# Tom versus Tom

Two of the more prominent actors over the last thirty years have been Tom Cruise and Tom Hanks. If you want to determine which actor has had a better career, what data might you consider? Present a case, based on data, outlining why one actor's career has been better than the other.

Trains of Thought

A train is composed of an engine and cars of various sizes, ranging in length from 1 unit to 10 units. The cars are placed end to end following the engine of the train. The train is described by the total length of the cars, not counting the engine. For example, all trains below have 4 units.

Note: Although all cars of the same length are indistinguishable from one another, the different arrangements of the cars make different trains.
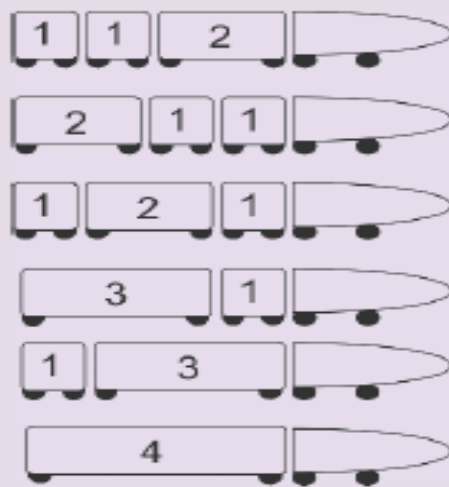
a. In how many ways can you create a 6-unit train?

b. In how many ways can you create a 10-unit train?

c. Determine the number of different ways that you can create a train of unit length n, using an unlimited number of cars of length 1 through length n.

Teacher's tip: To represent the cars of length 1 to 10, ask students to use Cuisenaire rods, copies of the template (*note:* see the template that is online with the April 2012 issue), or grid paper to outline cars of various lengths. The template can be printed and cut into separate "train cars" to help students count the possible trains.

Odd Shape Out

Consider the four quadrilaterals shown below. Which shape is the *odd shape out*? In other words, find and describe a property that three of the shapes have but the fourth shape does not.

Shape 1

Shape 3

Shape 2

Shape 4

Extension: Find and describe a way that each of the four shapes could be the odd shape out.

CCSSM: 5.G.B.3
Mathematical Practices: 1, 5, 6, and 7

Lemon Tea?

I have one large pitcher containing 1 quart of lemonade and a second large pitcher containing 1 quart of iced tea. I pour 1 pint of lemonade into the iced tea pitcher and thoroughly mix it, and then pour 1 pint of the mixture back into the lemonade pitcher.

1. Is there more iced tea in the lemonade or more lemonade in the iced tea? Justify your answer mathematically.

2. If the process is repeated a second time, how many ounces of lemonade will be in the original iced tea pitcher?

Extension: Is it possible to make both mixtures into 1/2 lemonade and 1/2 iced tea, using the same process repeatedly? Why or why not?

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Pruning Trees

It takes Keith 45 minutes to prune a tree. It takes Richard 70 minutes to prune a tree. They form a company and agree to prune 92 trees. If they both work at the same time and at constant rates, how long will it take them to prune all 92 trees? How many trees will each have pruned?

CCSSM: 6.RP.1, 2, 3; 7.RP2; 6.EE.2, 3, 4, 6; 7.EE.2
Standards for Mathematical Practice: 1, 2, 3

NCTM Regional Conference   Thursday, November 18, 9:45 – 11:00 am   Nashville, TN

Picking and Packing Strawberries

On Rhonda's family farm, the workers picked 5/12 of the strawberries in the first week. These strawberries were put in 10 crates, and 20 kg were leftover. The crates were sent off to the supermarket, and the other 20 kg were sold at the family's fruit stand. During the second week, the crew picked the remaining strawberries and placed them in 15 crates. How many kilograms of strawberries did the family's crew pick during those two weeks?

Solve the problem in two different ways.

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Patterns on a Calendar
By JoAnn Cady and Pam Wells

October, 2015						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

There are lots of interesting patterns on a calendar. Below are five conjectures middle school students found on the October, 2015 calendar (above). Will all of these patterns continue to work if you move the shape to a different spot on the calendar? Why or why not? If a pattern doesn't work when the shape is moved, can you modify it to make it work?

