Book 1

Investigation 4

Dividing With Fractions

In earlier investigations of this unit, you learned to use addition, subtraction, and multiplication of fractions in a variety of situations. There are times when you also need to divide fractions. To develop ideas about when and how to divide fractions, let's review the meaning of division in problems involving only whole numbers.

Getting Ready for Problem 4.1

Students at Lakeside Middle School raise funds to take a field trip each spring. In each of the following fundraising examples, explain how you recognize what operation(s) to use. Then write a number sentence to show the required calculations.

- The 24 members of the school swim team get dollar-per-mile pledges for a swim marathon they enter. The team goal is to swim 120 miles. How many miles should each swimmer swim?

- There are 360 students going on the field trip. Each school bus carries 30 students. How many buses are needed?

- The school band plans to sell 600 boxes of cookies. There are 20 members in the band. How many boxes should each member sell to reach the goal if each sells the same number of boxes?

Compare your number sentences and reasoning about these problems with classmates. Decide which are correct and why.

4.1 Preparing Food

There are times when the amounts given in a division situation are not whole numbers but fractions. First, you need to understand what division of fractions means. Then you can learn how to calculate quotients when the divisor or the dividend, or both, is a fraction.
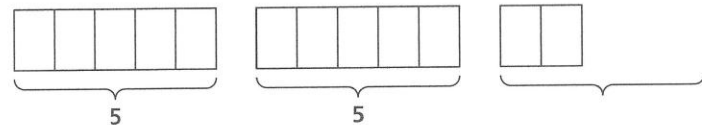
When you do the division $12 \div 5$, what does the answer mean?

The answer should tell you how many fives are in 12 wholes. Because there is not a whole number of fives in 12, you might write:

$$12 \div 5 = 2\frac{2}{5}$$

Now the question is, what does the *fractional part* of the answer mean?

The answer means you can make 2 fives and $\frac{2}{5}$ *of another five.*

Suppose you ask, "How many $\frac{3}{4}$'s are in 14?" You can write this as a division problem, $14 \div \frac{3}{4}$.

Can you make a whole number of $\frac{3}{4}$'s out of 14 wholes?

If not, what does the fractional part of the answer mean?

As you work through the problems in this investigation, keep these two questions in mind.

What does the answer to a division problem mean?

What does the fractional part of the answer to a division problem mean?

Note: Book 1 © 2009. Book 2 © 1998. Homework problems are not included.

Book 1

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculatiòn(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. 1. Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

Problem 4.2. Dividing a Fraction by a Whole Number
Problem 4.3. Dividing a Fration by a Fraction

4.4 Writing a Division Algorithm

You are ready now to develop an algorithm for dividing fractions. To get started, you will break division problems into categories and write steps for each kind of problem. Then you can see whether there is one "big" algorithm that will solve them all.

Problem 4.4 Writing a Division Algorithm

A. 1. Find the quotients in each group below.

Group 1	Group 2	Group 3	Group 4
$\frac{1}{3} \div 9$	$12 \div \frac{1}{6}$	$\frac{5}{6} \div \frac{1}{12}$	$5 \div 1\frac{1}{2}$
$\frac{1}{6} \div 12$	$5 \div \frac{2}{3}$	$\frac{3}{4} \div \frac{3}{4}$	$\frac{1}{2} \div 3\frac{2}{3}$
$\frac{3}{5} \div 6$	$3 \div \frac{2}{5}$	$\frac{9}{5} \div \frac{1}{2}$	$3\frac{1}{3} \div \frac{2}{3}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that works for dividing *any* two fractions, including mixed numbers. Test your algorithm on the problems in the table. If necessary, change your algorithm until you think it will work all the time.

B. Use your algorithm to divide.

1. $9 \div \frac{4}{5}$ **2.** $1\frac{7}{8} \div 3$ **3.** $1\frac{2}{3} \div \frac{1}{5}$ **4.** $2\frac{5}{6} \div 1\frac{1}{3}$

C. Here is a multiplication-division fact family for whole numbers:

$5 \times 8 = 40$ $8 \times 5 = 40$ $40 \div 5 = 8$ $40 \div 8 = 5$

1. Complete this multiplication-division fact family for fractions.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

2. Check the division answers by using your algorithm.

D. For each number sentence, find a value for N that makes the sentence true. If needed, use fact families.

1. $\frac{2}{3} \div \frac{4}{5} = N$ **2.** $\frac{3}{4} \div N = \frac{7}{8}$ **3.** $N \div \frac{1}{4} = 3$

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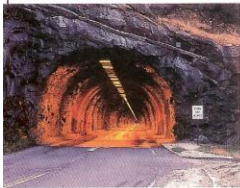
Book 2

7-4 Dividing Whole Numbers by Fractions

You'll Learn ...

■ to divide a whole number by a fraction

... How It's Used

Structural engineers divide whole numbers by fractions when building tunnels.

