered a disagreement by 77% from the survey participants. For those who disagreed with the former statement, they thought of math as a non-static, changing field, because there are multiple pathways to solving problems. They characterized mathematics in terms of the range of solution methods one can use in problem solving. The participants who were indecisive and both agreed and disagreed with the statement also held this view; however, although they considered the pathways to be diverse and changing, they thought that the answers to the problems would always be the same. For example, Ms. Bardole stated,

> Two plus two is always going to be four, so certain things like that are not going to change. But maybe how we help kids arrive at answers you know, I think that changes the strategies for how kids can arrive at answers.

Regarding the latter statement, teachers drew on their own personal experiences with curricula and standards in responding. For those who disagreed, they referred to the changing standards (e.g., CCSSM) and curricula series they had experienced in recent years as evidence that school mathematics was not static. This was clearly expressed by Ms. Gomez, "I disagree. It changes, like curriculums change; the Common Core all that stuff's changing all the time, standards and all of that." Unlike those who agreed with the statement, participants who disagreed thought there were a set of core concepts they were expected to teach each year, so in this regard school mathematics was unchanging.

One interesting observation was the teachers' views of mathematics in contrast to school mathematics. In the examples of mathematics that many teachers provided in their responses, it appeared that they considered mathematics to embody more non-routine, ill-structured problems and engaging with mathematics involved pattern-seeking and reasoning. In contrast, they seemed to conceptualize school mathematics as the content and problems in the curriculum they taught which tended to include tasks that targeted the use of algorithms and routine computations.

Tools & Technology: You "still need to instruct"

In terms of tools and technology, teachers were asked to consider the following statement in the interview: "Using technology and other tools to teach is easy. Just launch the app or website, or hand out the manipulatives, and let the students work on their own."

Teachers tended to interpret the statement in four different ways: First, some teachers focused on the easiness of the technology from the word "easy" in the statement. Second, some teachers focused on the effect of free exploration and guided exploration from the words "On their own" in the statement. Third, some teachers focused on the different purposes to use the apps or software such as promoting mathematical reasoning and practicing procedural skills from the words "the app or website" in the statement. Finally, some teachers focused on the different points in time such as before teaching new content or technology or after teaching them, which is not explicitly expressed in the statement.

More specifically, similar to the result of the survey, a majority of teachers (19 of the 35 interviewed teachers) responded they disagreed with the statement in the interview. Fifteen (15) teachers explained that it is important for teachers to provide students with some guidance to make using technology a meaningful experience, including modeling how to use the technology, setting the rules, tasks and expectations before letting students use it. Teachers also emphasized using technology during instruction by going through problems, explaining what students see, and giving examples.

In addition to the 15 teachers, four teachers disagreed with the statements for different reasons. One teacher said that she disagreed with the statement because technology cannot take the place of teachers and it should be used as resources. Another teacher disagreed because of multiple reasons that the way of using technology depends on teachers' experiences with technology, the amount of guidance students need, and balancing between different teaching methods such as hands-on, paper and pencil, and technology is important. One teacher disagreed with the statements because technology is not easy for the teacher herself or some students.

Below are examples of responses from teachers who disagreed with the statement:

> Ms. Tooley: I do not agree with this, I'm a teacher that use several, I like to use a lot of videos and things like that but I like to go through and I'll stop and expound what we just saw, give a few examples, it can be used as a supplemental but you would still need to instruct your kids and give your own examples and things like that.

> Mr. Mounts: No. There has to be some guidance there, you know it doesn't matter what the website is they have to know what the expectation is, they have to know what the task is, you know so you can't just, you can't just sit at the desk and put in NCTM and say there you go, go to town on it you know? So no, can it be effective yes, if it's done the right way.

On the other side of the argument, two teachers agreed with this statement. Ms. Knowles explained that technology is good for practicing procedural skills after students learn the skills. That is, as her response below indicates, she did not seem to consider technology as a cognitive tool to learn mathematical concepts,

> Ms. Knowles: I agree with that because that's what technology does. I mean it's all there for you. Once the student has learned the skill, then it's their time just to work on the skill and that's what to me technology does you know. Most of your websites (are) already programed for that, so to me that's what I use the website for, for reinforcement.

Also, ten teachers were indecisive answering "yes and no," or "only to a certain extent." Among them, 3 teachers said that technology is easy but they need to give some guidance to their students, which is the similar reason with the teachers who disagreed with the statement. One teacher disagreed by focusing on the word "easy" in the statement, because she was not confident with technology. However, she stated that if she knows the apps well, then she would agree with the statement. In a similar vein, Ms. Bardole said that it could be if she utilized the apps together a couple times, although she is still not comfortable with just letting students use technology on their own.

> Ms. Bardole: So just give it to them and they're on their own. It may be but I still would feel, maybe after we've utilized it together a couple times, but I don't, I don't think I would feel comfortable just saying ok go on this website and I'm just going to keep back.

Three teachers emphasized the balance between free exploration and guidance explaining that some apps promote students' mathematical reasoning while other apps are designed for drill and practice. As such, the teacher should pay attention to the different functions of apps or software.

__Assessment: Like a checkup at the doctor__

To further understand teachers' beliefs about assessment, we asked them to respond to the statement "Assessment in the classroom is an interruption of the instructional process," for which 78% of the teachers had disagreed or strongly disagreed during the initial survey. Overwhelmingly, the teachers' comments during the interviews focused on 1) the necessity of assessment, 2) the frequency of assessment, and 3) the advantages and disadvantages of different forms that assessment.

Several teachers emphasized that assessment was necessary in the learning process, because it provides relevant feedback for them as teachers.

