Teaching and Assessing Addition Fact Fluency

National Council of Teachers of Mathematics
(NCTM) Regional Meeting
Nashville, TN
November 20, 2015

Jennifer Bay-Williams & Amy Stokes
University of Louisville

Procedural Fluency is "skill in carrying out procedures flexibly, accurately, efficiently and appropriately."
(CCSSO, 2010, p. 6)

Phases of Basic Fact Mastery (Baroody, 2006)

Phase 1: Counting
(counts with objects or mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
(efficient production of answers)

CCSSM Expectations Related to Basic Facts

Grade K Standard K.OA.A.4: "For any number from 1 to 9, find the number that **makes 10** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation."

Grade 1 Standard 1.OA.C.6: "Add and subtract within 20, **demonstrating fluency*** for addition and subtraction within 10. **Use strategies** such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums."

Grade 2 Standard 2.OA.B.2 : "**Fluently** add and subtract within 20 using mental strategies (reference to 1.OA.6). By end of Grade 2, **know from memory** all sums of two one-digit numbers."

Note: This list illustrates that CCSS-M recognizes the importance of reasoning strategies (Phase 2) before expecting automaticity with their facts (Phase 3).

Selected Games for Addition and Subtraction Facts

Finding Combinations of 5

Use a deck of Ten Frame cards, using only cards 0 – 5. This is like concentration. Spread the cards out face down.

Each player turns over two cards and adds the numbers together. They say the equation using appropriate language. For example, if he/she turns over a 2 and a 5, player 1 says, "2 plus 5 equals 7." If the two cards do not equal 5, they are turned back upside down. If the two cards do equal 5, the player keeps the pair.

Fixed Addend "War"

Use a deck of cards, removing face cards. Identify an addend (e.g., 9) and place one of those values in the middle. Deal the rest of the cards to the two players, face down. Each player turns up a card, says the sum, followed by the addition sentence, taking turns with who goes first. The player with the larger sum gets the cards.
Example: Fixed Addend is 9. Player one turns up a 2 and says "Eleven – Nine plus two equals eleven (or two plus nine is eleven)." Player 2 two turns up a 6 and says, "15 – Nine plus six equals fifteen." If there is a tie – it is a 'war' and they repeat the process, but the winner of the next round wins both sets of cards.

Double to Connect 4

Children take turns flipping over a number card, doubling the value of the card, then placing an X or O anywhere in that double column. A player with four in a row, column or diagonal wins.

| 2	4	6	8	10	12	14	16	18
2	4	6	8	10	12	14	16	18
2	4	6	8	10	12	14	16	18
2	4	6	8	10	12	14	16	18
2	4	6	8	10	12	14	16	18
2	4	6	8	10	12	14	16	18

Salute!

Place students in groups of three, and give each group a deck of cards (omitting face cards and using aces as ones, jacks as zeros). Two students draw a card without looking at it and place it on their forehead facing outward (so the others can see it). The student with no card tells the sum. The other two players determine the value of their cards. Once both players have done so, they look at their cards and then students rotate roles before starting the next hand. This can be differentiated by including only certain cards (e.g., addition facts using only the numbers 1 through 5).

Subtraction Stacks

Each player needs 10 counters. Each player places the 10 counters on the number line (below the numbers) wherever they want to place them.

On a players turn, she rolls both dice, subtracts the smaller from the larger. If the difference is 2, she removes a counter from the 2. If she has nothing at the 2, she does not remove a counter. The winner is the person who is able to remove all of their counters first.
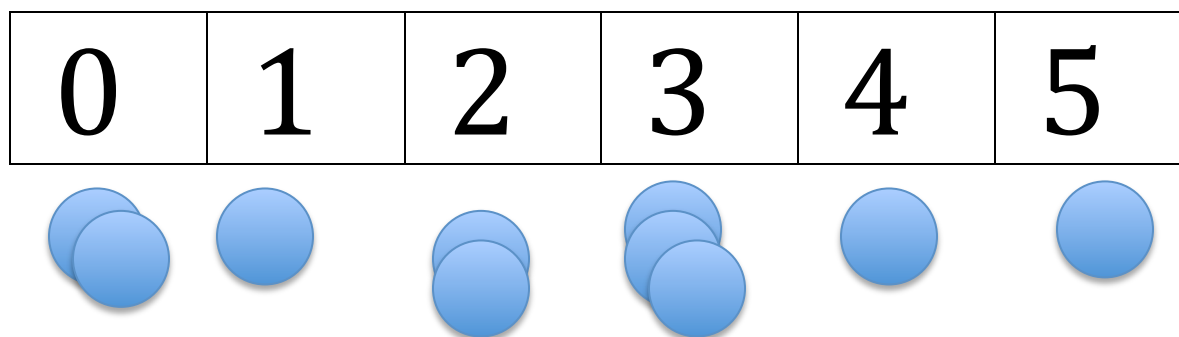
What is One More?

