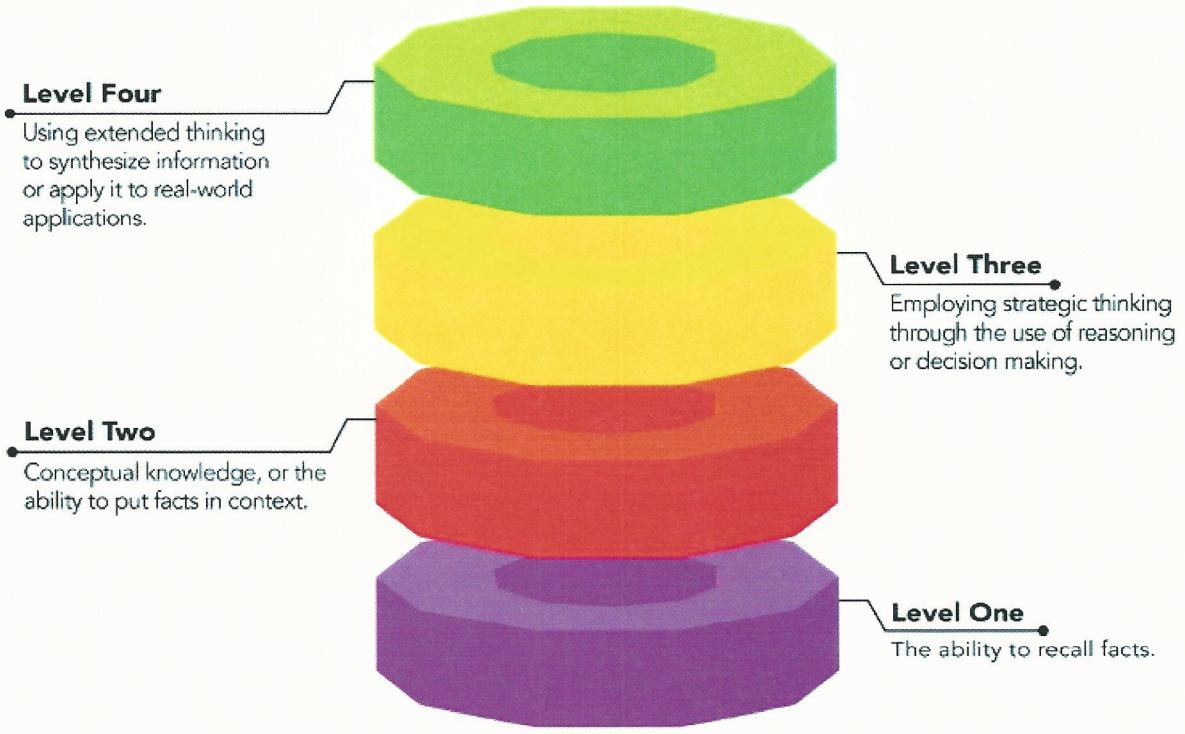
Are We There Yet? Increasing Rigor in the Mathematics Classroom



