## I Fall To Pieces

Staying Rational Dealing With Fraction Models


## What's up with all the fraction models?

- Models build conceptual understanding.

- Models create a cognitive connection...just in case one cannot recall the algorithm
- Computation without connections works with the "good memorizers."



## It's easy in the beginning with part-whole.....

And we show equivalence...

The part-whole relationship is easily illustrated.


## Fraction as a number? Not so much, but we're getting better....

Number lines illustrate a fraction as a numerical value...

...and that there can be fractions larger than one!
a. Mark and label points on the number line for $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}$, and $\frac{6}{2}$.

b. Mark and label a point on the number line for $\frac{11}{3}$. Be as exact as possible.


Source: illustrativemathematics.org
(and that equivalent fractions can have more than one name)

## Now to the operations......

Manipulatives are not new when teaching fractions
For example, find the sum of $\frac{2}{6}$ and $\frac{3}{6}$

$=1$, then:


$$
=\frac{1}{6}
$$



Added to


Gives us $\frac{5}{6}$

And if denominators are different....

$$
\frac{5}{6}-\frac{2}{3}
$$


$\square$

## Number Lines?

$$
\frac{2}{6}+\frac{3}{6}
$$

$$
\frac{5}{6}-\frac{2}{3}
$$

$$
\frac{5}{6}+\frac{7}{6}
$$

## Adding Pieces and Parts...

- Develop the concept with manipulatives
- Illustrate "fraction as a number," using number lines
- Move to the algorithm...why?...because it's efficient and works well for all numbers, not just the pretty ones
- Beware of tricks that get the answer. How we get the answer DOES matter. If you can't explain the math behind the procedure, it's best not to use that procedure.


## Multiplication and Division

Multiplication begins by multiplication of a fraction by a whole number.

$$
\begin{gathered}
\frac{2}{3} \times 5 \\
5 \text { groups of } \frac{2}{3}
\end{gathered}
$$



Multiplication on a number line...

## On the other hand...

$$
5 \times \frac{2}{3} \quad \text { Means if you have } 5 \text { wholes, what is } \frac{2}{3} \text { of that amount? }
$$



Count the blue parallelograms, and you will see that there are 10 of them, resulting in $\frac{10}{3}$


While $5 \times \frac{2}{3}$ can be illustrated on a number line, it is more useful for showing reasonableness of an answer after learning the traditional algorithm.

## Division of fractions and whole numbers... Whole number divided by a fraction

- $3 \div \frac{1}{2} \quad$ How many one-halves are
there in ${ }^{3}$ ?

$3 \div \frac{2}{3}$
How many two-thirds are there in ${ }^{3}$ ?



## Bar Models are also effective in illustrating...

- $3 \div \frac{1}{2} \quad$ How many one-halves are there in 3?
$3 \div \frac{2}{3} \quad$ How many two-thirds are there in 3?

Number lines can also work, but choose units carefully. Essentially, bar models work similarly to number lines.

# Division of fractions...fraction divided by a whole number. 

1
$\frac{1}{2} \div 3$
How many threes are there in one-half?

## After a party, there were two-thirds of a pizza left over. This pizza was to be shared equally between 4 kids. What fraction of a pizza did each child receive?



$$
\frac{2}{3} \div 4
$$

Take the $2 / 3$ of the pizza that remains into 4 equal parts. Each of those parts is what fraction of a whole pizza?

A bar model would also work in this type of problem.

# And to finish up $5^{\text {th }}$ grade, let's multiply some fractions...area model works well... 

What is two-thirds of three fourths?

## But so can a number line...

## And it works for fractions larger than one...

$\frac{7}{2} \times \frac{2}{5}$

And here's the number line...
$\frac{7}{2} \times \frac{2}{5}$

## Can multiple models work for fractions?

- Yes, most models will work for most operations, but not all are clear and simple to understand.
- The purpose of a model is to make a connection and to develop conceptual understanding
- If the model complicates, rather than assists, it is probably not an effective model.
- The ULTIMATE GOAL is efficient computation, most likely through the traditional algorithms that we know and love.


## Stay Rational

- Use modeling wisely, and our students will stop "falling to pieces" over fractions.



## Information and References..



## References:

- Developing Essential Understanding of Rational Numbers Grades 3-5; Barnet-Clarke, Fisher, Marks, Ross; NCTM; 2010
- Uncomplicating FRACTIONS to Meet Common Core Standards in Math, K-7; Small; NCTM, Nelson Education; 2014

