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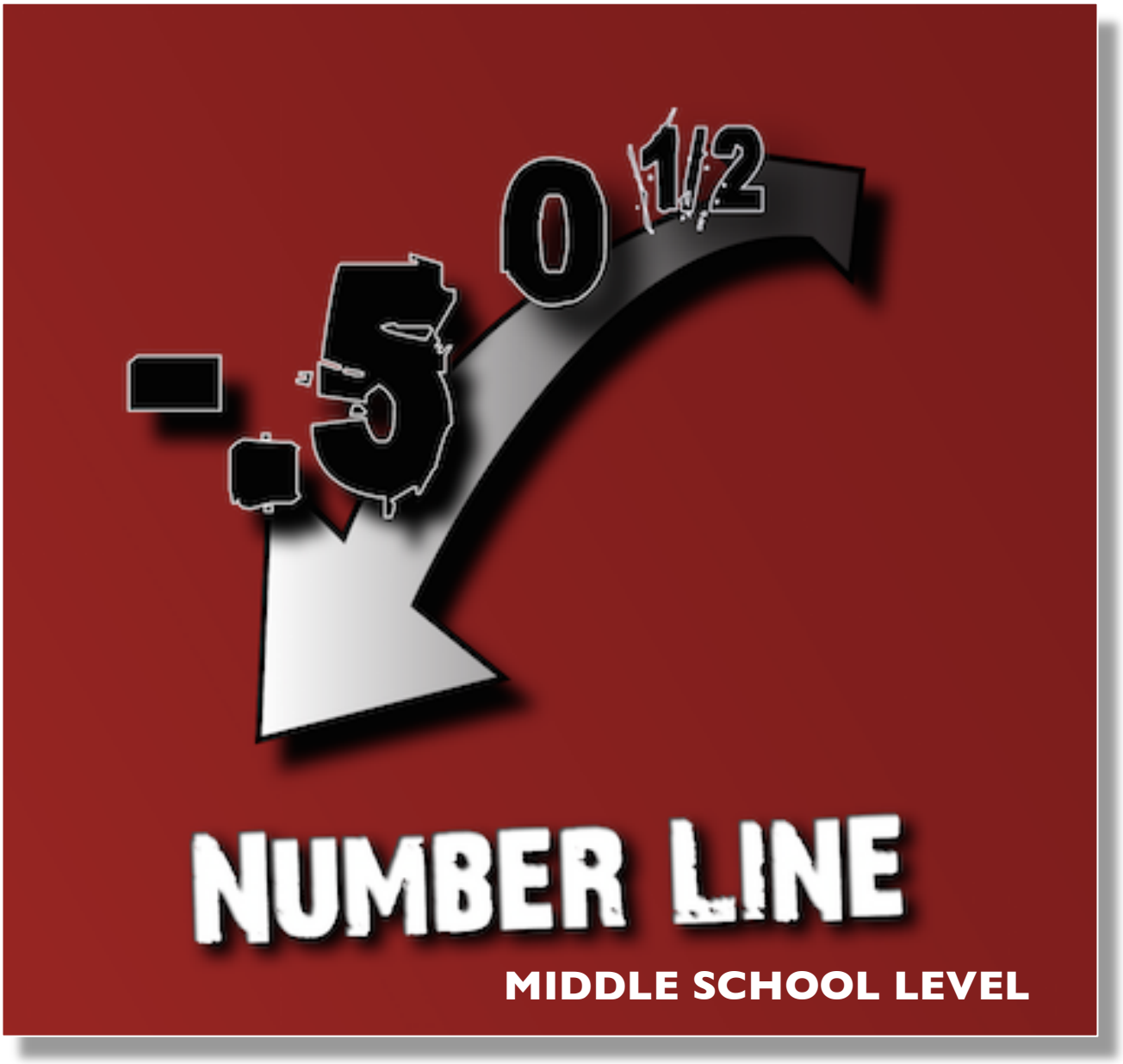

