Delving Deeper Into The Derivative, The Central Concept of Calculus

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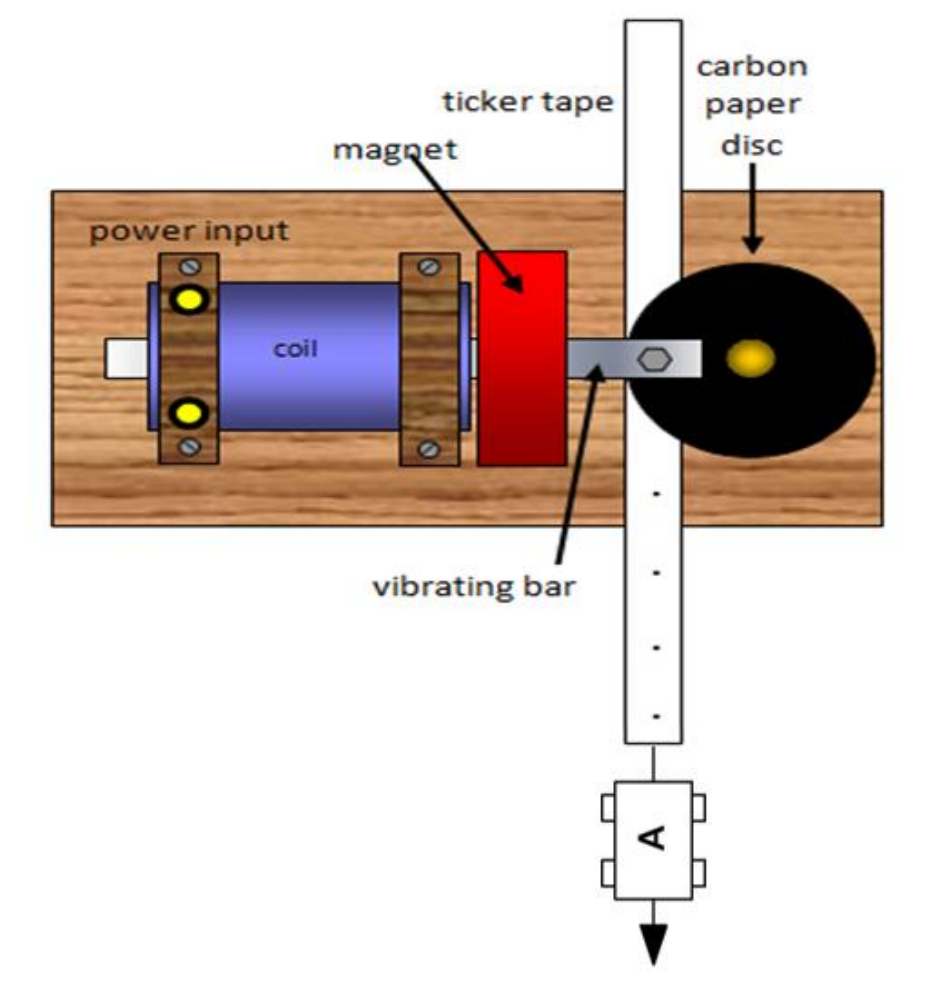
Introduction

- Find the function task
  - Motivation and background
- The ticker tape timer
  - A practical context for exploring the derivative
- Delving deeper into the derivative
  - Exploring the mathematical concept of the derivative

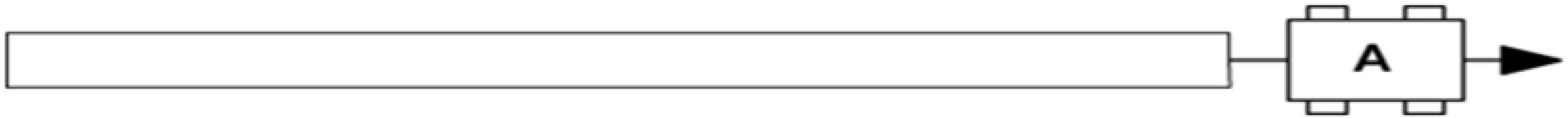
The Ticker Tape Timer



Typical frequencies:

- 10 Hz – 0.1 second increments
- **40 Hz – 0.025 second increments**
- 50 Hz – 0.02 second increments
- 100 Hz – 0.01 second increments

Object Under No Acceleration or Initial Velocity



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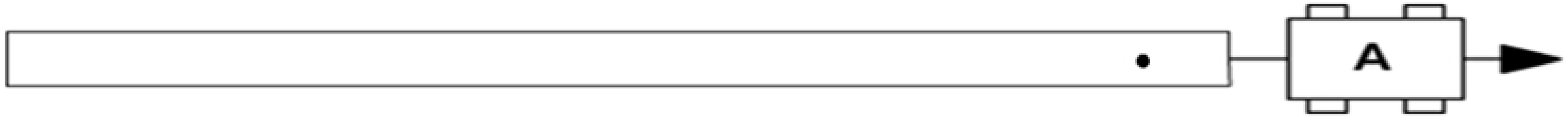

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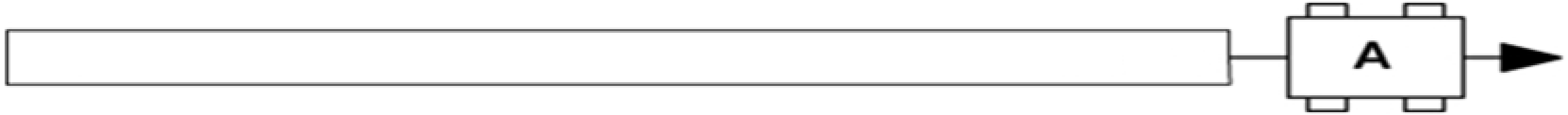
Object Under No Acceleration with Initial Velocity



