# Hands-on Trigonometry

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## What is a Radian?

Definition for radian:
A unit of measure for angles; one radian is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.
Source: http://www.mathopenref.com/radians.html

Let's investigate this definition a bit more. On the back of this paper, draw a circle using your compass. Clearly mark the center.

Take a Twizzler and form a Twizzler line segment that is the length of the radius of your circle. Now, go around the circle with your Twizzler radius. How many times does the Twizzler radius go?

Does this number look familiar?

Go around half the circle with your Twizzler radius. How many times does the Twizzler radius go?

Does this number look familiar?

So, let's look at relationships.

$360° =$ _______ radians

$180° =$ _______ radians

Now, can we convert from degrees to radians and vice versa?
Let's use proportions.

Convert $135°$ to radians.

Convert $205°$ to radians.

Convert $\frac{7\pi}{6}$ radians to degrees.

Convert $\frac{2\pi}{9}$ radians to degrees.

## The Unit Circle Paper Plate

You will need one paper plate (the cheapest kind you can find—NOT the coated ones) for each student.

1. Fold the paper plate in half and crease well. Trace the crease and label this diameter as the *x*-axis.
2. Fold the paper plate in half again but DO NOT CREASE. Instead pinch the paper plate to locate the center.
3. Fold the paper plate on the *x*-axis. Using the pinched center, fold the paper plate into thirds. This will give your 60° rotations. Trace these folds and label appropriately (60°, $\frac{\pi}{3}$, etc.). You may want to connect the endpoint of these diameters with a chord that passes through the *x*-axis to see the triangles (30-60-90). Use what you know about the triangles to label the ordered pairs at the endpoints of the diagonals appropriately.
4. Fold the paper plate again on the *x*-axis. From the *x*-axis fold, fold halfway up on each side to the 60° fold. This will give you the 30° rotations. Trace these folds and label appropriately (30°, $\frac{\pi}{6}$, etc.). Again, you may want to connect the endpoint of these diameters with a chord that passes through the *x*-axis to see the triangles (30-60-90).
5. Fold the paper plate again on the *x*-axis. Now fold in half again. This will give you the *y*-axis. Trace and label this diameter as the *y*-axis.
6. Fold the paper plate again on the *x*-axis. Now fold up to the *y*-axis fold on each half. This will give your 45° rotations. Trace these folds and label appropriately (45°, $\frac{\pi}{4}$, etc.).

As you are making your folds, take time to fill in the Big Trig Chart that accompanies this activity. Students may want to color code lines with the same reference angles (e.g., 30°, 150°, 210°, 330° with one color; then the 60° reference angles with another color; then the 45° reference angles with another color). After this activity has been completed, students can use their charts to make graphs of the trigonometric functions.

Name____________________________

## Basic Trigonometric Values

| | sin | cos | tan | csc | sec | cot |
|---|---|---|---|---|---|---|
| 30°<br>Radians: | | | | | | |
| 45°<br>Radians: | | | | | | |
| 60°<br>Radians: | | | | | | |
| 90°<br>Radians: | | | | | | |
| 120°<br>Radians: | | | | | | |
| 135°<br>Radians: | | | | | | |
| 150°<br>Radians: | | | | | | |
| 180°<br>Radians: | | | | | | |
| 210°<br>Radians: | | | | | | |
| 225°<br>Radians: | | | | | | |
| 240°<br>Radians: | | | | | | |
| 270°<br>Radians: | | | | | | |
| 300°<br>Radians: | | | | | | |
| 315°<br>Radians: | | | | | | |
| 330°<br>Radians: | | | | | | |
| 360°<br>Radians: | | | | | | |

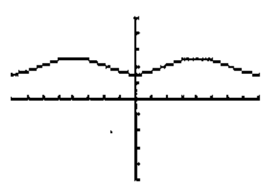
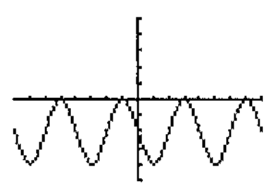
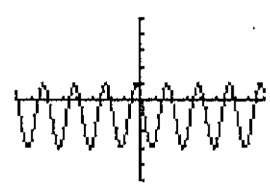
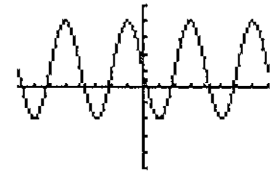
General Rules for Signs:

Quadrant 1: sin cos tan csc sec cot

Quadrant II: sin cos tan csc sec cot

Quadrant III: sin cos tan csc sec cot

Quadrant IV: sin cos tan csc sec cot

| | | | | |
|---|---|---|---|---|
| **Equation** | $y = 3\sin(x - \pi) + 1$ | $y = -2\sin\left(2x - \frac{\pi}{4}\right) - 1$ | $y = \frac{1}{2}\cos\left(\frac{x}{2} + \pi\right) + 2$ | $y = 2\cos\left(x + \frac{\pi}{2}\right) - 2$ |
| **Amplitude** | $\frac{1}{2}$ | 2 | 2 | 3 |
| **Period** | $\pi$ | $2\pi$ | $2\pi$ | $4\pi$ |
| **Phase Shift** | $-2\pi$ | $\frac{-\pi}{2}$ | $\frac{\pi}{8}$ | $\pi$ |
| **Midline** | $y = -2$ | $y = -1$ | $y = 1$ | $y = 2$ |
| **Graph** |  |  |  |  |

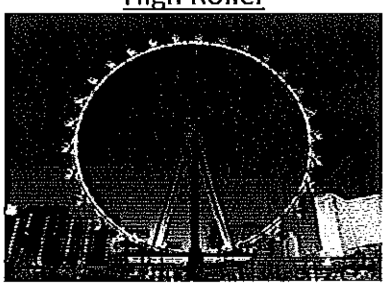
| Equation | $y = 3\sin(x - \pi) + 1$ | $y = -2\sin\left(2x - \frac{\pi}{4}\right) - 1$ | $y = \frac{1}{2}\cos\left(\frac{x}{2} + \pi\right) + 2$ | $y = 2\cos\left(x + \frac{\pi}{2}\right) - 2$ |
|---|---|---|---|---|
| **Amplitude** | | | | |
| **Period** | | | | |
| **Phase Shift** | | | | |
| **Midline** | | | | |
| **Graph** | | | | |

## Around and Around We Go!!!

You may have heard of the famous London Eye Ferris wheel, but have you heard of the High Roller Ferris wheel in Las Vegas? The London Eye, once the largest Ferris wheel in the world, was surpassed by the High Roller in 2014. Although I want to ride both Ferris wheels, my fear of heights may cause me to change my mind. I need you to help me with my decision. I want to know, at any given point in time, how high I will be off the ground. (I want to put in my call to Superman now and tell him where I will be in case I have a panic attack.) Use the information below to draw a graph of my height off the ground. Assume that I decide to stay on each Ferris wheel and go around twice. Then, try to convince me (in a paragraph) of why I need to ride these Ferris wheels or why I need to keep my feet firmly planted on the ground.

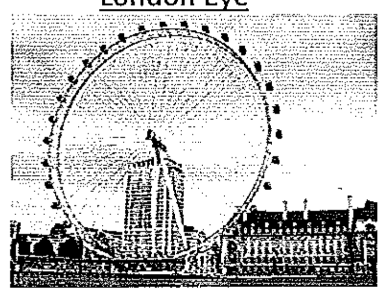
Source:
http://www.telegraph.co.uk/travel/destinations/northamerica/usa/lasvegas/10736177/Worlds-tallest-Ferris-wheel-opens-in-Las-Vegas.html

High Roller



Opened: March 31, 2014
Height: 550 feet
Diameter: 520 feet
Ride Length: 30 minutes
Capsules: 28
People per Capsule: 40

London Eye



Opened: December 31, 1999
Height: 443 feet
Diameter: 394 feet
Ride Length: 30 minutes
Capsules: 32
People per Capsule: 25