Chelsea: Red box – I noticed that the sum of any two adjacent squares is always odd. 19+26=45

Megan: Brown box: I noticed that the sum of any three squares in a row is always even. 29+30+31=90.

Jessie: Purple "L" shape—I noticed that the sum of the 4 numbers in my L-shape is 48. That is eight times as large as the smallest number in my pattern (6x8=48).

Nathan: Blue box—I noticed that the products of the diagonals of my square seven apart (3x11=33 and 4x10=40).

Devrah: Orange diagonal—I noticed that the sum of my three numbers is a multiple of 3 (1+9+17=27).

Find and justify your own calendar pattern.

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Answers:

Chelsea: When looking at adjacent squares, one is odd and one is even. The sum of an odd and even will always result in an odd number, since there will always be one left over when dividing by two.

Megan: This works if two of the three numbers are odd. Each odd number will have one left over; these leftovers will combine to make another even number. It will not work if two of the three numbers are even. There will be one left over from the odd number, which would make the sum odd.

Jesse: The sum would be four times the sum of the smallest number and 6 or 4(x+6) where x is the smallest number. [If the smallest number is x, then the next numbers in the pattern would be x +7, x + 8, and x + 9. Adding these together gives one x + x + 7 + x + 8 + x + 9 or 4x + 24.]

Nathan: This will not always work. Answers may vary as to how students may modify this conjecture so that it will work.

Devrah: If you choose a number, the number on the diagonal would be the chosen number + 8. The number on the diagonal from this number would be the chosen number + 16. If you added these three numbers you would get 3 times the chosen number + 24. Since 24 is a multiple of 3, the sum of the 3 numbers on the diagonal is also a multiple of 3.

CCSSM: 6.EE.2, 3, 4, & 6; 7.EE. 2
Mathematical Practices: 1, 2, 3, 7, & 8.

We encourage classroom teachers to pose this problem to their students and submit creative solutions to share with our readers. Please include a brief analysis of the specific strategy; examples of original student work or high quality digital images; and your name, the school name and address, and your email address. Email submissions to JoAnn Cady at jcady@utk.edu. The name and school of each participating teacher, but only the first name of the students whose work is being examined, will be cited. Release forms for students' work should also be submitted (http://www.nctm.org/WorkArea/DownloadAsset.aspx?id=23295) with the student work sample. Submission of student release form does not guarantee that the student's work will be published.

Puppy Love

By JoAnn Cady and Pam Wells

Ian's golden retriever just had a litter of puppies. Ian is a budding scientist and decides to keep track of some data about the puppies. Here is some information Ian collected about his puppies.

- There are 4 female and 3 male puppies in the litter.
- The median birth weight of the puppies is 14oz
- The mean birth weight of the puppies is 15 oz.
- The mode birth weight of the puppies is 14 oz.
- The range of the birth weights is 8 oz.

1. Make a list or a plot of possible weights for Ian's puppies. Find another set of puppy weights that matches the information given. Explain your strategy.

2. Ian weighs the puppies one week after they are born and discovers that all of the puppies have gained exactly 8 oz. How will the median, mean, mode, and range of the puppies' weights compare to those of the puppies' weights at birth?

3. Ian weighs the puppies again after another week and discovers that 3 of the puppies have each gained 10 oz., 2 puppies have gained 16 oz., and 2 puppies were sick and each lost 3oz. What is the mean weight of the puppies now?

CCSS: 6.SP. 2; 6.SP.3; 6.SP.5.c; 6.SP.5.d; 7.SP.3; & 7.SP.4
Mathematical Practices: 1, 2, 3, 7, & 8

Answers

1. Answers will vary. One possible set is 11, 13, 14, 14, 16, 18, & 19.

2. The range will remain the same. The mean, median, and mode will each increase by 8 to be 22.

3. The mean will increase by 8 again to be 31.

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Thank all of you for coming to our session. Sorry I have been delayed in posting the problems. However, I am glad I was delayed. One of our session attendees stopped me later and offered a fantastic idea. He purchased laminated notebook paper, punched holes in the top, and placed hooks on the walls of his room. His students use dry erase markers to show their work on this laminated notebook paper. The work is shared by hanging the laminated notebook paper on the hooks. He takes pictures then erases the laminated paper and saves to reuse. Great idea – thanks for sharing.