Vocabulary

reciprocal

▶ **Lesson Link** In the last section, you learned to multiply whole numbers by fractions. Now you'll divide whole numbers by fractions. ◀

Circles and Strips Forever

Dividing a Whole Number by a Fraction

$4 \div \frac{2}{3} = 6$

- Draw a number of strips equal to the whole number.
- Divide the strips into equal pieces. The number of pieces in each strip should be equal to the fraction denominator.
- Circle groups of equal pieces. The number of pieces in each circled group should equal the numerator.
- Describe the number of groups circled.
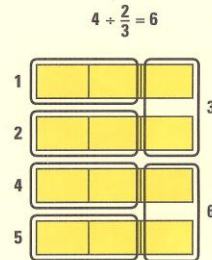
1. Model these problems.

   a. $6 \div \frac{2}{3}$ **b.** $7 \div \frac{1}{2}$ **c.** $5 \div \frac{5}{6}$ **d.** $4 \div \frac{3}{6}$ **e.** $2 \div \frac{2}{7}$

2. When you divide a whole number by a fraction less than 1, is the quotient larger or smaller than the original whole number? Why?

3. Will $3 \div \frac{2}{5}$ have a whole-number answer? Explain.

Learn Dividing Whole Numbers by Fractions

You can think of division as taking a given amount and breaking it down into groups of a certain size. For example, $6 \div 2$ can be modeled as 6 loaves of bread divided into groups of 2. The quotient, 3, is the number of groups you have.
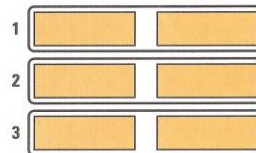
You can think of dividing by fractions in the same way. For example, $6 \div \frac{2}{3}$ is the same as 6 loaves of bread divided into groups of $\frac{2}{3}$. The number of groups you have, 9, is the quotient.
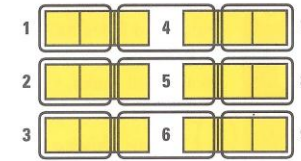
Notice that to find the answer, you first found the number of thirds by multiplying the number of loaves, 6, by the denominator, 3. Then, you divided the number of thirds by the numerator, 2.

$$6 \div \frac{2}{3} = 6 \times 3 \div 2 = 9$$

Dividing by a fraction is the same as multiplying by its **reciprocal** . Reciprocals are numbers whose numerators and denominators have been switched. When two numbers are reciprocals, their product is 1.

Dividing	Multiplying by reciprocal
$6 \div \frac{2}{3} = 9$	$6 \times \frac{3}{2} = \frac{6}{1} \times \frac{3}{2}$
	$= \frac{18}{2}$
	$= 9$

Remember

The numerator is the number on top of a fraction. The denominator is the number on the bottom. **[Page 287]**
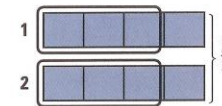
Examples

1 Divide: $2 \div \frac{3}{4}$

$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$ Multiply by the reciprocal of the fraction.

$= \frac{2 \times 4}{1 \times 3}$

$= \frac{8}{3}$ or $2\frac{2}{3}$

$2 \div \frac{3}{4} = 2\frac{2}{3}$

2 1 *nail* $= \frac{9}{4}$ in. of cloth. Find the length of 5 in. of cloth in nails.

$5 \div \frac{9}{4} = \frac{5}{1} \times \frac{4}{9}$ Multiply by the reciprocal.

$= \frac{20}{9}$ or $2\frac{2}{9}$ Simplify.

A 5-inch piece of cloth is $2\frac{2}{9}$ nails long.

Try It

Divide.   **a.** $4 \div \frac{3}{5}$ **b.** $1 \div \frac{4}{7}$ **c.** $10 \div \frac{17}{4}$ **d.** $3 \div \frac{3}{5}$

DID YOU KNOW?

Three measurements used primarily for cloth include the *nail*, the *finger*, and the *span*. A *finger* is equal to $4\frac{1}{2}$ inches. A *span* is equal to 9 inches.

Book 2

WHAT DO YOU THINK?

Peter and Erica have a recipe that makes 9 quarts of punch. They want to know how many $\frac{3}{4}$-quart (3-cup) servings the recipe will make.

Peter thinks ...

I'll use mental math.

How many groups of $\frac{3}{4}$ quart are in 9 quarts?

Every whole quart has one $\frac{3}{4}$ in it, plus $\frac{1}{4}$ left over.

In 9 quarts, there are nine $\frac{3}{4}$ quarts, which is 9 servings.

There are nine $\frac{1}{4}$ quarts left over. They can be regrouped as three $\frac{3}{4}$ quarts, which is another 3 servings.

That equals $9 + 3$ or 12 groups of $\frac{3}{4}$ quart.