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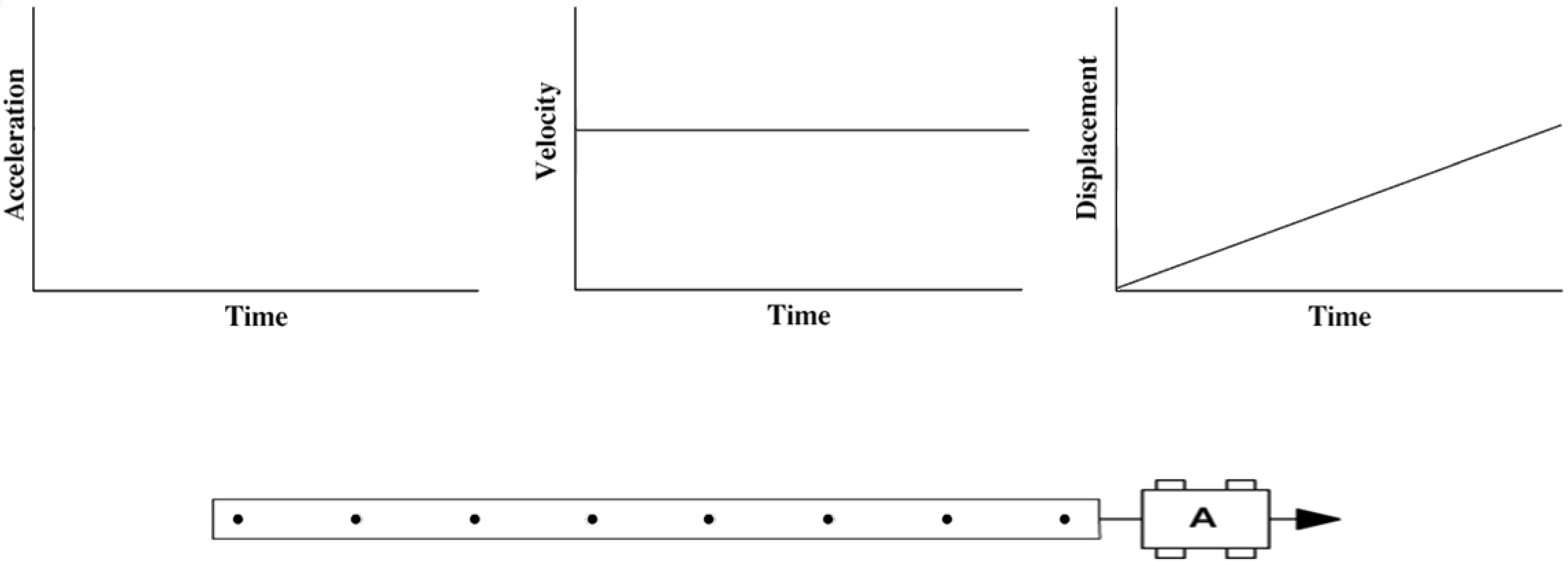
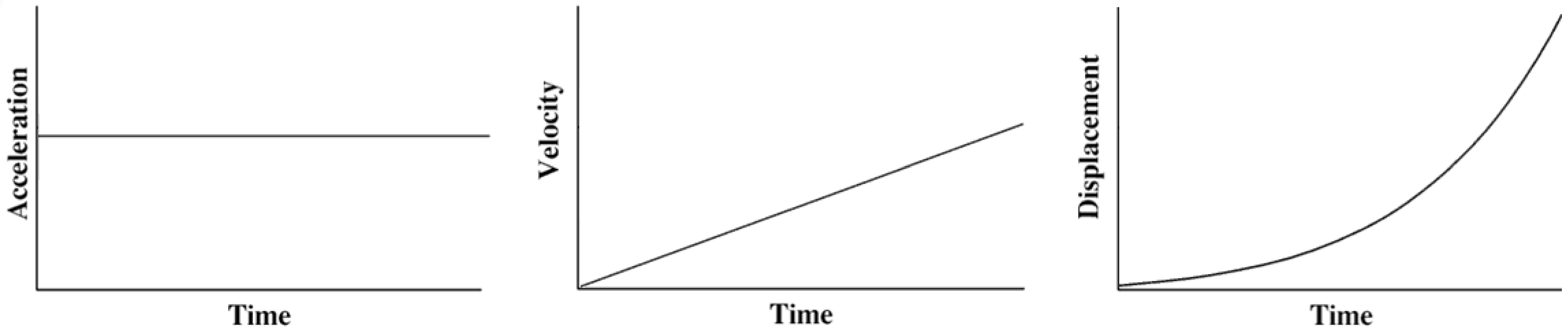
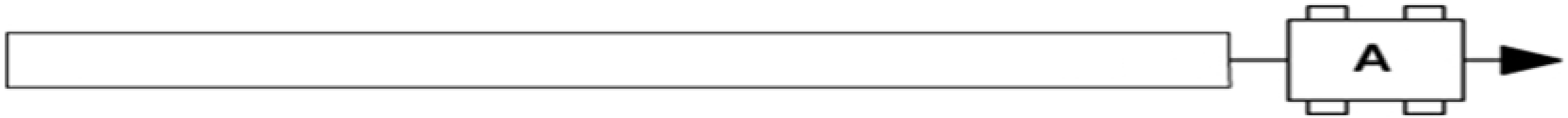