(Britt, 2014)

The goal to this game is to have four counters in a vertical, horizontal, or diagonal row. Play with a partner. Roll the number cube. Cover a square that is One More than what you rolled.

2	3	4	5	6	7
3	4	5	6	7	2
4	5	6	7	2	3
5	6	7	2	3	4
6	7	2	3	4	5
7	2	3	4	5	6

Questions to Ask While Playing Facts Games

Use questions such as the following to encourage good mathematical thinking during game play:

- *How did you figure it out?*
- *Can you say out loud how you thought about it in your head?*
- *Is there another way you could figure it out?*
- *Can you think of another fact that strategy would work well for?*
- *If someone didn't know the answer to _____, how would you tell them to figure it out?*

Addition Fact Fluency Quiz

Solve these problems and tell how you solved out.

4 + 5 = _____ Check one: ___ I used this strategy: _____
          ___ I just knew.

10 + 6 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

6 + 2 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

5 + 3 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

2 + 9 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

3 + 10 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

5 + 7 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

8 + 10 = _____ Check one: ___ I used this strategy: _____
         ___ I just knew.

Facts Assessment: First-grade journal responses to
"If your friend didn't know the answer to 4 + 5, how could he figure it out?"

I would tell my friend to yoos a double plus 1. 4+4=8 so count 1 up. now you get your ancer

I would tell my friend to takaway one number from ten. And that is nine. I know that five plus five equals ten.

MAY 10, 2012 I would tell my friend to take 5 and count 4 in your hand

I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.

Assessing Basic Fact Mastery

(Kling & Bay-Williams, 2014)

Interviews

Focus on fluency:

1. Write 4 + 5 on a card. [point at card] What does 4 + 5 mean?

2. What is the answer to 4 + 5?

3. How did you find the answer to 4 + 5? Could you find it another way?

4. If your friend was having trouble remembering this fact, what strategy might you suggest to her/him?

Focus on flexibility and **strategy selection.**

1. What is 8 + 5?

2. How can you use 8 + 2 to help you solve 8 + 5?

Writing Prompts

Flexibility	Accuracy
Solve 8 + 7 using one strategy. Now try solving it using a different strategy.	What is the answer to 9 + 4? How do you know it is correct (how might you check it)?
Efficiency	**Appropriate Strategy Selection**
Which facts do you "just know"? Which facts do you use a strategy to solve?	Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.

Strategy Tracking Table: Addition Facts

Names ☞	Addition Fact Strategies						
	1 more/ 2 more	Combinations that Make 10	Making 10	Doubles	Near Doubles	Find Fives	Applies Commutative Property

Kling, G. & Bay-Williams. J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics*.

Additional Resources for Developing and Assessing Addition Fact Fluency

Please see additional ideas for games and assessment tools in the following articles and their related More4U sections:

Bay-Williams, Jennifer M. & Gina Kling (2015). Developing Fact Fluency. Turn Off Timers, Turn Up Formative Assessment. In NCTM Annual Perspectives in Mathematics Education (APME) 2015: Assessment to enhance learning and teaching. Chris Suurtamm, (Ed.) National Council of Teachers of Mathematics, Reston, VA.

> *Contains examples of different types of fact assessments that can be used across the three phases of fact mastery.*

Bay-Williams, Jennifer M. and Gina Kling (2014). Enriching Addition and Subtraction Fact Mastery through Games. *Teaching Children Mathematics* 21(4): 238-247.

> *Contains game directions in the main article as well as detail on how to use the games to help children progress through the three phases of fact mastery.*

Britt, B.A. (2014). Mastering Basic Math Skills: Games for Kindergarten through Second Grade. Reston, VA: NCTM.

> *Contains many games for many of the concepts in K-2, including basic facts.*

Kling, Gina and Jennifer M. Bay-Williams (2014). Assessing Basic Fact *Fluency*. *Teaching Children Mathematics 20(8):* 488-497.

> *Contains examples of different types of fact assessments that can be used to more accurately capture the four components of fluency.*

Kling, Gina (2011). Fluency with Basic Addition. *Teaching Children Mathematics 18(2):* 80-88.

> *Contains a summary of basic addition strategies as well as game directions, including games focused on developing foundational fact fluency.*