Comparison of Student and Teacher Perceptions

of Mathematics Teaching Practices

DeAnn Huinker & Melissa Hedges

Center for Mathematics and Science Education Research (CMSER)
University of Wisconsin-Milwaukee
Milwaukee, WI

huinker@uwm.edu ~ mhedges@uwm.edu

CMSER

Comparison of Student and Teacher Perceptions of Mathematics Teaching Practices

DeAnn Huinker & Melissa Hedges
University of Wisconsin-Milwaukee

For a number of years, the major emphasis in professional development through large-scale projects, such as the *Mathematics and Science Partnerships* through the National Science Foundation and the Department of Education has been to develop teachers' mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). While it is essential that teachers understand deeply the mathematics they are teaching their students, attention also needs to be given to the development of specific, pedagogical content strategies to support deeper student learning of mathematical content (Lampert, 2010; Richardson, 1990). The National Council of Teachers of Mathematics (NCTM, 2014) in their recently released publication, *Principles to Actions: Ensuring Mathematical Success for All*, highlighted the essential work of teachers in classrooms as they interact with students engaged in learning mathematics content.

Our current work with teachers in a professional development project has offered us the opportunity to utilize the NCTM core set of mathematics teaching practices as a framework in our expectations for instructional practice. We are now explicit about our goals for teachers to not only deepen their understanding of mathematics, but to become "skilled" (Ball & Forzani, 2010) in using this core set of teaching practices in their classrooms.

Problem Statement

Assessment of teaching is not only problematic from logistics and financial perspectives, but also from issues of content validity (Banilower, Boyd, Pasley, & Weiss, 2006; Hill et al., 2008). Many approaches attempt to use generic, content-neutral assessment instruments, whether for classroom observations, student surveys, or teacher surveys. However, the teaching of mathematics is not the same as teaching reading or other subject areas. In order to evaluate the impact of the professional development project, we needed to assess whether teachers were implementing and becoming more skilled in using specific mathematics teaching practices to support student learning of mathematics. While we considered conducting classroom observations, evidence supports using student perceptions as an important source of information on the quality of teaching and the learning environment in individual classrooms (Burniske & Meibaum, 2012; Crow, 2011; Ferguson, 2010, 2012; Kunter et al., 2008; Measures of Effective Teaching [MET] Study, 2012). Thus, we decided not to conduct classroom observations as part of the project evaluation, but rather to pursue viewing teaching through the eyes of students.

The purpose of this study was to contribute to our understanding of ways to examine the implementation of high-leverage mathematics teaching practices in classrooms. We developed and revised instruments to examine teachers' perceptions on their own enactment of the mathematics teaching practices and developed instruments to assess students' perceptions of the use of these same teaching practices in their mathematics classrooms. The study is framed around the following research question: How do teacher and student perceptions compare on the implementation of the mathematics teaching practices?

Mathematics Teaching Framework

Ball and Forzani (2011) and other researchers (e.g., Sleep, Boerst, & Bass, 2009; Hlas & Hlas, 2012; McDonald, Kazemi, & Kavanagh, 2013) have argued that the profession of teaching needs to identify and work together toward the implementation of a common set of high-leverage practices that underlie effective teaching. In response, NCTM (2014) identified eight teaching practices as a framework for strengthening the teaching and learning of mathematics (see Figure 1). NCTM posited that this set of eight mathematics teaching practices "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9).

We utilized these eight mathematics teaching practices as the framework for our current work with teachers in the professional development project. The teachers are studying this set of teaching practices and collaboratively working to implement them in their mathematics classrooms. As a component of the project evaluation plan, we used the eight teaching practices as constructs to design the teacher and student surveys.

---

1. **Establish mathematics goals to focus learning**. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

2. **Implement tasks that promote reasoning and problem solving**. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

3. **Use and connect mathematical representations**. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

4. **Facilitate meaningful mathematical discourse**. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

5. **Pose purposeful questions**. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

6. **Build procedural fluency from conceptual understanding**. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

7. **Support productive struggle in learning mathematics**. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

8. **Elicit and use evidence of student thinking**. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*

Figure 1. Mathematics Teaching Practices (NCTM, 2014)

Methodology

The teacher professional development project aims to deepen teachers' mathematical knowledge for teaching and strengthen those teaching practices most effective in furthering student achievement and success in mathematics. It seeks to (1) deepen teachers' mathematical knowledge and understanding of content progressions aligned to the *Common Core State Standards for Mathematics*; (2) strengthen teachers' instruction through the implementation of high-leverage instructional practices for mathematics, and (3) increase collaboration among teachers within and across schools in moving to common and coherent practice in mathematics.

Participants

The project involves K-8 teachers (n=70) and their students (n=1720) from three school districts. Student achievement in mathematics is below the state average (49%) in all three districts with 33%, 41%, and 19%, respectively, proficient in mathematics. All three school districts have high percentages of students who are economically disadvantaged (i.e., 51%, 49%, 82%, respectively). The student populations are comprised of 31%, 23%, and 86% minority students, respectively.

The teachers comprise two groups: 39 in the professional development intervention and 31 in a comparison group. Of these 70 teachers, 19 teach at the middle school level in Grades 6-8, 37 teach at the upper elementary school level in Grades 2-5, nine teach at the primary level in Grades K-1, and 5 hold other positions (e.g.,, math coach, math interventionist, or principal).

Instruments

The project team developed teacher and student surveys using the mathematics teaching practices framework. We developed one survey for teachers. In order to accommodate the range of students, from Kindergarten to Grade 8, three variations of the student survey were developed.

Teacher Survey. The project team began with the statement of each teaching practice (see Figure 1) and then utilized the "Teacher and Student Actions" tables listed in *Principles to Actions* (NCTM, 2014) to develop an initial set of items for the survey. We included items for each of the eight teaching practice constructs to establish eight subscales on the instrument. After many iterations and rewording of the items, a version of the survey was given to a panel of mathematics educators and teachers to examine the content validity. The panel was asked to comment on the wording of each item for clarity, to identify the alignment of each item to one of the eight constructs, and to suggest additional items, if needed, to capture further aspects of each construct. After further revisions based on this review, the initial teacher survey contained 59 items with eight subscales, one for each of the eight teaching practices. The survey utilized a six-point Likert scale.

Student Surveys. The student surveys were modeled on those used in the Measures of Effective Teaching project (MET, 2010). We developed an initial set of items for each of the eight teaching practice constructs and considered variations of the items for each of three grade bands—primary (Grades K-1), elementary (Grades 2-5), and middle (Grades 6-8). It became quickly apparent that it would be difficult to develop an instrument for Grades K-1 that included enough items for a reliable assessment of separate constructs. Thus, we decided to only develop one overall scale for the primary survey. Given this situation, we will only discuss the elementary and middle school surveys in the remainder of this paper.

A project team member individually administered the elementary and middle school surveys to a small sample of students in the respective grade bands to examine each item for clarity. This lead to revisions of both surveys which were again individually administered to respective students. A panel of mathematics educators and teachers were also asked to review the student surveys and identify the alignment of each item to one of the eight constructs and to suggest revisions for clarity or additional items, if needed, to capture further aspects of each construct. After further revisions, the initial Grades 2-5 elementary survey contained 24 items with six subscales. We decided not to include the Fluency subscale due to difficulties in crafting items that were clear to students at this grade band. We also eliminated the Questions subscale due to the difficulty in writing items that were clearly identifiable as part of this scale. The initial Grades 6-8 middle school survey contained 33 items with seven subscales. At this grade band, we retained the Fluency subscale, but eliminated the Questions subscale for the same reasons as the elementary survey. The elementary and middle school surveys both utilized a five-point Likert scale.

Procedures

The teacher survey was administered as a paper-and-pencil survey to all teacher participants by project staff prior to the start of the professional development program in summer 2014. The student surveys were administered electronically by a district representative (not the classroom teacher) at the end of the school year in May 2014. This baseline data was used to conduct an analysis of the internal consistency of the instruments by examining the Cronbach alpha coefficients for each of the subscales and the overall total scale. The analysis had three goals. First, we wanted to determine the level of internal consistency for each subscale and identify whether any items needed to be removed. Second, we hoped to shorten each instrument while retaining strong reliability of each subscale. Third, we hoped to identify any subscales that needed further revision due to low reliability.

After the reliability analysis and revisions to some of the subscales (reported below in the results section), mean scores for each subscale were determined for each teacher and each student. Using the teacher as the unit of analysis, student scores were aggregated by teacher. Given that each survey used a different scale, the teacher mean scores were transformed to a comparable five-point scale to ease comparison of teacher and student perceptions on the implementation of the mathematics teaching practices in their classrooms.

Results

The evaluation of the teacher professional development project seeks to examine whether teachers become more skillful in implementing the high-leverage mathematics teaching practices (NCTM, 2014) in their classrooms. The teachers are being asked to report on their own level of implementation of the mathematics teaching practices and their students are also being asked about their perceptions of mathematics instruction in these teachers' classrooms. In this section, we first discuss the results of the reliability analysis of the teacher survey and the student surveys at the elementary (Grades 2-5) and middle school (Grades 6-8) grade bands. Then we present a comparison of the baseline teacher and student perceptions of teaching practices in their mathematics classrooms.

Reliability Analyses

The purpose of the reliability analyses was to examine the internal consistency of the subscales on the teacher and student surveys and use the results to inform further revisions to the instruments. Table 1 shows the results for the teacher survey. The original survey contained 59 items. The Questions scale was problematic as reflected by the Cronbach alpha coefficient of .643. Given the removal of this scale from the student surveys, it was decided to also eliminate it from the teacher survey. The other seven scales were examined for internal consistency. The removal of items from the Tasks and Discourse scales increased their internal consistency and removal of items from the other five scales had minimal impact on the internal consistency of each scale. The revised survey (included in Appendix A) contained 41 items and seven subscales with Cronbach alpha coefficients ranging from .820 to .865. The total scale had an overall Cronbach alpha coefficient of .961.

Table 1. Teacher Survey – Mathematics Teaching Practices Inventory

	Original Instrument Scales				Revised Instrument Scales			
	n	Number of Items	Cronbach's α	Item-total Correlations	n	Number of Items	Cronbach's α	Item-total Correlations
Goals	66	6	.851	.532 to .778	66	5	.850	.621 to .711
Tasks	66	7	.820	.412 to .678	66	5	.821	.566 to .663
Representations	66	9	.878	.483 to .757	66	6	.865	.595 to .759
Discourse	66	9	.834	.235 to .673	66	6	.847	.473 to .752
Questions	66	4	.643	.291 to .536	66	0	deleted scale	
Fluency	66	6	.828	.492 to .713	66	5	.820	.534 to .723
Struggle	66	10	.869	.361 to .762	66	7	.862	.528 to .790
Evidence	66	8	.857	.534 to .705	66	7	.851	.522 to .745
Total Scale	66	59	.968	.336 to .739	66	41	.961	.337 to .756

The results of the reliability analysis for the elementary student survey (Grades 2-5) are shown in Table 2. The original survey contained 24 items and six subscales. The subscales were examined for internal consistency. The removal of two items from the Discourse scale raised the Cronbach alpha coefficient to .822. The Evidence scale showed strong internal consistency with a Cronbach alpha coefficient of .794. The other four scales were problematic. The Goals and Representations scales had acceptable Cronbach alpha coefficients of .622 and .661, but we would like both scales to be stronger. The low reliability of the Tasks and Struggle scales were most likely due to the low number of items per scale (i.e., 2 items).

For the purpose of examining the baseline views of students, the revised survey (included in Appendix B) contained 22 items and six subscales with a total Cronbach alpha coefficient of .899. We acknowledge that some of the results for the subscales will be specious, but also feel that it will be informative for the project to examine the results by subscales. In the future, we intend to revisit and revise this instrument by further examining the wording of each item for clarity and to include additional items.

Table 2. Student Survey – Student Perceptions of Math Class: Grades 2-5

Scale	Original Instrument Scales				Revised Instrument Scales			
	n	Number of Items	Cronbach's α	Item-total Correlations	n	Number of Items	Cronbach's α	Item-total Correlations
Goals	642	3	.622	.415 to .458	642	3	.622	.415 to .458
Tasks	646	2	.451	.298 to .298	646	2	.451	.298 to .298
Representations	614	5	.661	.379 to .485	614	5	.661	.379 to .485
Discourse	608	7	.810	.330 to .695	544	5	.822	.574 to .711
Struggle	626	2	.478	.323 to .323	626	2	.478	.323 to .323
Evidence	602	5	.794	.463 to .642	602	5	.794	.463 to .642
Total Scale	539	24	.908	.416 to .638	544	22	.899	.406 to .665

The results for the middle school student survey (Grades 6-8) are shown in Table 3. The original survey contained 33 items and seven subscales. We examined the subscales for internal consistency. The removal of items from the Tasks, Discourse, and Evidence scales raised the Cronbach alpha coefficients to .639, .789, and .855, respectively. We also removed items that had minimal impact on the internal consistency of the Representations and Struggle scales. The Goals and Fluency scales were not changed. The Goals, Representations, Discourse, Struggle, and Evidence scales had good internal consistency. The Tasks and Fluency scales were problematic, most likely due to the low number of items (i.e., 3 and 2 items, respectively).

For the purpose of examining the baseline views of students, the revised survey (included in Appendix C) contained 27 items and six subscales with a total Cronbach alpha coefficient of .943. In the future, we intend to revisit and revise this instrument by further examining the wording of each item for clarity and to include more items for some scales, particularly for Goals and Tasks. We have also decided to eliminate the Fluency scale due to its low internal consistency and to have the same subscales as the elementary survey.

Table 3. Student Survey – Student Perceptions of Math Class: Grades 6-8

	Original Instrument Scales				Revised Instrument Scales			
	n	Number of Items	Cronbach's α	Item-total Correlations	n	Number of Items	Cronbach's α	Item-total Correlations
Goals	842	4	.724	.467 to .586	842	4	.724	.467 to .586
Tasks	836	3	.529	.208 to .416	838	2	.639	.471 to .471
Representations	820	6	.766	.408 to .602	821	5	.758	.465 to .611
Discourse	820	7	.785	.380 to .649	823	5	.789	.481 to .644
Fluency	836	2	.599	.430 to .430	836	2	.599	.430 to .430
Struggle	822	6	.829	.537 to .654	824	5	.815	.541 to .635
Evidence	823	5	.851	.512 to .728	828	4	.855	.614 to .751
Total Scale	768	33	.946	.269 to .713	778	27	.943	.482 to .724

Perceptions of Teachers and Students

Based on the revised teacher and student surveys, we compared the perceptions of teachers and students in Grades 2-8 on the implementation of the mathematics teaching practices. This analysis included 47 teachers and the aggregated student views on mathematics instruction in the classrooms of these 47 teachers. The mean number of students per teacher was 16.0 (SD=26.2), ranging from 3 to 109 students. While some teachers did have fewer than 15 students (e.g., special education teacher, split classrooms, co-taught classes), other teachers unfortunately did not have results for the majority of their students due to scheduling and technical issues. We chose to retain all teachers in the sample that did have student data from three or more students. A total of 25 teachers were in the Intervention Group with 17 teaching Grades 2-5 and eight teaching Grades 6-8. A total of 22 teachers were in the Comparison Group with 13 teaching Grades 2-5 and nine teaching Grades 6-8.

The mean scores for teacher and student perceptions on implementation of the mathematics teaching practices are displayed in Table 4 by treatment group for the six subscales. The students in the Intervention Group rated the implementation of Goals, Tasks, Representations, Struggle, and Evidence significantly higher than their teachers; no difference was found for Discourse. The lowest mean rating by the students was for Discourse. In fact, the rating for Discourse was strikingly lower than the other teaching practices. This suggests that students do not have as many opportunities to interact with their peers and to discuss the mathematics they are learning. The highest mean ratings by the students were for Struggle and Evidence. This seems to suggest that learning from mistakes and perseverance were valued in these classrooms.

The students in the Comparison Group rated the implementation of Goals, Struggle, and Evidence significantly higher than their teachers; no differences were found for Tasks, Representations, and Discourse. The lowest mean rating, and strikingly lower, by the students in the Comparison Group was also for Discourse. This suggests that students in these classrooms do not have as many opportunities as they would like to interact with their peers and to discuss the mathematics they are learning. The highest mean ratings were reported for Struggle and Evidence, the same as for the Intervention Group

Table 4. Mathematics Teaching Practices by Treatment Group

Scale	Intervention Group (n=25)					Comparison Group (n=22)				
	Teachers		Students		sig	Teachers		Students		sig
	Mean	SD	Mean	SD		Mean	SD	Mean	SD	
Goals	3.21	0.524	4.13	0.275	.000	3.71	0.236	4.13	0.595	.016
Tasks	3.73	0.575	4.12	0.319	.008	4.02	0.279	4.17	0.458	.646
Representations	3.51	0.621	4.17	0.237	.000	3.96	0.241	4.17	0.529	.430
Discourse	3.51	0.621	3.71	0.489	.619	3.96	0.634	3.69	0.595	.414
Struggle	3.29	0.531	4.36	0.253	.000	3.60	0.251	4.42	0.646	.000
Evidence	3.31	0.517	4.24	0.241	.000	3.87	0.284	4.25	0.556	.014

The comparative results for the four groups are shown in Figure 2. Most striking are the similarities of the students in both groups. No significant differences were found when comparing the students in the Intervention Group to the Comparison Group. In other words, the students viewed their mathematics instruction similarly across the two treatment groups. This provides a clear baseline indicator of the mathematics teaching practices from the student viewpoint. It will be informative to observe whether or not student views of their mathematics instruction changes over time and whether this can be attributed to the participation of the teachers in the professional development project.

In contrast, the teachers in the comparison group rated themselves higher than the intervention group teachers on all six scales. Significant differences were found for Goals ($p = .002$), Representations ($p=.006$), Discourse ($p=.045$), and Evidence ($p = .000$), but not for Tasks ($p=.093$) nor Struggle ($p=.063$). It is interesting to note that while the overall scores differ, the trend in responses are similar across the two groups. The highest mean ratings for both groups were reported for Tasks, followed by Representations and Discourse. The lowest mean ratings for both groups were for Goals and Struggle. These initial differences in teacher views of their mathematics instruction will be controlled for in further analyses as the project progresses.
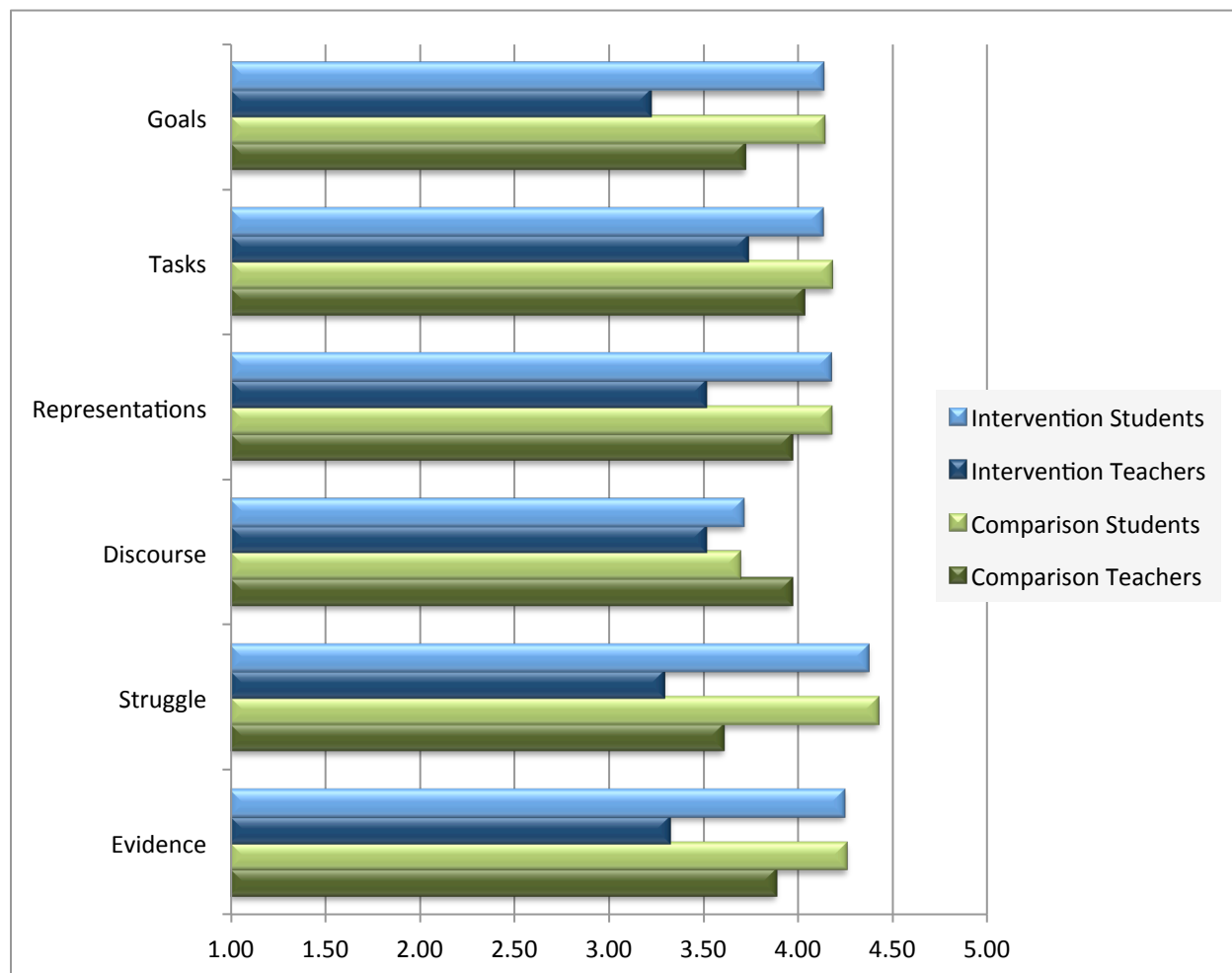

Figure 2. Comparison of Teacher and Student Perceptions by Treatment Group

Discussion and Conclusions

The project developed a teacher perceptions survey to measure the implementation of a set of high-leverage mathematics teaching practices that had good reliability for the overall total scale and seven subscales—Goals, Tasks, Representations, Discourse, Fluency, Struggle, and Evidence. The project also developed student surveys for Grades 2-5 and Grades 6-8 that had good reliability for the overall total scale, but not for all subscales. On the Grades 2-5 student survey, only the Discourse and Evidence subscales had sufficiently strong internal consistency. On the Grades 6-8 student survey, the Discourse, Struggle, and Evidence subscales had strong internal consistency. The student surveys need further revision to increase the reliability of the subscales to provide stronger measures of their teachers' implementation of the mathematics teaching practices. This will involve reviewing the wording of items for clarity and increasing the number of items on the weaker subscales.

Our baseline comparison of teacher and student perceptions allowed us to examine how teachers viewed their own mathematics instruction as aligned to the high-leverage mathematics teaching practices and to compare their results to that of their students. It was clear that teachers and students held differing views about mathematics instruction in their classrooms. In general, students rated their teachers higher than the teachers rated themselves. Specifically, the students rated the Goals, Struggle, and Evidence subscales significantly higher than their teachers in both the intervention and comparison groups. The students in the intervention group also rated Tasks and Representations significantly higher than their teachers. The implementation of Discourse was viewed similarly by teachers and students in both groups.

The results also revealed that students across both the intervention and comparison groups had strikingly similar views of the mathematics teaching they experience in their classrooms. No significant differences were found for any of the subscales. In contrast, the perceptions of the teachers revealed differences in mathematics instruction between the intervention and comparison groups with the comparison group teachers rating themselves significantly higher on the Goals, Representations, Discourse, and Evidence subscales.

It was surprising that students generally rated their teachers' mathematics instruction higher than the teachers rated it themselves. This may show that students in Grades 2-8 have a favorable view of their mathematics instruction or it may reflect the problematic nature of some of the subscales on the student surveys. However, the similarity of student perceptions across both treatment groups gives us confidence in moving forward and using this data as a baseline indicator of the teachers' implementation of the mathematics teaching practices in their classrooms. We also wonder whether the lower self-ratings of the teachers in the intervention group reflected a more critical view of their own mathematics instruction and perhaps a reason for joining the project. Whereas the higher scores of the teachers in the comparison group likely reflected more self-efficacy in their own mathematics instruction given the fact that they volunteered to participate as a member of the comparison group. The use of teacher and student surveys aligned to the core set of high-leverage mathematics teaching practices (NCTM, 2014) is providing us with insight into the mathematics instruction occurring in these classrooms. We will continue to further develop the teacher and student surveys and examine their use to further understand and evaluate the impact of the professional development project.

References

Ball, D. L, & Forzani, F. M. (2010). Teaching skillful teaching. *Educational Leadership, 68*(4), 40-45.

Ball, D. L, & Forzani, F. M. (2011). Building a common core for learning to teach, and connecting professional learning to practice. *American Educator, 35*(2), 17-21.

Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal, 109*, 458-474.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special? *Journal of Teacher Education, 59*, 389-407.