Object Under Constant Acceleration





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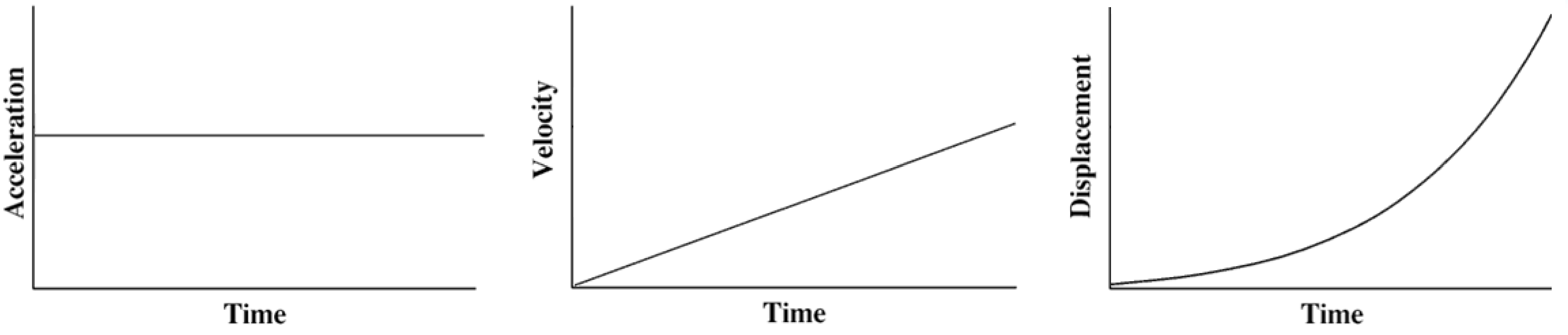
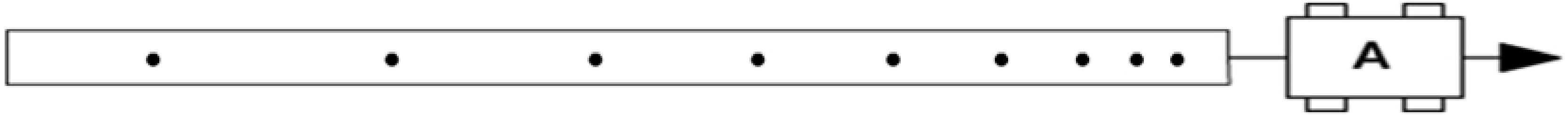
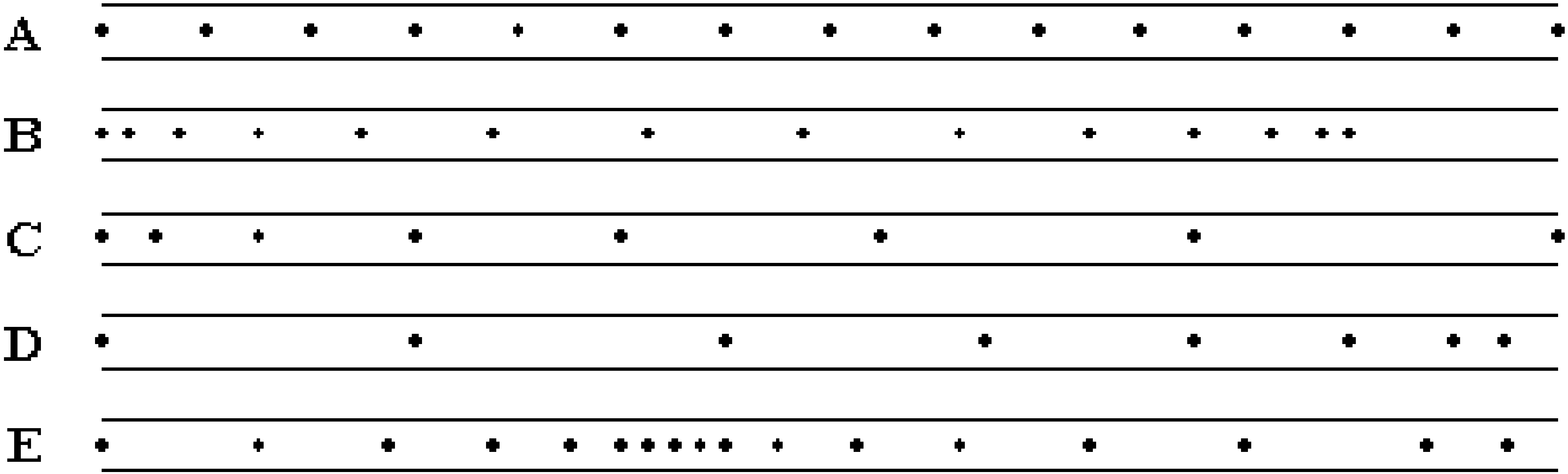