By Brad Fulton
Educator of the Year, 2005
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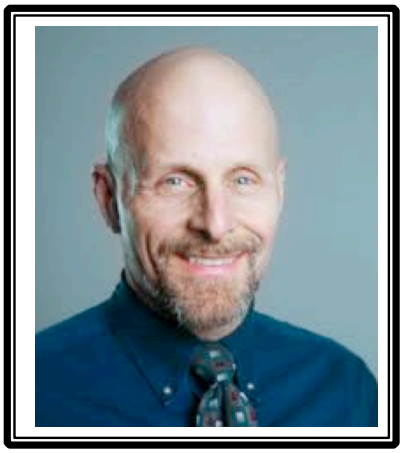
Brad Fulton

Educator of the Year



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- ♦ Consultant
- ♦ Educator
- ♦ Author
- ♦ Keynote presenter
- ♦ Teacher trainer
- ♦ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

References available upon request

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Thanks and happy teaching,

Brad 

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- ☐ b) Affordable staff development
- ☐ c) Ongoing staff development
- ☒ d) *ALL OF THE ABOVE!*

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- **Ongoing because** when you hire new staff, simply hit "play" and the training begins. There's no need to bring back the consultant.

Number Line

Middle School Version

Overview:

Students can develop incredible fluency with fraction, decimal, and percent concepts using this easy-to-implement activity. Adaptations make the lesson a perfect fit for arithmetic through algebra. Use this as a class warm up or a whole period activity. Return to this throughout the year with more and more complex numbers to develop and reinforce number concepts. Introduce variables to give students a real understanding of algebraic concepts.

Required Materials:

- ☒ Blank or preprinted number cards (included)

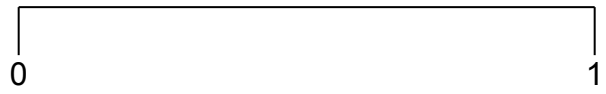
Optional Materials:

- ☐ Tape, or sticky notes, or magnets, or string and paper clips
- ☐ Calculators
- ☐ Paper

View a video vignette of the activity at:
https://www.youtube.com/watch?v=MUL0Csl2QAM

Procedure:

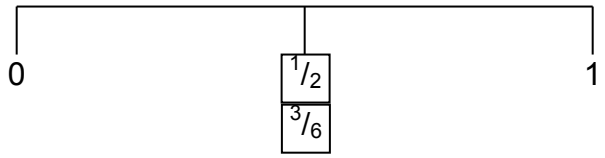
1. You will need to draw a number line on the board. It should run the length of the entire board. It should only include the numbers zero and one as shown:



2. Give each student a card from the simplified fraction set. (You can copy these onto card stock to make them more durable.) Each student can come to the board and tape their fraction where it should go. You may wish to have them do this one at a time, in small groups, or as a whole class. Students should be able to explain their reasoning for the placement. Ask other students if they agree with a card's location and have them explain why.
3. For example, let's assume $^1/_4$ and $^1/_3$ have been placed on the number line. The next student has to place $^1/_5$ on the line. He or she might use common denominators to determine that $^1/_5$ is the smallest of the three. On the other hand, the student could

reason that larger denominators mean a whole has been cut into smaller parts, so the one fifth is smaller than the one fourth or the one third. The student might use division to convert the three fractions to decimals or use percent equivalents to tell that the fifth was the smallest. Encouraging students to look at these fractions in multiple ways will foster the conceptual fraction sense that students need.

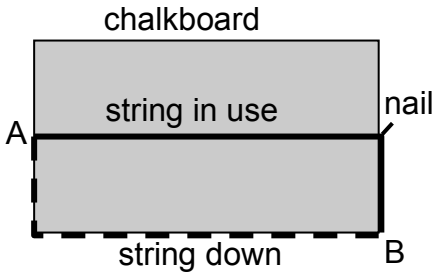
4. Once students have completed this task, they are ready for the equivalent fraction cards. There are two ways to present this activity. The first is to pass out the cards from this set as before. The second is to mix these cards with the simplified set. This will allow students to see the equivalency of the fractions as shown here:

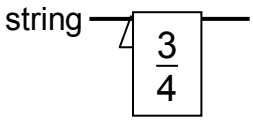


5. The third activity that can be done with the cards involves the decimal cards. Again these may be presented individually or mixed with the fraction cards to show equivalency. Both approaches are conceptually rich, and doing the activity one way does not preclude doing it the other.