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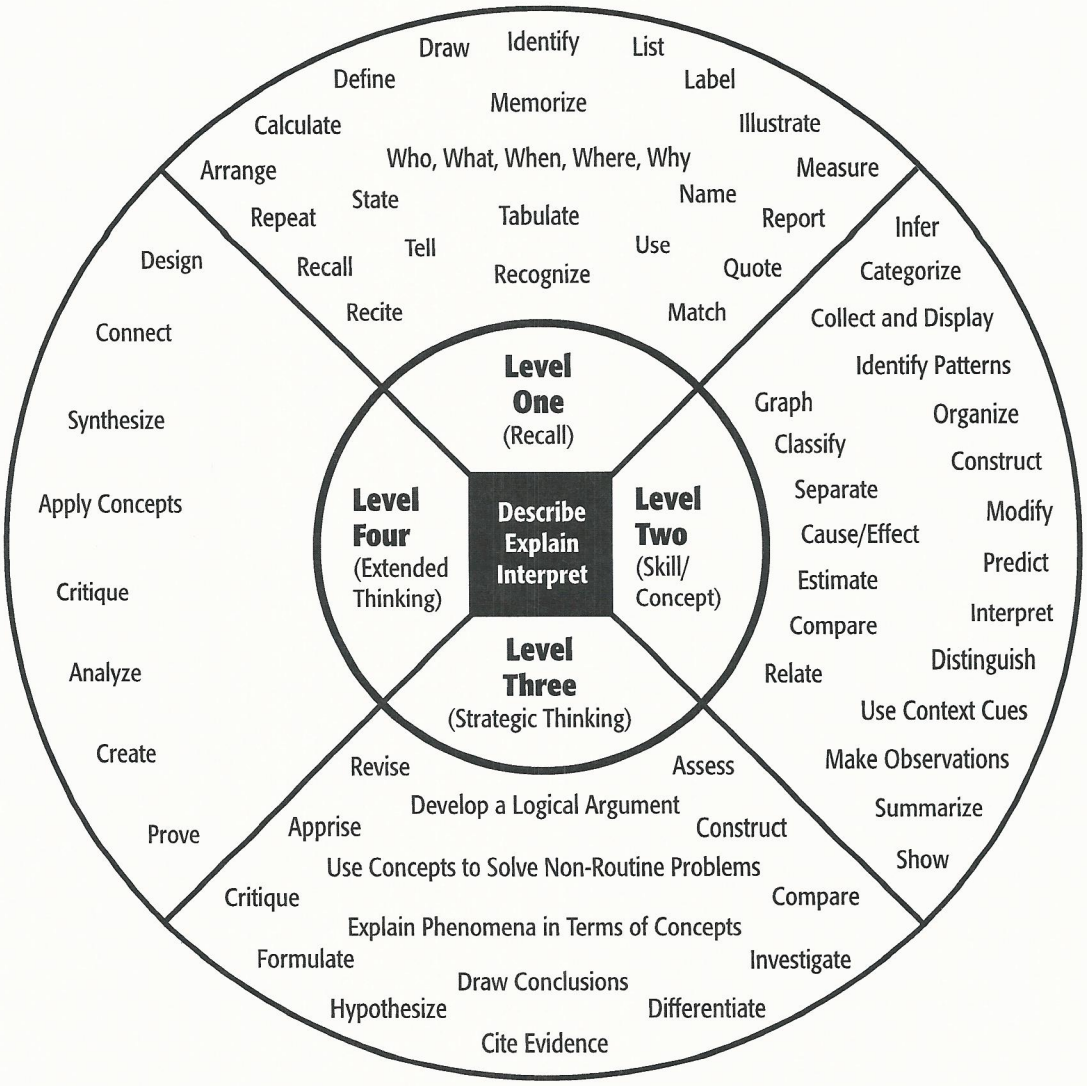
Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

DOK Question Stems

DOK 1

- Can you recall_____?
- When did ____ happen?
- Who was ____?
- How can you recognize____?
- What is____?
- How can you find the meaning of____?
- Can you recall____?
- Can you select____?
- How would you write___?
- What might you include on a list about___?
- Who discovered___?
- What is the formula for___?
- Can you identify___?
- How would you describe___?

DOK 2

- Can you explain how ____ affected ____?
- How would you apply what you learned to develop ____?
- How would you compare ____? Contrast_____?
- How would you classify____?
- How are____alike? Different?
- How would you classify the type of____?
- What can you say about____?
- How would you summarize____?
- How would you summarize___?
- What steps are needed to edit___?
- When would you use an outline to ___?
- How would you estimate___?
- How could you organize___?
- What would you use to classify___?
- What do you notice about___?

DOK 3

- How is ____ related to ____?
- What conclusions can you draw _____?
- How would you adapt____to create a different____?
- How would you test____?
- Can you predict the outcome if____?
- What is the best answer? Why?
- What conclusion can be drawn from these three texts?
- What is your interpretation of this text? Support your rationale.
- How would you describe the sequence of____?
- What facts would you select to support____?
- Can you elaborate on the reason____?
- What would happen if___?
- Can you formulate a theory for___?
- How would you test___?
- Can you elaborate on the reason___?

DOK 4

- Write a thesis, drawing conclusions from multiple sources.
- Design and conduct an experiment. Gather information to develop alternative explanations for the results of an experiment.
- Write a research paper on a topic.
- Apply information from one text to another text to develop a persuasive argument.
- What information can you gather to support your idea about___?
- DOK 4 would most likely be the writing of a research paper or applying information from one text to another text to develop a persuasive argument.
- DOK 4 requires time for extended thinking.

From Depth of Knowledge – Descriptors, Examples and Question Stems for Increasing Depth of Knowledge in the Classroom Developed by Dr. Norman Webb and Flip Chart developed by Myra Collins

Table 1: *Math Descriptors* – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity (M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

Level 1 Recall	Level 2 Skills/Concepts	Level 3 Strategic Thinking	Level 4 Extended Thinking
a. Recall, observe, or recognize a fact, definition, term, or property b. Apply/compute a well-known algorithm (e.g., sum, quotient) c. Apply a formula d. Determine the area or perimeter of rectangles or triangles given a drawing and labels e. Identify a plane or three dimensional figure f. Measure g. Perform a specified or routine procedure (e.g., apply rules for rounding) h. Evaluate an expression i. Solve a one-step word problem j. Retrieve information from a table or graph k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures l. Locate numbers on a number line, or points on a coordinate grid m. Solve linear equations n. Represent math relationships in words, pictures, or symbols o. Read, write, and compare decimals in scientific notation	a. Classify plane and three dimensional figures b. Interpret information from a simple graph c. Use models to represent mathematical concepts d. **Solve a routine problem** requiring multiple steps/decision points, or the application of multiple concepts e. Compare and/or contrast figures or statements f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones g. Provide justifications for steps in a solution process h. Extend a pattern i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps j. Translate between tables, graphs, words and symbolic notation k. Make direct translations between problem situations and symbolic notation l. Select a procedure according to criteria and perform it m. Specify and explain relationships between facts, terms, properties, or operations n. Compare, classify, organize, estimate, or order data	a) Interpret information from a complex graph b) Explain thinking when more than one response is possible c) Make and/or justify conjectures d) Use evidence to develop logical arguments for a concept e) Use concepts to solve non-routine problems f) Perform procedure with multiple steps and multiple decision points g) Generalize a pattern h) Describe, compare, and contrast solution methods i) Formulate a mathematical model for a complex situation j) Provide mathematical justifications k) Solve a multiple- step problem and provide support with a mathematical explanation that justifies the answer l) Solve 2-step linear equations/inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results m) Translate between a problem situation and symbolic notation that is not a direct translation n) Formulate an original problem, given a situation o) Analyze the similarities and differences between procedures p) Draw conclusion from observations or data, citing evidence	a) Relate mathematical concepts to other content areas b) Relate mathematical concepts to real-world applications in new situations c) Apply a mathematical model to illuminate a problem, situation d) Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results e) Design a mathematical model to inform and solve a practical or abstract situation f) Develop generalizations of the results obtained and the strategies used and apply them to new problem situations g) Apply one approach among many to solve problems h) Apply understanding in a novel way, providing an argument/justification for the application ***NOTE**: Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.*

Level Ground Math

Max asked 50 students in his school which breakfast cereal they prefer. The table below shows the results of his survey.

Cereal Survey

Breakfast Cereal	Number of Students
Yummy Flakes	12
Choco Crunch	25
Fruit Crunchies	13

1 What decimal represents the fraction of students who prefer Fruit Crunchies?

2 Explain the process you used to convert the fraction to a decimal.

3 Predict what would happen to the results if 100 students were surveyed?

On the back, write a level 1, level 2 and level 3 question about the graph. Be sure you know the answers.

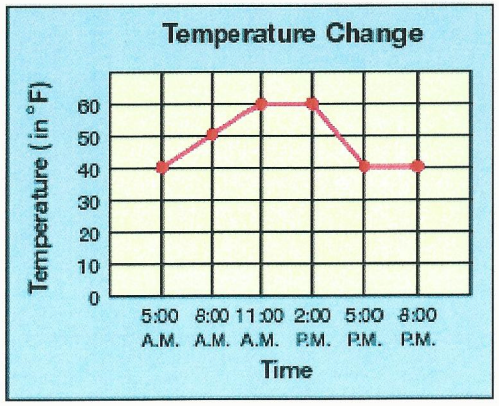
Level One
- Define
- Describe
- Identify
- List
- Name
- Observe
- Recite

Level Two
- Analyze
- Compare
- Contrast
- Group
- Infer
- Report
- Classify
- Sequence
- Rank

Level Three
- Apply
- Evaluate
- Hypothesize
- Imagine
- Judge
- Predict
- Speculate

read...interpret...think...write

Level Ground Math



1

2

3

4

Level One
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Describe
Identify
List
Name
Observe
Recite

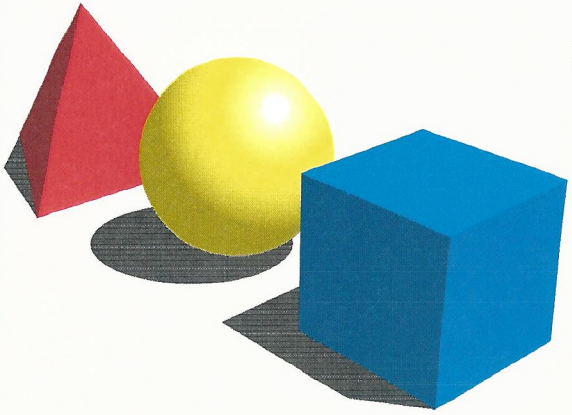
Level Two
Analyze
Compare
Contrast
Group
Infer
Report
Classify
Sequence
Rank

Level Three
Apply
Evaluate
Hypothesize
Imagine
Judge
Predict
Speculate

Write leveled questions based on the graph.

read...interpret...think...write

Level Ground Math



1

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4

Level One
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Question 1:

From any vertex of a 4-sided polygon, 1 diagonal can be drawn.
From any vertex of a 5-sided polygon, 1 diagonal can be drawn.
From any vertex of a 6-sided polygon, 1 diagonal can be drawn.
From any vertex of a 7-sided polygon, 1 diagonal can be drawn.