> Ms. Wildt: Assessment in the classroom is basically like checkups at the doctor, if we don't get checkups then we don't know, and you can be a ticking time bomb, and don't know it, so we need the information to guide us…assessment should help us, it should guide us through our teaching, so I don't think it's an interruption, but too much can be.

As demonstrated by Ms. Wildt's last sentence, frequency was another concern the teachers expressed about assessments. In one of the participating school districts, students were required to take three assessments each grading period (referred to the teachers as a "pre," "mid," and "post"). The statements from the two teachers below describe some of the challenges they face in attempting to implement an assessment program with such a regimented and frequent assessment program.

> Ms. Howder: You know, every three weeks you have to give a cycle test, and you have to be through these three things. And then on this date you have to give a test, and you know what, if I'm not there yet, you know, and you know what if my kids took longer to do this and I've got to stop and take this test when first of all I have, probably might not have even gotten to everything. And you know, why can't I just assess it when I complete the topic and I feel like they're ready for that?

> Ms. Zeller: There's so many assessments; I feel sorry for (the students), and the standardized testing is just horrible. So yes in that respect, those are assessments.

> Yes, I think they're a hinder, but…we have to give them pre-assessments every three weeks, and I try to let them know that, you're not graded. This is just so I know what you're doing. So, I try to alleviate any anxiety they feel. Some kids don't like all the assessments, but you can also do formative assessments and just observe.

Ms. Zeller’s final comment embodies the third common aspect that teachers mentioned about assessment, which deals with the advantages and disadvantages of different types of assessment. As she stated, Ms. Zeller believed that formative assessments that involve observation are less intrusive and do not cause the same type of anxiety as standardized assessments. Other teachers mentioned specific types of assessments that they utilize, and that formative assessments initiated by the teacher can be helpful guides in understanding how students think and how to adapt instruction to further students’ thinking. For example, Ms. Finn described using checklist and “exit tickets” as ways to learn “where they are and where they need to go.”

Professionalism: Sharing the wealth

There was little variation in the teachers’ responses to the statement “Effective teachers can work in isolation. As long as the students in one’s own classroom are successful, all is well.” One of the teachers however disagreed that isolation should be part of an effective teacher’s practice, even if students are successful. The only teacher who agreed with the statement referenced teacher evaluation as the reason she agreed. Ms. Overton stated,

> …because of the evaluation process I think that it's more of ownership of these are my kids, this is what I do, this is what I'm choosing to make better and I think sharing, collaboration has been more of a competitive issue.

Although a majority of the teachers disagreed with this statement, their reasons lay in four categories. The majority of the teachers described the importance of sharing, help, and collaboration as a characteristic of an effective teacher. As Ms. Gonzalez noted, “I personally believe that effective teachers collaborate and share what works and what didn't or look for advice and share what's successful.” Another category that emerged was the participants’ reasoning that you are just one part of the process for a child in their long-term learning success as is evidenced by Ms. Kerry, “No because those aren't going to be your students next year.” A few teachers referred to isolation as an act of selfishness. Ms. Garrison was one of thee teachers who stated, “that's kind of a selfish approach, but I think that we need to share the knowledge, share the wealth or whatever. I think of students as all, the whole school really.” The last subcategory included responses that identified reflection as an important aspect of being an effective teacher, thus positioning isolation as a barrier to developing that ability. Both teachers who remarked about the use of the “others” in the classroom to support reflection, including a video camera, which could be used in isolation, suggested that collaborative reflection was important. In summary, others are needed to help you see what you can’t see. Ms. Knowles commented,

> The videotape makes sure that I understand that the children are understanding the skills through someone else's eyes you know. It's one thing of being effective teaching, you're teaching all this stuff and you think that they are learning and they're gaining things, but when you bring in someone else it makes it a little bit different.

Similarly, Ms. Cornell argued,

> I don't think so, only because you need to bounce ideas off of other people, you need to see that sometimes someone standing outside of your teaching expertise can say, you know how did you think that lesson went? And you may have thought the lesson worked fantastically but they may have seen things that you didn't.

Although community becomes the common link between all four of these subcategories of agreement, each takes a unique interpretation of the connection between isolation and success as experienced by an "effective" teacher.

It is important to note that the research team had conversations about the connectedness of the two statements presented simultaneously and the word "can" versus an alternative word like "should" being used in the first statement. When addressed in tandem, it veiled the fact that a teacher can be successful working in isolation, as certainly there are such teachers present in schools, however, it is not the message promoted by NCTM's Principles to Action. Rewording the statement could prevent such misinterpretations. Furthermore, Ms. Wildt even questioned the meaning of the word success in the statement, "it depends on what you measure as success, because I mean there's some teachers that work on their own and it's considered that their classes look successful but it depends on what you measure as success."

Discussion

The results of our study have three important implications. First, the results showed the teachers responded with overwhelming support for most of the beliefs identified by NCTM as productive and disagreement with NCTM's unproductive beliefs. Given the belief-action relationship, having a majority of the teachers holding productive beliefs bodes well for the success of professional development initiatives geared towards improving instructional quality. Second, there were a few statements identified where a majority of teachers' responses did not indicate agreement with the belief posited by NCTM. Although NCTM identified beliefs that could present barriers to the implementation of their vision for school mathematics, our results suggest the specific beliefs may be most problematic for teachers to overcome. As such, they may require more attention from the mathematics education community. Finally, the collection of statements from *PtA* that we identified presents a wide range of potential beliefs about mathematics education. We believe this collection can be used as a formative assessment of teacher beliefs in preservice and inservice settings, which can help professional developers identify the greatest needs. By understanding how teachers think about the beliefs posited in *Principles to Actions,* we are better prepared to lead professional development in ways that support teacher reflection on their beliefs, especially when those beliefs counter those described in *PtA*.

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