6. Lastly, mix the percent cards with the decimal and fraction cards so students will become familiar with all three forms of fraction notation.

7. This activity is most powerful if presented over a period of days throughout the year. More learning and more retention will occur that way. If you would like to make it easier to use this activity on a regular basis, consider putting up a permanent number line. To do this, run a string from a nail or tack on one side of your chalkboard (A) to the other side, over another nail, and down to the chalk tray (B) as shown. Fasten it there with a third nail. When you want to remove the number line, unhook it from the nail and run it under the chalk tray as shown by the dotted line. Each fraction card has a dashed line indicating where to fold it backward so it can be hung on such a string. You may wish to use a





paper clip to secure the card.

8. Two blank masters are included to make cards of your own. You can also use 3 x 5 index cards or sticky notes.

9. Students invent creative and sophisticated methods for analyzing fractions in this activity. For example, I have had students note that $^{13}/_{18}$ is greater than $^{9}/_{14}$ because the first fraction lacks $^{5}/_{18}$ from being a whole while the other lacks $^{5}/_{14}$. Since eighteenths are smaller than fourteenths, the first fraction is missing five smaller parts" while the second is missing five larger parts.

10. Another strategy students have invented involves finding common numerators. I have seen students reason that $^{5}/_{9}$ is less than $^{10}/_{17}$ because $^{5}/_{9}$ is equal to $^{10}/_{18}$. They then reason that since seventeenths are bigger than eighteenths, $^{10}/_{18}$ is the smaller of the two.

11. Yet another clever observation has led students to conclude that the following fractions are in the correct order:

$$^{6}/_{11} \qquad ^{7}/_{12} \qquad ^{8}/_{13}$$

They note that each numerator is increasing by one as is each denominator. Also, each numerator is five less than its denominator. If you continued this pattern indefinitely, you would eventually you would eventually encounter $^{95}/_{100}$ and $^{995}/_{1000}$. These fractions are approaching a value near 1. Taking the pattern backward would eventually yield $^{1}/_{6}$ and $^{0}/_{5}$. Thus the pattern increases in value as you move to the right.

Algebra Extension:

12. For algebra students, draw a blank number line on the board. Don't put in a zero or a one. Give out the card with the "x" on it and ask a student to place it. Some students may hesitate and claim that it can't be placed until you tell them the value of x. However, other students will realize that since x is unknown, it can go anywhere on the number line. Ask them, "What do we currently know about the value of x?" Since there are no reference numbers on the line, all we know is that it is a real number.

13. Now give a student or the class another card such as $-x$. Most students will place this to the left of the x card. Ask them why they did this. They may say that since it has a negative sign, it must be less than zero. Ask them if there is ever a case when $-x$ could go to the *right* of x. They should realize that if $x < 0$, then this would mean that

$-x$ would be positive. Allow them to place the $-x$ on whichever side they prefer and ask them, "What do we now know about the value of x? If they put the $-x$ to the left of the x, then we know that $x > 0$. If it is to the right of ˆxˆ, then $x < 0$. The third case would be if they placed the x and the $-x$ vertically, then we know that $x = -x$ and x must have a value of zero.

14. Continue to pass out cards and ask these types of probing questions. This activity will go *much* more slowly than the activity using fractions, decimals, and percents because the thinking is so much more intense. It is not uncommon for me to get to only five or six cards in a single period. After each card, I ask the students to write down what they know about the value of x. I also ask them if the most recent card could possibly go in any other location. Other examples follow.

15. Ask them to place x^2. Is there ever a case when it could be to the *left* of x? (Yes). What do you know about the value of x if this is the case? ($0 < x < 1$)

16. Where does $\sqrt{x}$ go in relation to x? What do we know about the value of x if $\sqrt{x}$ is to the left of x? ($x > 1$).