Banilower, E. R., Boyd, S. E., Pasley, J. D., & Weiss, I. R. (2006). *Lessons from a decade of mathematics and science reform: A capstone report for the local systemic change through teacher enhancement initiative*. Chapel Hill, NC: Horizon Research.

Burniske, J., & Meibaum, D. (2012). *The use of student perceptual data as a measure of teaching effectiveness*. Austin, TX: Texas Comprehensive Center at SEDL.

Crow, T. (2011). The view from the seats: Student input provides a clearer picture of what works in schools. *Journal of Staff Development, 32*(6), 24–30.

Ferguson, R. F. (2010). *Student perceptions of teaching effectiveness*. Discussion brief. Cambridge, MA: National Center for Teacher Effectiveness and the Achievement Gap Initiative.

Ferguson, R. F. (2012). Can student surveys measure teaching quality? *Kappan, 94*(3), 24-28.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*, 430-511.

Hlas, A. C., & Hlas, C. S. (2012). A review of high-leverage teaching practices: Making connections between mathematics and foreign languages. *Foreign Language Annals, 45*(s1), s76-s97.

Lampert, M. (2010). Learning teaching in, from, and for practice: what do we mean? *Journal of Teacher Education, 61*, 21-34.

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education, 64*, 378-386.

Measures of Effective Teaching Study. (2010). *Learning about teaching: Initial findings from the measures of effective teaching project*. Seattle, WA: Bill & Melinda Gates Foundation.

Measures of Effective Teaching Study. (2012). *Asking students about teaching: Student perception surveys and their implementation*. Seattle, WA: Bill and Melinda Gates Foundation.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

Richardson, V. (1990). Significant and worthwhile change in teaching practice. *Educational researcher, 19*, 10-18.

Appendix A
Mathematics Teaching Practice Inventory (MTPI)

Pease indicate which response best describes your current instructional practice.					
1	2	3	4	5	6
Never	Rarely	Occasionally	Frequently	Usually	Always

Mathematics Goals

1. I discuss the math learning goal for each lesson with my students.

2. I refer to the stated goal throughout each lesson to further focus students on the math they are learning.

3. My students know what they are learning about in math class.

4. My students can tell others the purpose of what they are learning in math class.

5. My students use the stated math learning goal to monitor their own understanding and progress.

Mathematics Tasks

6. I encourage my students to use varied approaches and strategies to solve math problems.

7. I regularly give students math tasks that engage them in mathematical reasoning and problem solving.

8. I purposefully select math tasks that allow multiple entry points for student learning.

9. I purposefully select math tasks that build on and extend student learning from our previous work.

10. I give my students math tasks on a regular basis that require a high level of cognitive demand.

Mathematical Representations

11. My students use tools and math representations as needed to support their thinking and problem solving.

12. I often allow students to decide which representations they want to use in making sense of the problems.

13. I ask students to make math drawings or use other visual supports to explain and justify their reasoning.

14. I assess students' abilities to use representations meaningfully to solve math problems.

15. My students often use diagrams to make sense of math problem situations.

16. My students make choices about which representations to use as tools for solving math problems.

Mathematical Discourse

17. I purposefully select and sequence the strategies students use in solving math problems for whole-class analysis and discussion.

18. I allow students to work in small groups or pairs to discuss math ideas and solve problems.

19. I engage students in whole-class discussions in which they must explain and justify their strategies for solving math problems.

20. I ask students to analyze and compare the various strategies used by their peers to solve math problems.

21. My students often discuss their ideas for solving math problems with each other in pairs or small-groups.

22. My students listen carefully to and critique the reasoning each other during whole-group discussions in math class.

Fluency from Understanding

23. I always make sure students understand why the math procedures and formulas they are using work.

24. I have students discuss why specific procedures seem to work well to solve particular math problems.

25. I use visual models to support students' understanding of mathematical procedures and formulas.

26. I carefully help students move from using inefficient to more efficient procedures without sacrificing the connection to their conceptual understanding.

27. My students demonstrate flexible use of strategies and methods for solving math problems.

Productive Struggle

28. I anticipate what students might struggle with as I prepare for teaching my math lessons.

29. I prepare for ways to address student misconceptions that might appear in a math lesson.

30. My students realize that mistakes are a natural part of learning mathematics.

31. My students see mistakes as opportunities to further their understanding of the mathematics.

32. My students know that breakthroughs in learning math often emerge from confusion and struggle.

33. My students ask questions in math class that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.

34. My students persevere in solving math problems and do not give up.

Evidence of Student Thinking

35. I am clear on what counts as evidence of student progress toward the math learning goal of each lesson.

36. I carefully plan for ways to elicit and gather evidence of student understanding at strategic points during a lesson.

37. I use evidence of student thinking to make instructional decisions as I interact with students during each math lesson.

38. I use evidence of student math learning to inform my planning and preparation for subsequent lessons.

39. My students expect me to ask them to explain their mathematical understanding, reasoning, and methods in written work and classroom discourse.

40. My students regularly reflect on their mistakes and misconceptions to improve their math understanding.

41. My students can describe specific math topics and areas in which they need to improve.

Appendix B
Student Perceptions of Math Class Survey: Grades 2–5

Please indicate how true each of the following statements is of Ms. X's math class.
Totally Untrue   Mostly Untrue   Sometimes True   Mostly True   Totally True

Goals

1. Everybody knows what they should be learning in math class.
2. My teacher tells us the purpose of what we are learning in math class.
3. I understand what I am supposed to be learning in math class.

Tasks

4. Ms. X encourages us to compare different strategies in math class.
5. The math problems we do in math class are interesting.

Representations

6. In math class, we use tools to help us solve math problems.
7. In math class, we use pictures and diagrams to show how we solved math problems.
8. The math I learn in school is useful in my everyday life.
9. Ms. X uses different ways to explain math ideas.
10. Ms. X explains difficult math problems clearly.

Discourse

11. Ms. X asks us to talk about our ideas in math class with each other.
12. Ms. X has me turn and talk with others during math class.
13. We get to work in pairs or small groups on math problems in math class.
14. Ms. X gives us time to share our thinking in small groups during math class.
15. Ms. X asks us to share our ideas with a partner during math class.

Struggle

16. When the work is hard in math class, I keep trying.
17. In math class, we learn from our mistakes.

Evidence

18. Ms. X knows when we understand the math lesson and when we do not.
19. Ms. X asks students to explain more about the answers they give in math class.
20. Ms. X asks me questions to be sure I am understanding the math we are learning.
21. Ms. X talks to me about my math work to help me understand my mistakes.
22. Ms. X helps me understand why my math answers are right or wrong.

Appendix C
Student Perceptions of Math Class Survey: Grades 6–8

Please indicate how true each of the following statements is of Ms. X's math class.

Totally Untrue   Mostly Untrue   Sometimes True   Mostly True   Totally True

Goals

1. Everybody knows what they should be learning in math class.
2. Ms. X tells us the purpose of what we are learning in math class.
3. I understand what I am supposed to be learning in math class.
4. Ms. X tells us why the math we are learning is important.

Tasks

5. Ms. X encourages us to consider different strategies or points of view in math class.
6. Ms. X helps me think about math in new ways.

Representations

7. In math class, we use tools to help us solve problems.
8. Ms. X uses drawings or diagrams to explain math concepts so I understand.
9. What I learn in math class is useful to me in my everyday life.
10. Ms. X uses different ways to help us learn new math concepts.
11. Ms. X explains difficult math problems clearly.

Discourse

12. Ms. X asks us to talk about our ideas in math class with each other.
13. Ms. X has me turn and talk with others during math class.
14. We get to work in pairs or small groups on math problems.
15. Ms. X wants me to explain and justify my answers.
16. In math class, my teacher wants us to share our strategies for solving math problems with each other.

Fluency

17. In math class, it is more important to understand math concepts than to memorize formulas.
18. Ms. X wants us to become better mathematical thinkers, not just memorize things.

Struggle

19. When the work is hard in math class, I keep trying.
20. Ms. X encourages us to learn from our mistakes in math class.
21. In math class, we view our mistakes as part of learning.
22. I work hard to do my best in math class.
23. Ms. X doesn't let students give up when the work gets hard in math class.

Evidence

24. Ms. X knows when we understand the lesson and when we do not in math class.
25. Ms. X asks me questions to make sure I understand what we are learning in math class.
26. Ms. X talks to me about my work to help me understand my mistakes in math class.
27. Ms. X gives me specific suggestions about how I can improve my work in math class.