We will have 12 servings.
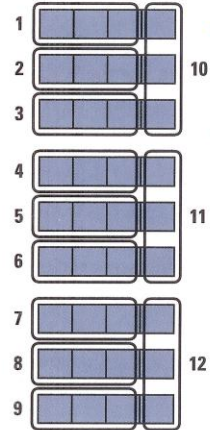
Erica thinks ...

I'll divide 9 by $\frac{3}{4}$. To do that, I'll multiply $\frac{9}{1}$ by the reciprocal of $\frac{3}{4}$.

$$9 \div \frac{3}{4} = \frac{9}{1} \times \frac{4}{3} = 12$$

We will have 12 servings.

What do you think?

1. Whose method is easier to use without paper and pencil? Explain.

2. How could you find the answer by writing 9 quarts as $\frac{36}{4}$ quarts?

Check Your Understanding

1. How could you use the "multiply by the reciprocal" rule to divide 20 by 5?

2. If you divide a whole number by a proper fraction, is the quotient larger or smaller than the whole number? Explain.

Book 2

Dividing Fractions by Fractions

7-5

► **Lesson Link** In the last lesson, you learned to divide whole numbers by fractions. Now you'll divide fractions by fractions. ◄

You'll Learn …

■ to divide a fraction by a fraction or a whole number

… How It's Used

Choreographers divide fractions by fractions when fitting a dance routine to a particular tempo of music.

Explore | Dividing Fractions by Fractions

Wish Upon a Bar

Materials: Fraction Bars®

Dividing a Fraction by a Fraction

$\frac{2}{3} \div \frac{1}{12} = 8$

- Using a Fraction Bar®, draw and label the first fraction.

- Under that, use a Fraction Bar® to draw as many diagrams of the second fraction as will fit.

| $\frac{1}{3}$ | $\frac{1}{3}$ |

$\frac{1}{12}$

- Describe the number of diagrams below the first fraction.
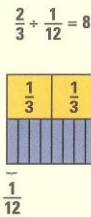
1. Model each problem.

  a. $\frac{3}{6} \div \frac{1}{12}$ **b.** $\frac{1}{2} \div \frac{1}{4}$ **c.** $\frac{2}{3} \div \frac{1}{6}$ **d.** $\frac{2}{4} \div \frac{2}{12}$
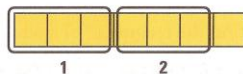
2. When you divide a fraction by a fraction less than 1, why is the answer bigger than the fraction you started with?

3. How is dividing a fraction by a fraction similar to dividing a whole number by a fraction?

4. Can you use Fraction Bars® to divide $\frac{1}{2} \div \frac{1}{5}$? Explain.

Learn | Dividing Fractions by Fractions

When you divide a whole number by a fraction, you get the same result as if you had multiplied the whole number by the fraction's reciprocal. This is also true when you divide a fraction by a fraction.

Dividing   Multiplying by Reciprocal

$\frac{6}{7} \div \frac{3}{7} = 2$ $\frac{6}{7} \times \frac{7}{3} = \frac{42}{21}$ or 2

1   2

► **History Link**

The term *furlong* originated in the Middle Ages. The furlong was originally a "furrow long," the length of a plowed strip of land on a standard-size field.

Remember

You can write a fraction in lowest terms when the numerator and the denominator share a common factor.
[Page 292]

Example 1

Horse races are measured in *furlongs* (1 furlong = $\frac{1}{8}$ mi). The Kentucky Derby is $1\frac{1}{4}$ mi long. How long is the Kentucky Derby in furlongs?

$1\frac{1}{4} \div \frac{1}{8} = \frac{5}{4} \div \frac{1}{8}$ Write the numbers as fractions.

$= \frac{5}{4} \times \frac{8}{1}$ Multiply by the reciprocal.

$= 10$

The Kentucky Derby is 10 furlongs long.

When dividing a fraction by a whole number, you can write the whole number as a fraction with 1 as its denominator. You can then multiply the fraction by the reciprocal of the whole number.

Example 2

What is $\frac{3}{5} \div 6$?

$\frac{3}{5} \div 6 = \frac{3}{5} \div \frac{6}{1}$ Write the whole number as a fraction with a denominator of 1.

$= \frac{3}{5} \times \frac{1}{6}$ Multiply by the reciprocal of $\frac{6}{1}$.

$= \frac{3}{30}$ or $\frac{1}{10}$ Simplify.

Try It

Divide.   **a.** $\frac{4}{5} \div \frac{5}{8}$ **b.** $\frac{3}{7} \div \frac{2}{7}$ **c.** $\frac{1}{5} \div 2$ **d.** $\frac{2}{5} \div 10$

Check | Your Understanding

1. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$. Does $\frac{1}{2} \div \frac{1}{4} = \frac{1}{4} \div \frac{1}{2}$? Explain.

2. If you find the reciprocal of a whole number, what will the numerator of that reciprocal be equal to?