17. If $x - 1$ is placed vertically with $2x$, what value(s) of x would satisfy the equation?

$$x - 1 = 2x$$

$$-1 = x$$

18. Is there ever a case in which x and x^2 could be aligned vertically, that is $x = x^2$? (Yes, there are two cases: $x = \{0,1\}$ This can be shown as an algebraic equation solved here by factoring:

$$x = x^2$$

$$0 = x^2 - x$$

$$0 = x(x - 1)$$

$$x = \{0, 1\}$$

19. For which values of x is $x-1$ greater than $-x$?

$$x-1 > -x$$

$$x > -x + 1$$

$$2x > 1$$

$$x > \tfrac{1}{2}$$

Journal Prompts:

Put these three fractions in order from least to greatest. Explain your reasoning.

$^{3}/_{5}$ $^{7}/_{10}$ $^{6}/_{11}$

The fractions $^{1}/_{4}$ and $^{2}/_{5}$ have been placed on the number line. What fraction might go in between them? How do you know?

Homework:

You can use the enclosed activity master for homework, or assign a page from a text or workbook.

Taking a Closer Look:

Include mixed numbers and improper fractions. Or ask students to place the reciprocals of their cards on the number line.

Assessment:

This activity can be checked as the students are putting up their cards. you can also check them by converting all the cards to decimals. If you assign the homework master, use the following key to check it.

Answer Key:

Set 1:

$^{1}/_{4}$ $^{3}/_{8}$ $^{1}/_{2}$ $^{3}/_{4}$ $^{5}/_{6}$

Set 2:

$^{1}/_{4} = {}^{2}/_{8}$ $^{7}/_{12}$ $^{9}/_{15} = {}^{3}/_{5}$ $^{2}/_{3}$

Set 3:

$^{1}/_{2}$ $^{2}/_{3}$ $^{3}/_{4}$ $^{4}/_{5}$ $^{5}/_{6}$

Set 4:

$^{1}/_{5}$ = .2 $^{1}/_{4}$ 30% $^{1}/_{3}$ .35

Set 5:

.5% .05 $^{1}/_{10}$ $^{1}/_{4}$ 50% 100%

Activity master

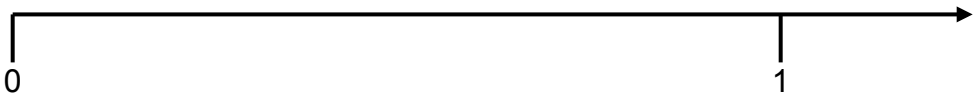
Number Line Fractions

Name_____

Put the following fractions, decimals or percents on their number lines in the correct locations.

Set 1: $\frac{3}{8}$ $\frac{5}{6}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$

