

***Real* Problem Solving Adventures!**

Presented by:

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at the

**National Council of Teachers of Mathematics Annual
Meeting and Exposition**

Nashville, TN

November 2015



Polya

(from a lecture on teaching)

“Mathematics is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean to be doing mathematics? In the first place, it means to be able to solve mathematical problems.”



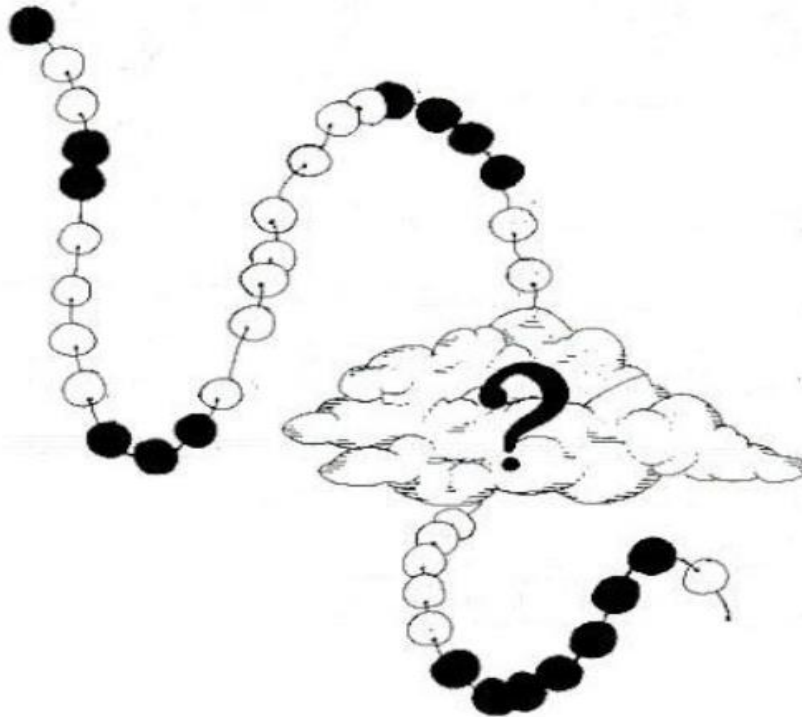
Let's Discuss

- When is solving a problem really problem solving?
- What are characteristics of a 'good' problem solver?



Problem 1: Beads Under the Cloud

How many beads are hidden under the cloud?



NOTE - the cloud is not drawn to scale, and

HINT - remember planes often fly through clouds....

Problem from: Kentucky Department of Education: Secondary Differentiation Resources

<http://education.ky.gov/educational/diff/Documents/BeadsUnderTheCloud.pdf>



Problem 1: Beads Under the Cloud

How many beads are hidden under the cloud?



NOTE - the cloud is not drawn to scale, and
HINT - remember planes often fly through clouds....

Are there multiple solutions to this problem?

What are some potential student misconceptions?

Is this a good problem solving problem? Why or why not?

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Student Example



Problem from: Kentucky Department of Education: Secondary Differentiation Resources
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Common Core State Standards

<http://www.corestandards.org/the-standards/mathematics>

- Standards for Mathematical Practice
 - Make sense of problems and persevere in solving them.
 - Reason abstractly and quantitatively.
 - Construct viable arguments and critique the reasoning of others.
 - Model with mathematics.
 - Use appropriate tools strategically.
 - Attend to precision.
 - Look for and make use of structure.
 - Look for and express regularity in repeated reasoning.



Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL

Specific, research-based teaching practices that are essential for a high-quality mathematics education for all students are combined with core principles to build a successful mathematics program at all levels.

Principles to Actions offers guidance to teachers, mathematics coaches, administrators, parents, and policymakers.

It includes:

- Eight research-based essential Mathematics Teaching Practices
- Conditions, structures, and policies necessary to support the Effective Teaching Practices
- Implementation strategies for the Common Core State Standards for Mathematics and the *Principles and Standards for School Mathematics* and designed to attain high levels of mathematics achievement for all students
- Unproductive and productive beliefs, obstacles, and key actions that must be acknowledged, and addressed by all stakeholders
- Strategies for teachers to engage students in mathematical thinking, reasoning, and problem making to significantly strengthen teaching and learning

[Executive Summary](#) [Resumen Ejecutivo](#) [Overview](#)

<http://www.nctm.org/PtA/>

Professional Learning Materials

- Exploring Proportional Relationships: Candy Jar (Frasco de dulces)
 - [The Case of Mr. Donnelly](#) [El caso del profesor Donnelly](#) [Effective Teaching Practices](#)
- Exploring Exponential Relationships: Pay It Forward
 - [The Case of Ms. Culver](#) [Effective Teaching Practices](#)
- Exploring Representations for Multiplication: The Band Concert
 - [The Case of Mr. Harris](#) [Effective Teaching Practices](#)

Related Resources



Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

(NCTM, PtA 2014, p. 10)



Problem #2: The Square Pizza Puzzle

The Pizza Palace has started to make square pizzas. This sounds good to Myra, who wants her family to one for dinner.

“No way,” says her brother Kenji. “It costs too much. A 12-inch round pizza with four toppings costs \$11.00 at Mondo’s. The Pizza Palace wants \$13.00 for their 12-inch square pizza with four toppings.”

Show which pizza company charges more per identical size piece of pizza.



Engage

How would you engage your students with this problem?

I will have a picture of a pizza on the television for the students to see as they walk into the room. I will engage the students by passing around clippings of pizza coupons. We will begin to have a discussion about what they see on the coupons from different pizza companies. I will ask them how they truly know when they are getting the best deal. This will lead into them exploring the pizza puzzle.

What would you think about if I said pizza? I would think about Chicago because I am from Chicago; so when I think of pizza, I think of deep-dish pizza. The #1 deep-dish pizza happens to be from Chicago. Has anyone tried deep-dish pizza? You know deep-dish pizza can be circular or it can be square, which brings me to my problem. My family and I wanted to get some pizza, but we had some disagreements on price because my dad wanted the most pizza for the best price. Here was our problem:



Explore

When students explore, what is the teacher's role/responsibility? What is the student's role/responsibility?

On your own, you will try to figure out the problem. There are many different ways you can solve this problem.

(After 5 minutes)

Now, turn to your partner and discuss how to solve it or if you have already solved it, share your strategy.



Explain

How can we assist students in succeeding as a problem solver?

Walk around the classroom and monitor the students as they answer the problem. Pick a few students who solved the problem in different ways and ask them to present to the class how they found the answer.

Ask the students who presented how they solved the problem. After each student presents how they found the answer, ask the rest of the class if anyone solved the answer similarly to the student who presented.

After the students present, I will show how I solved the problem.



Would Students Do?

12-inch
\$11.00
\$2.75
\$13.75

12-inch
\$13.00
\$3.85
\$16.85

50

X	A	B
1	11	13
2	22	26
3	33	39
4	44	52
5	55	65
6	55	78

They are both 12-inch one is round one is squared. The square pizza cost more than the round pizza. The pizza that will cost the least is the round.

Show which pizza company charges more per identical size piece of pizza.

because of the 4 corners on the square pizza there is more pizza so the prices is reasonable

12 in

$$\begin{array}{r} 6^2 \\ \times 12 \\ \hline 36 \\ \times 12 \\ \hline 144 \end{array}$$

about 110

Show which pizza company charges more per identical size piece of pizza.

I need to know the amount of the slices to complete this. As it stands, I can't solve it.

... Well, actually...

12 inch
144 inches

6 inch 6 inch
About 113 inches

... I don't get it from here.

The PIZZA PALACE	Mondo's
9 slices	8 slices
13 dollars	11 dollars
\$1.44 per slice	\$1.38 per slice



Candy Jar Problem

Suppose you have a new candy jar with the same ratio of Jolly Ranchers (JR) to jawbreakers (JB) as shown in the picture, but it contains 100 Jolly Ranchers.

How many jawbreakers do you have?

Justify your answer.

Note: In the picture, Jolly Ranchers are represented by 5 rectangles, and jawbreakers are shown by 13 circles.



Fig. 12. The Candy Jar task. Adapted from Smith et al. (2005).



Candy Jar Problem-Potential Student answers!

- 260
- 360
- 40
- 26
- 340
- 50
- 240
- 270
- I do not know
- 2600
- 65
- 100
- 38.14
- 82.3
- 30
- 87
- 250
- I do not understand
- 108
- 325
- 35
- 61
- 6.9
- 38
- 160
- 13
- 20
- 36
- 7 R 9
- 50/50
- Less than 78 but more than 65



NCTM Classroom Resources

- <http://www.nctm.org/Classroom-Resources/Connect-with-NCTM-Illuminations/>



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Success Stories



- <http://www.nctm.org/Classroom-Resources/Core-Math-Tools/Core-Math-Tools/>
- Hirsch. Martin. Hopfensperger. Zbiek (AMTE 2013)



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Core Math Tools



Core Math Tools

General Purpose Tools

Custom Apps

Advanced Apps

Sample Lessons

Data Sets

How-To Pages

Core Math Tools is a [downloadable suite](#) of interactive software tools for algebra and functions, geometry and trigonometry, and statistics and probability. The tools are appropriate for use with any high school mathematics curriculum and compatible with the Common Core State Standards for Mathematics in terms of content and mathematical practices. Java required.



General Purpose Tools CAS, Spreadsheet, Geometry, Data Analysis, and Simulation	Custom Apps Focused explorations of specific topics	Advanced Apps Focused explorations of advanced topics
Sample Lessons Problem-based lessons that employ <i>Core Math Tools</i>	Data Sets Wealth of data sets organized by data type	How-To Pages Help, hints and steps to do basic tasks



NCTM Supporting Resources

- <http://www.nctm.org/Conferences-and-Professional-Development/Professional-Development-Resources/>

Professional Development Resources



Principles to Actions Toolkit

This toolkit consists of a series of grade-band specific modules that are focused on a subset of the effective teaching practices as well as professional learning resources that support implementation of the five guiding principles in the publication, *Principles to Actions: Ensuring Mathematical Success for All*, other than Teaching and Learning: Access and Equity, Curriculum, Tools and Technology, Assessment, and Professionalism.

Professional Development Guides

NCTM school journals and other publications are a rich resource for professional development. Professional Development Guides are detailed guides for using journal articles and books as professional development experiences.



Sample NCTM Resources

www.nctm.org

- *Implementing the CCSS through Mathematical Problem Solving: **For various grade bands***
- *Connecting the NCTM Process Standards and the CCSSM Practices*
- *5 Practices for Orchestrating Productive Mathematics Discussions*
- *Principles to Action: Ensuring Mathematical Success for All*
- *Essential Understandings*



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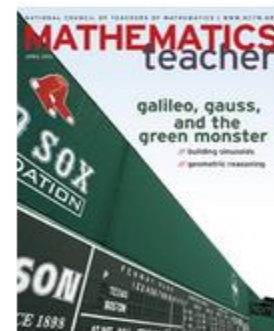
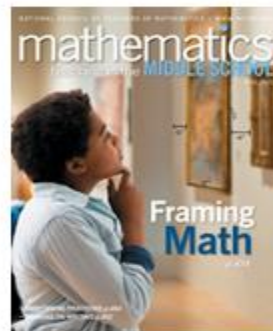
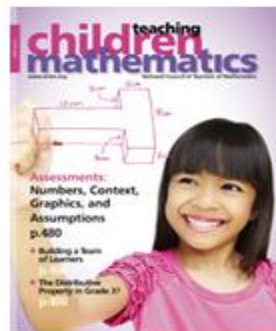


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April 13–16, 2016
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Scottsdale Plaza Resort, Phoenix, AZ



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<http://www.todos-math.org>

**Classroom Teacher
Scholarships Available!**

TODOS 2016 Conference is co-sponsored by NSF-funded Arizona Master Teachers of Mathematics (AZ-MTM), award #1035330, administered by the Department of Mathematics at The University of Arizona.



Thank you!

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