How many diagonals can be drawn from any vertex of a 20-sided polygon?

Answer: Level 1: This is a finding a simple, routine pattern which is immediately recognizable and requires no processing.

Question 2:

A triangle has 0 diagonals, a quadrilateral has 2 diagonals, a pentagon has 5 diagonals, and a hexagon has 9 diagonals. If the pattern continues, how many diagonals will an octagon have?

A 11
B 14
C 18
D 20

Answer: Level 2: Pattern recognition is required here but the pattern is non-routine, which brings this up to a higher DOK level. Some analysis and generalization is required to extend the pattern.

Question 3:

Order 0.6, 2/3, 7/10, and 64% from least to greatest.

Answer: Level 2: Students must not only be able to *identify* different numerical representations (Level 1), but also *manipulate* and *compare* the representations (Level 2).

Question 4:

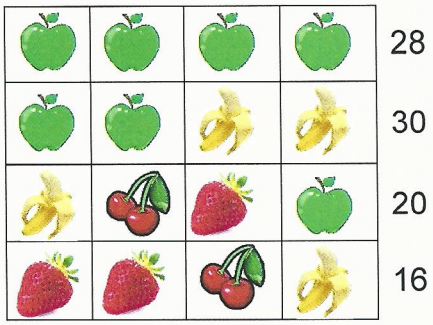
A sports-shop owner bought some baseball cards and then sold them for $7.50 each. He sold 150 cards on Monday and 82 cards on Tuesday. What piece of information is needed to find the amount of profit he made from the sale of the baseball cards on Monday and Tuesday?

A How much the shop owner paid for the baseball cards
B Number of cards sold on Wednesday
C Total number of cards sold
D Number of football cards bought by the shop owner

Answer: Level 2: Students must weigh the options to determine which provides information needed to solve the problem. *NOTE: If the choices were removed, this problems would be at DOK Level 3 because students would have to reason analytically about the missing information.*

Question 5:

Look at the drawing. The numbers alongside each column and row are the total of the values of the symbols within each column and row. What should replace the question mark?



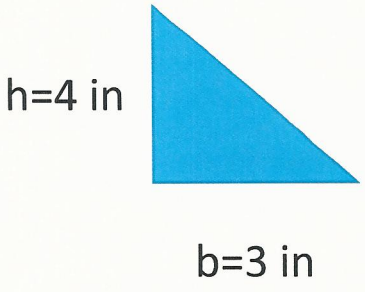
A 23
B 25
C 28
D 30
E 32

Answer: Level 3: A number of strategies can be used to solve this problem. This means students must make choices and assumptions, keeping track of a complex logical chain. The multiple choices do not make this task less complex.

Question 6:

Interpret the solution of the following problem using the formula for finding the area of a triangle:

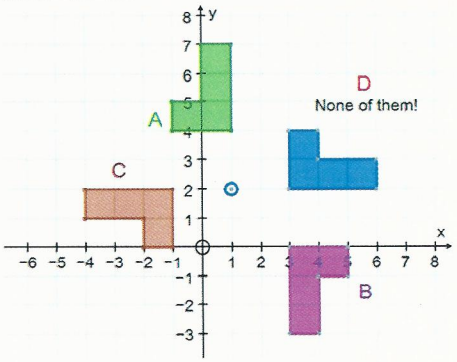
Height = 4 inches
Base = 3 inches



Answer: Level 1: This item requires students to simply follow routine procedures and perform simple calculations.

Question 7:

The blue object has been rotated 90^0 clockwise about the point (1, 2). Which is the correct image?



Answer: Level 1: This item requires students to simply recognize a rotation.

Question 8:

Two small children were playing a game. The goal of the game was to be the first one to reach the door. The children started off the game by standing 20 feet away from the door, and then each took a turn to do the following:

- Child A moved **one half** the distance between herself and the door on each move.
- Child B moved **one** foot toward the door on each move.

a. How far was each child from the door after the **first** move?
b. After **four** moves, which child was closer to the door? Show your work.
c. Child A claimed that the game was unfair because she would never reach the door. Explain why her statement is correct or incorrect.

Answer: Level 3: This problem involves an abstract idea requiring multipole steps supported with mathematical explanations and justifications.

Question 9:

Elizabeth dropped a ball from 50 feet off the ground. Each time the ball bounced, it rebounded half the distance it dropped. What was the height of rebound after the third bounce?

A 3.125 ft.
B 6.25 ft
C 12.5 ft
D 25 ft.

Answer: Level 2: The student must recognize and apply a real-world pattern using multiple steps.