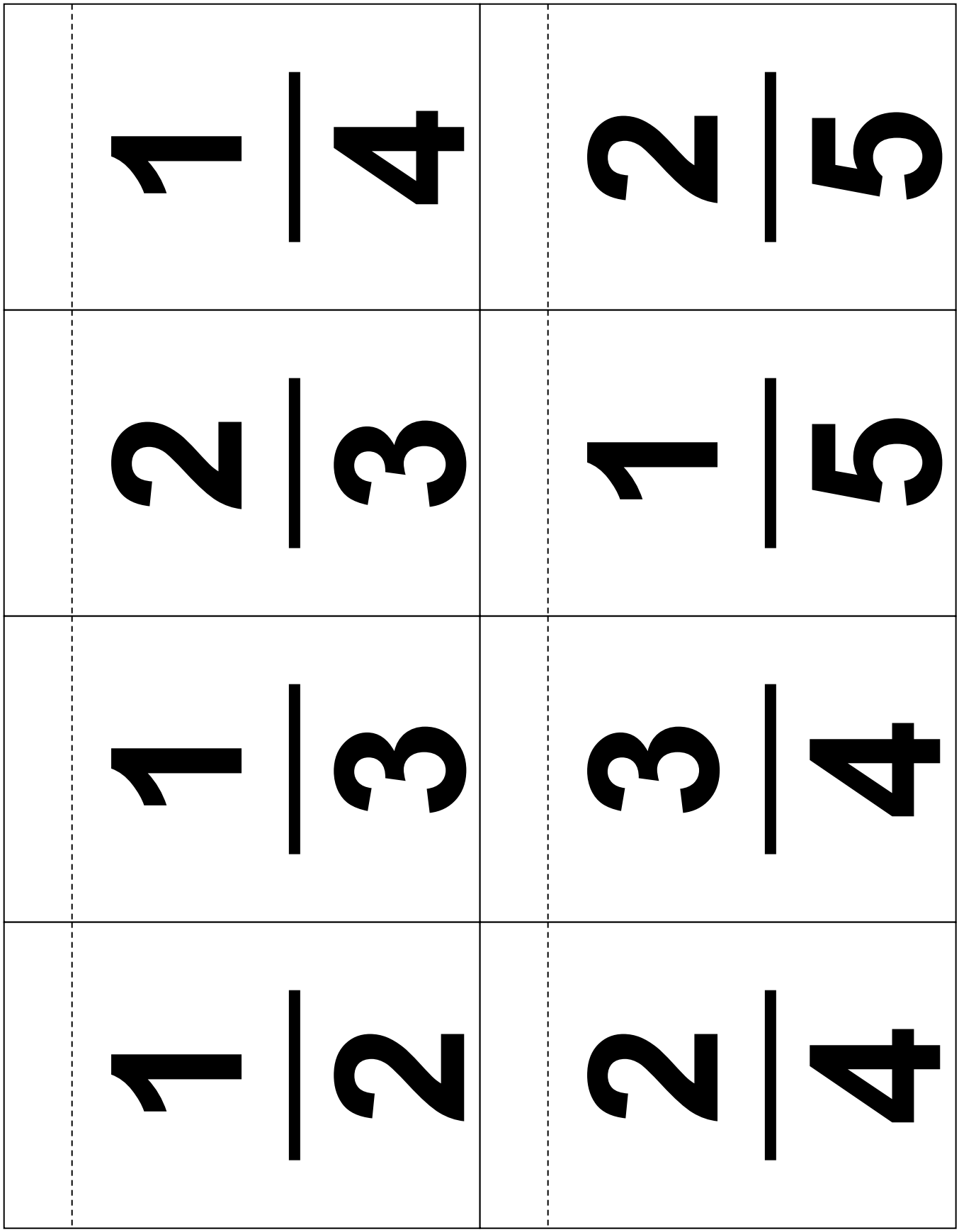
Set 2: $\frac{2}{8}$ $\frac{2}{3}$ $\frac{9}{15}$ $\frac{7}{12}$ $\frac{1}{4}$ $\frac{3}{5}$



Set 3: $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{4}{5}$

Set 4: $\frac{1}{3}$ $\frac{1}{5}$ .2 $\frac{1}{4}$ 30% .35

Set 5: 100% .05 $\frac{1}{10}$ 50% .5% $\frac{1}{4}$



2 | 6	1 | 7
1 | 6	5 | 6
4 | 5	4 | 6
3 | 5	3 | 6

5 | 7	3 | 8
4 | 7	2 | 8
3 | 7	1 | 8
2 | 7	6 | 7

7 | 8	4 | 8
6 | 8	3 | 8
5 | 8	2 | 8
4 | 8	1 | 8

$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$
$\frac{9}{10}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$

$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$
$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$