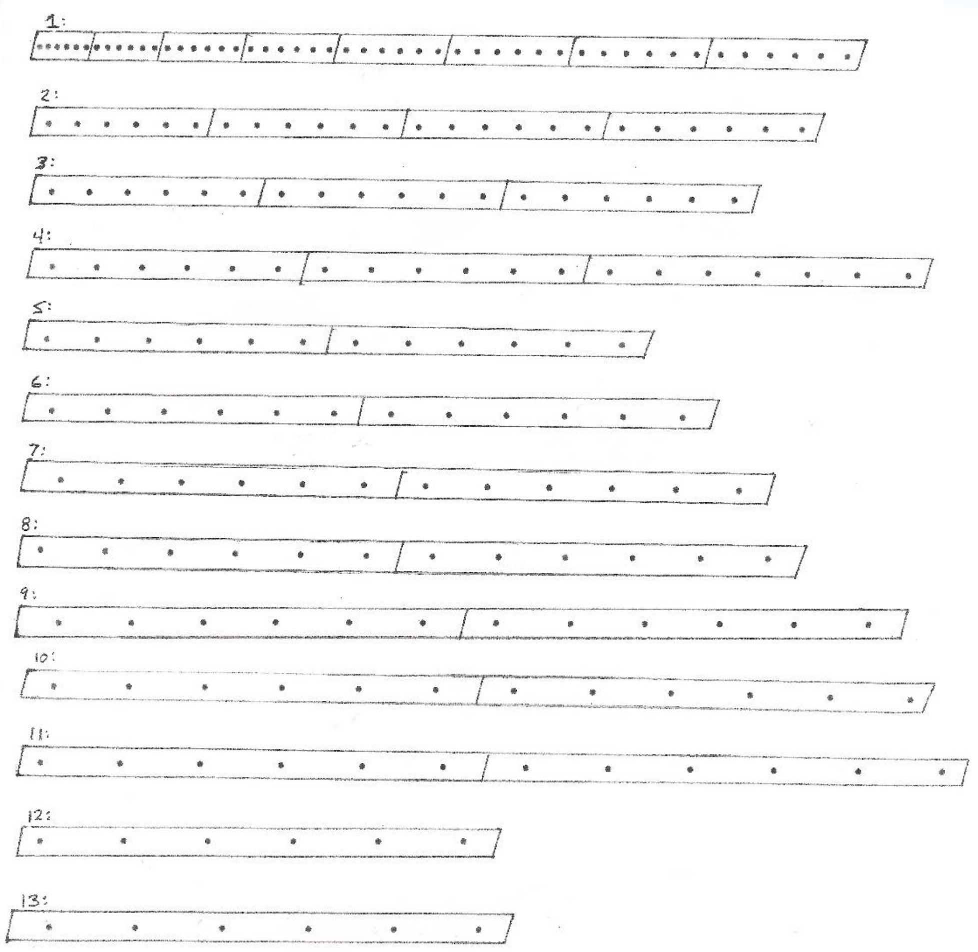
Ticker Tape Examples



Ticker Tape Samples

- How would you describe the motion of the object that created each strip?
- Order the strips from the least average velocity to the greatest average velocity.
- Assuming each increment is 0.025s, estimate the average velocity for each strip.

Ticker Tape Samples



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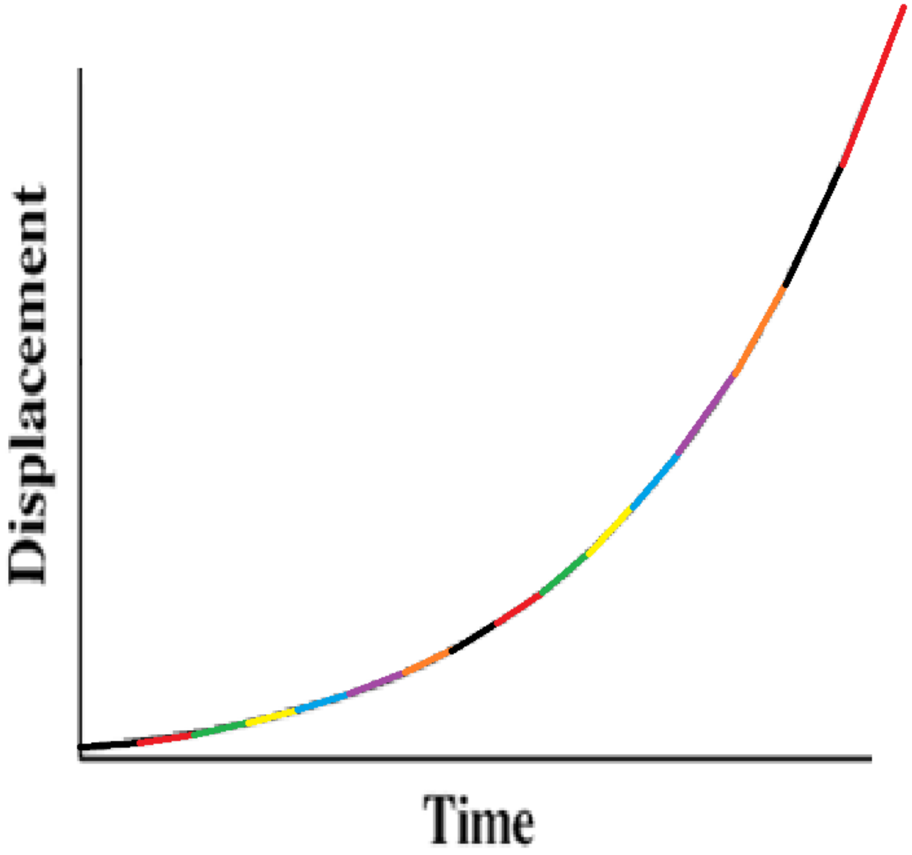
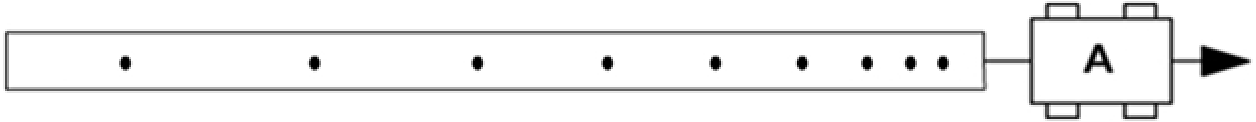
Alternate Representations





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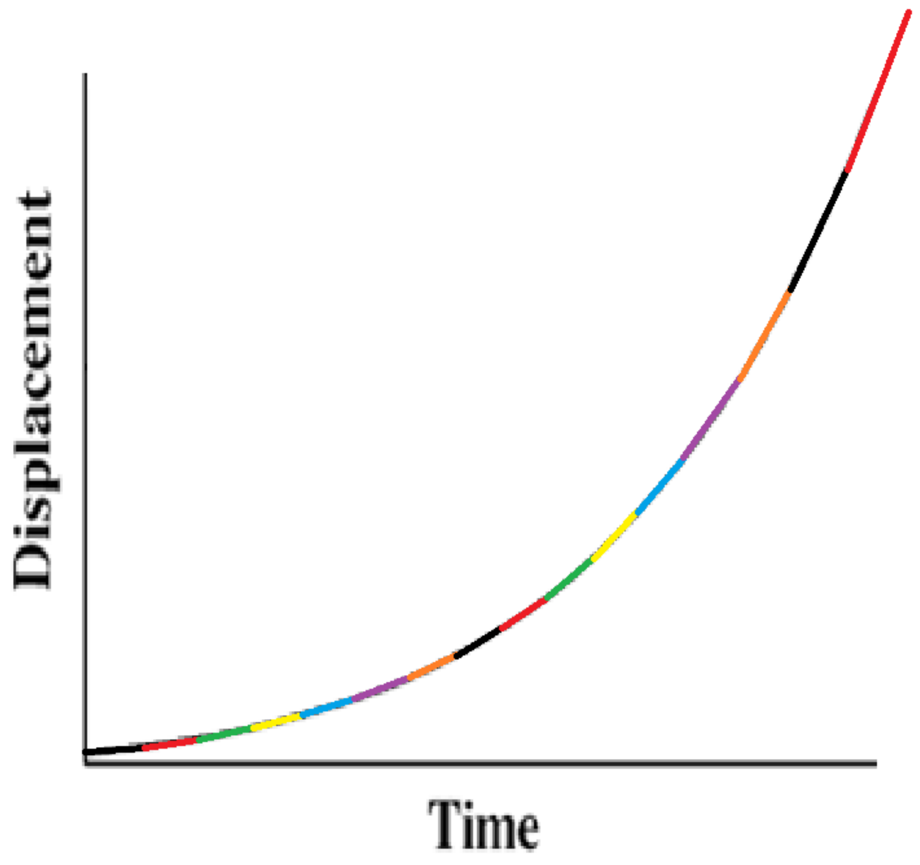

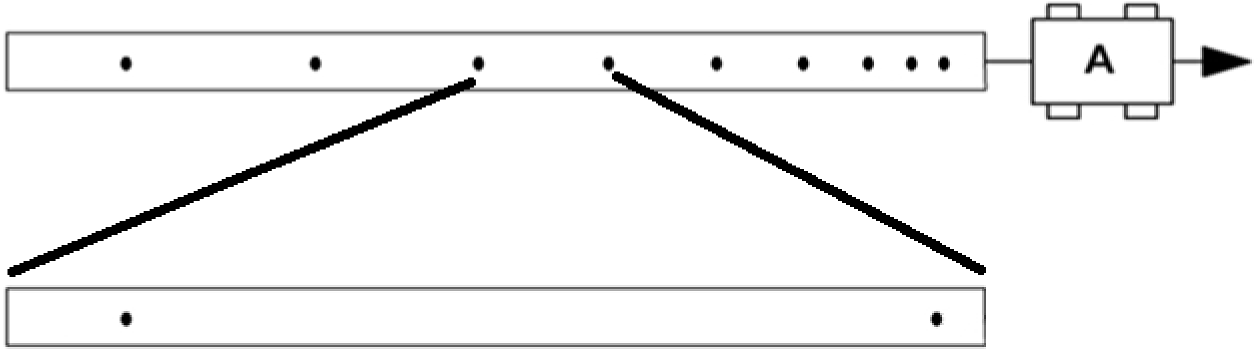

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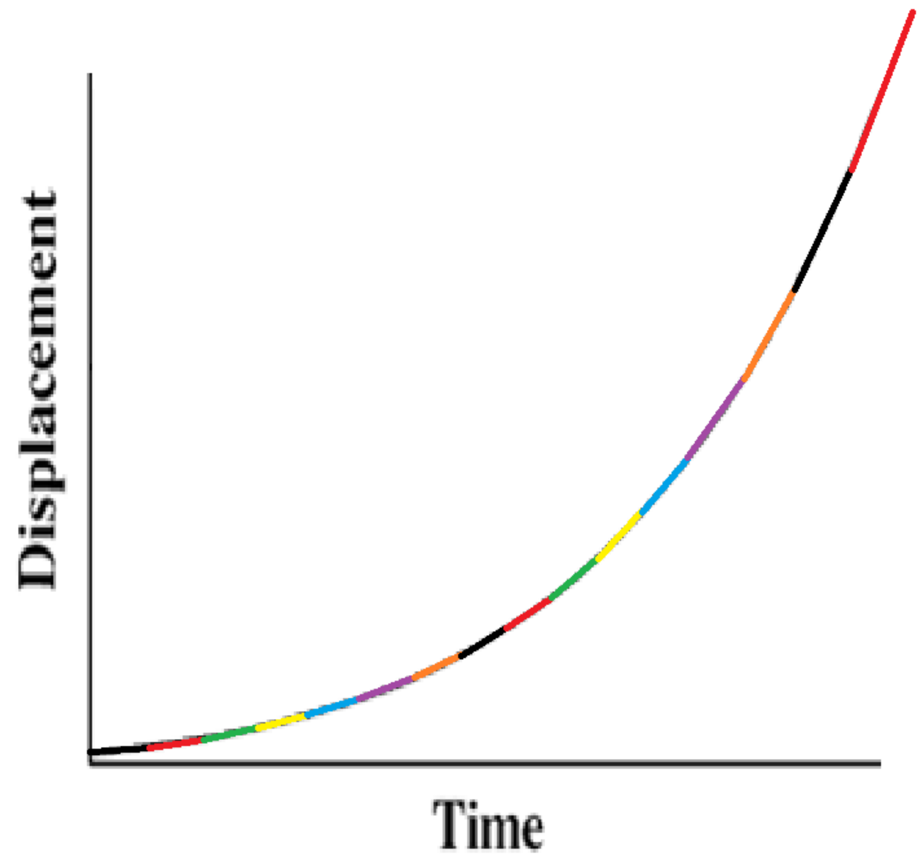

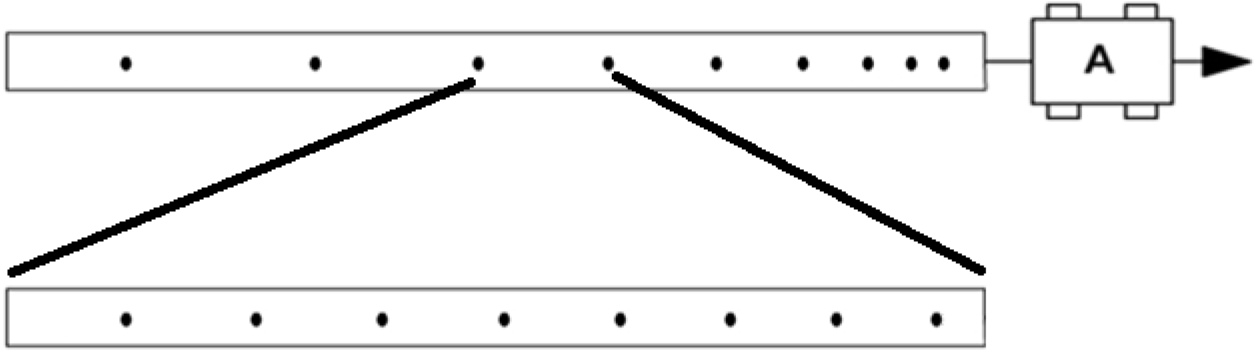

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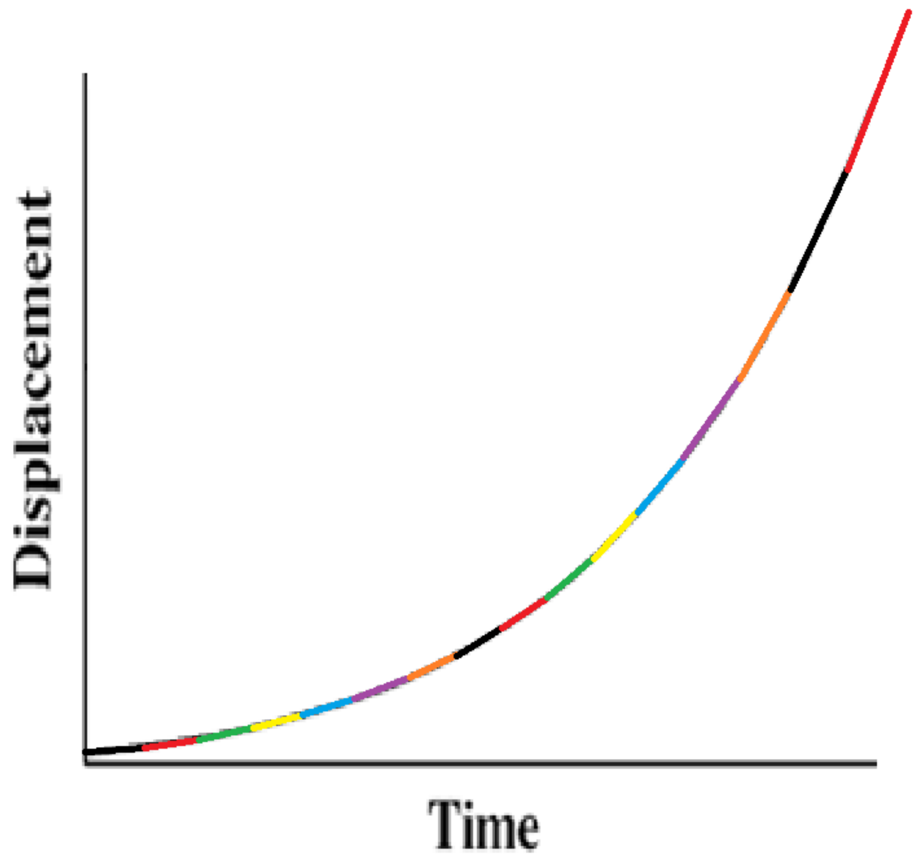

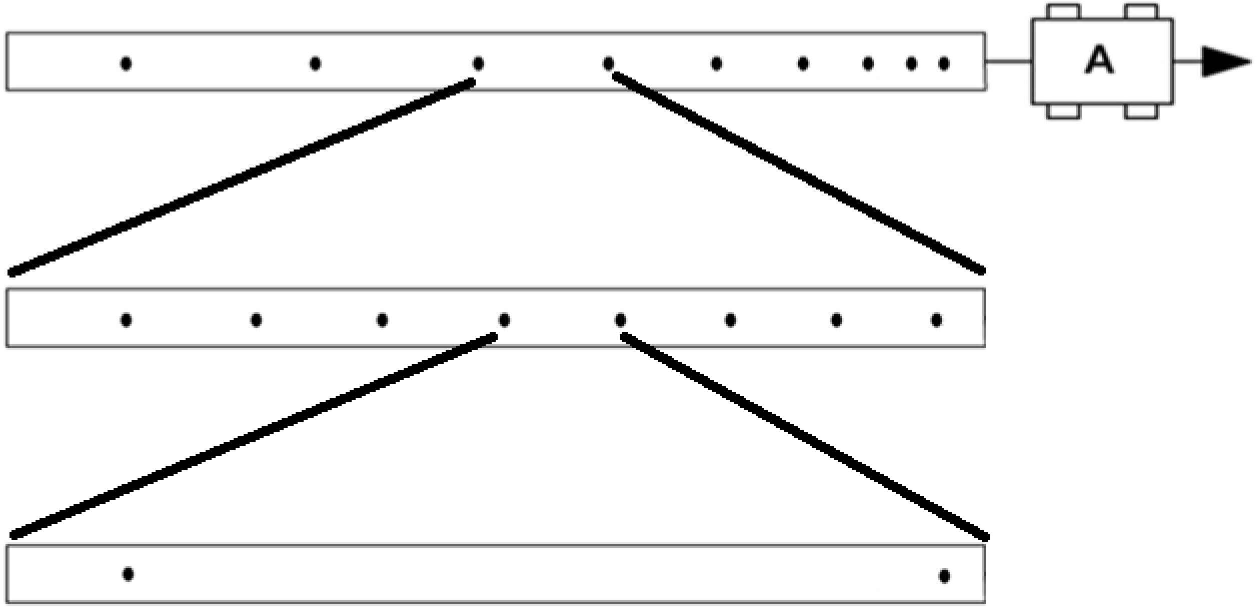

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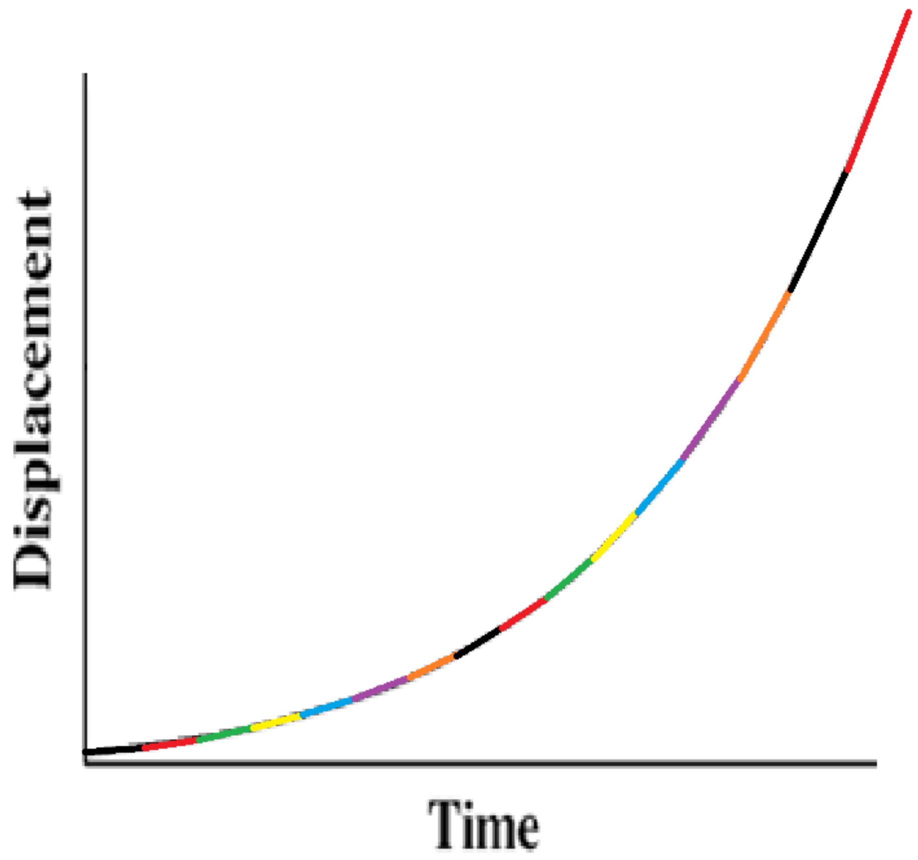

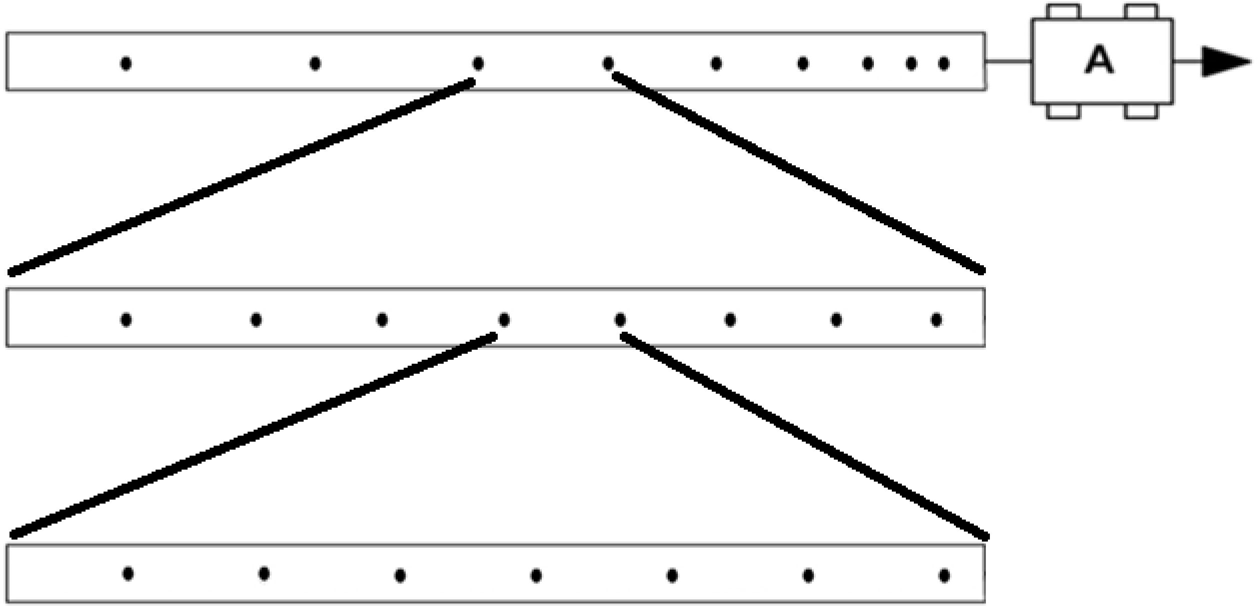

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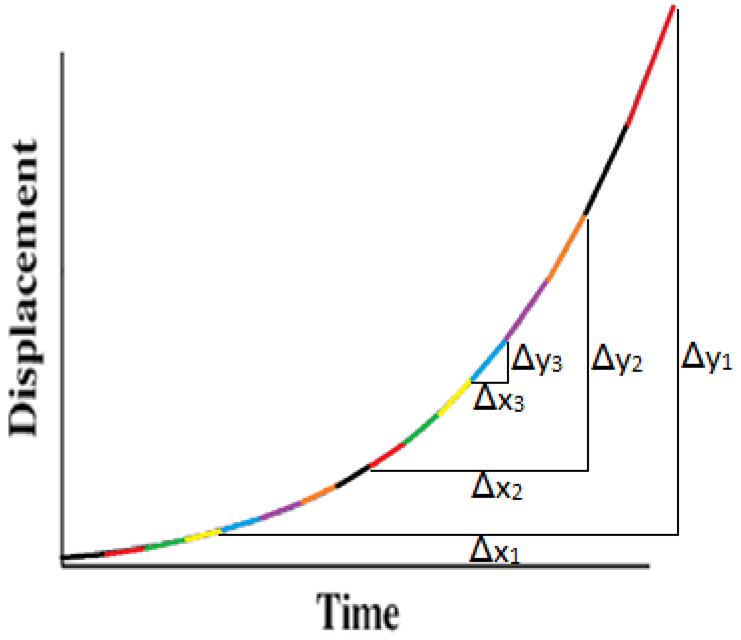

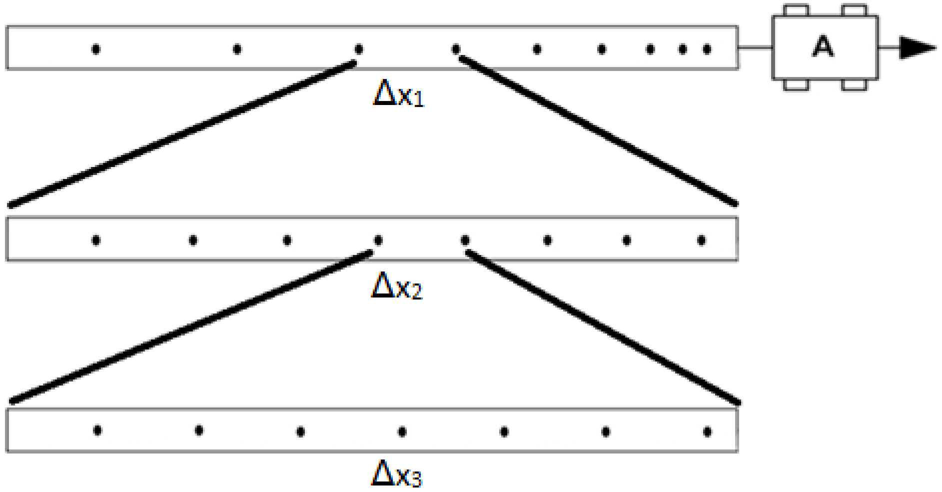

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

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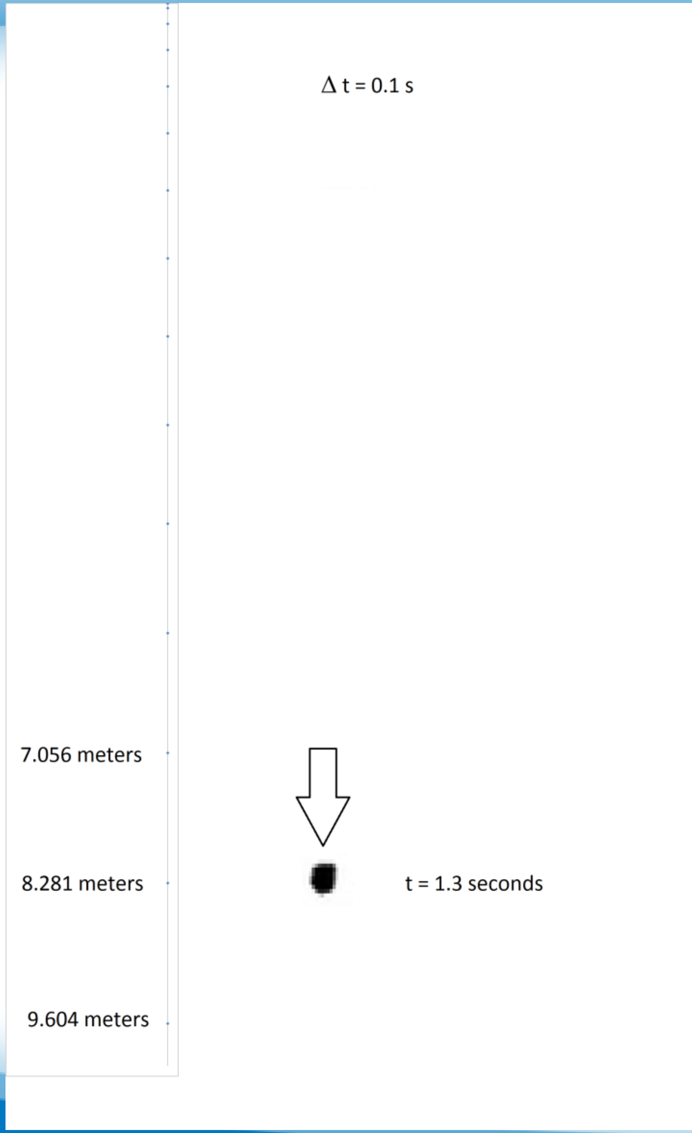
Determining the Speed of a Falling Object



- How would you use a ticker tape to determine the speed of a falling object after it had been falling for 1.3 seconds?
- What information would you need to get a better approximation of the speed at 1.3 seconds?

What Did You Find?

The function $d(t) = \frac{1}{2}\left(9.8\frac{m}{s^2}\right)t^2$ gives the distance the object falls after a given time t. By taking the derivative of this function we obtain the velocity function, $v(t) = (9.8\frac{m}{s^2})t$. Thus the theoretical speed at 1.3 seconds is.

$$v(1.3) = 9.8(1.3) = 12.74\frac{m}{s}$$

Using the ticker tape data, how close could you get to this value?

Our Intention

We expected to emphasize that the smaller our change in time The smaller our change in time Δt, the closer our calculations will approach the true speed of the object.

$$\boxed{f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

We Found Ourselves Doing Mathematics…

- We discovered that we could calculate the exact value of the speed using the ticker tapes. !?!?
- Under what conditions and for what functions would this be possible?

End

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