.1	.01
.2	.20
.25	.3
.33	.35

.4	.45
.5	.50
.05	.6
.65	.7

10%	1%
25%	50%
75%	$33\frac{1}{3}\%$
$66\frac{2}{3}\%$	11%

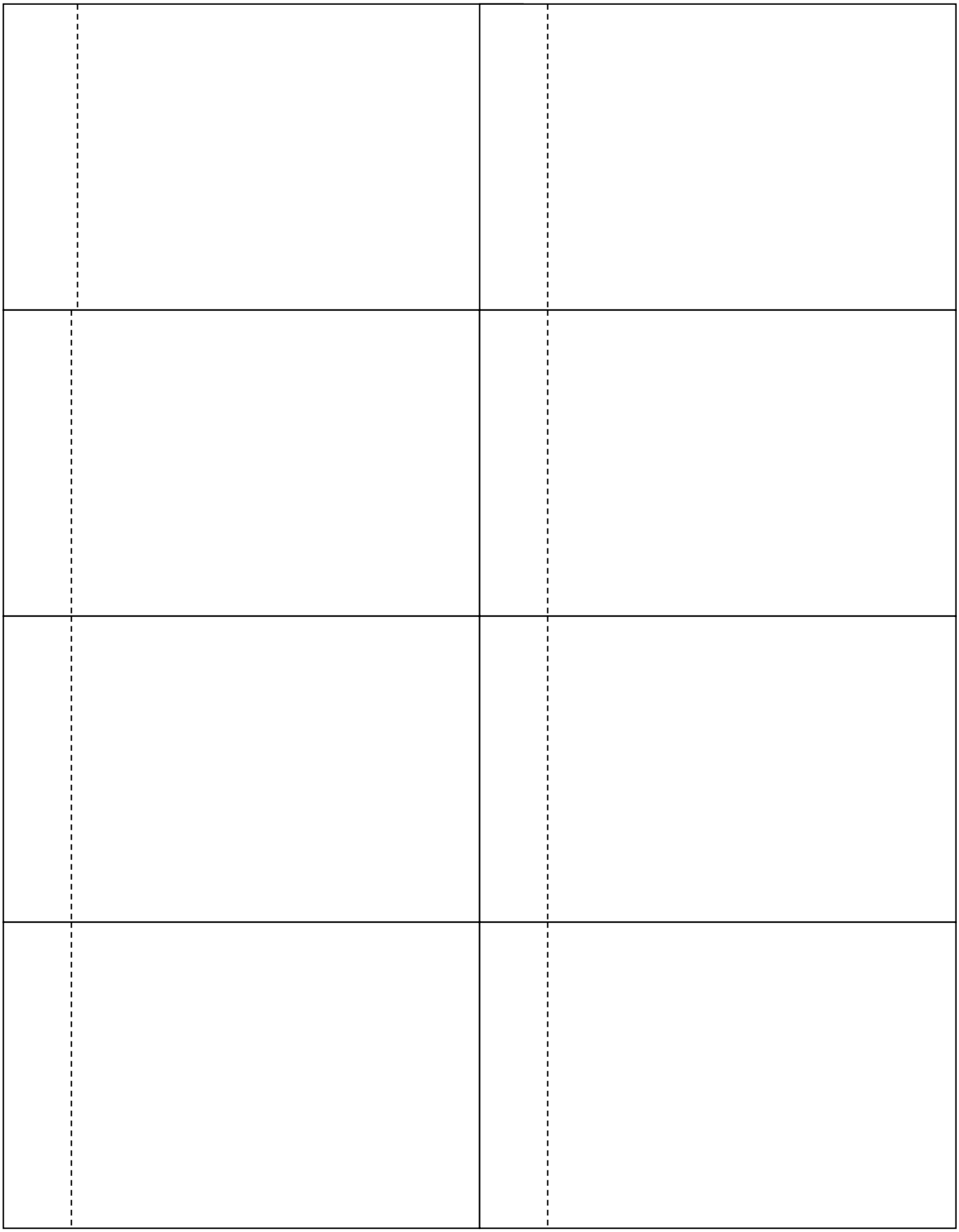
5%	9%
11%	72%
65%	70%
60%	80%

x	$2x$	x^2	x^3
$\frac{x}{x}$	$\frac{1}{x}$	$\frac{x}{1}$	$\frac{2x}{5}$

$x+1$	$x-1$
$x+2$	$-(x^2)$
$-x+1$	$(-x^2)$
$-2x$	$(-x)^2$

1	$\frac{x}{2}$
-1	$\frac{x}{.5}$
0	$\frac{2x}{x}$
$-x$	$\frac{x^2}{x}$

$\sqrt{x}$	$x-x$
$\sqrt{x^2}$	$x+x$
$-\sqrt{x^2}$	$2x-x$
$-.5x$	$x-2x$



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Similar activities include:

- *Number Line:* ***Elementary Version****:* Simpler fractions, decimals and percents and including tips for implementing Number Line across other content areas such as social studies and science.
- *Take Your Places: Activities 7–12*: Covering multiplication and division of fractions, order of operations, using parentheses
- *Pyramid Math:* A great way to practice addition and subtraction of whole numbers, decimals, and integers with extensions to algebra.
- *Math Maps:* Developing the Mathematical Practices
- *Developing Number Sense:* Great strategies to foster greater mathematical thinking, mental math, and number sense.

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad