Common Core Based Investigations in Geometry

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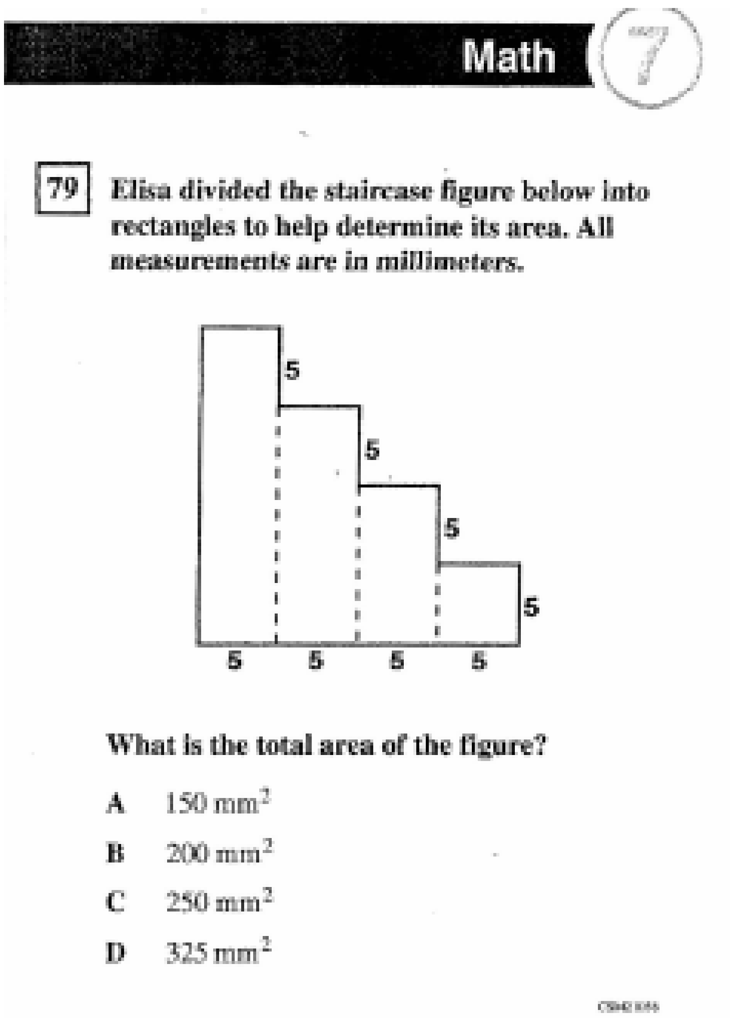
Standards of Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Using technology to initiate discussion



Math 7

79 Elisa divided the staircase figure below into rectangles to help determine its area. All measurements are in *millimeters*.

What is the total area of the figure?

A $150\ mm^2$

B $200\ mm^2$

C $250\ mm^2$

D $325\ mm^2$

How can we extend this problem?

Who Am I?

Investigation 1:

- Draw three non-collinear points A, B and D.
- Draw a vector from A to B.
- Translate point D by the vector to obtain point C.
- Hide the vector.
- Construct the polygon ABCD.

Who am I? Why?

Who Am I?

Investigation 2:

- Draw segment AC.
- Construct the perpendicular bisector of segment AC.
- Draw point B on the perpendicular bisector of segment AC.
- Reflect point B in Segment AC to obtain point D.
- Hide the segment and the perpendicular bisector.
- Construct the polygon ABCD.

Who am I? Why?

Who Am I?

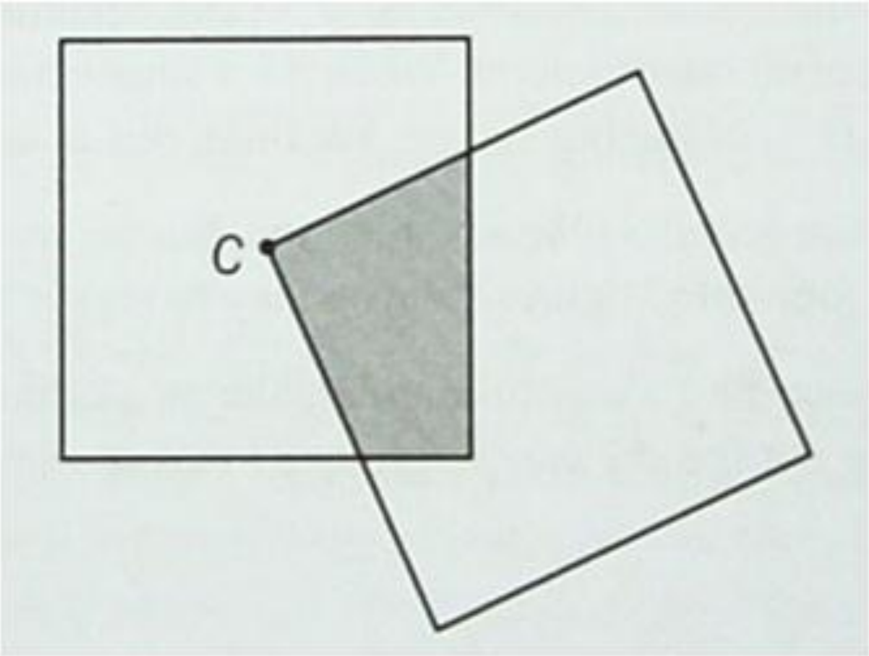
Investigation 4:

- Draw triangle ABD.
- Construct the midpoint M of side BD.
- Rotate the triangle 180° around the point M.
- Label the unnamed point C.
- Construct the polygon ABCD.

Who am I? Why?

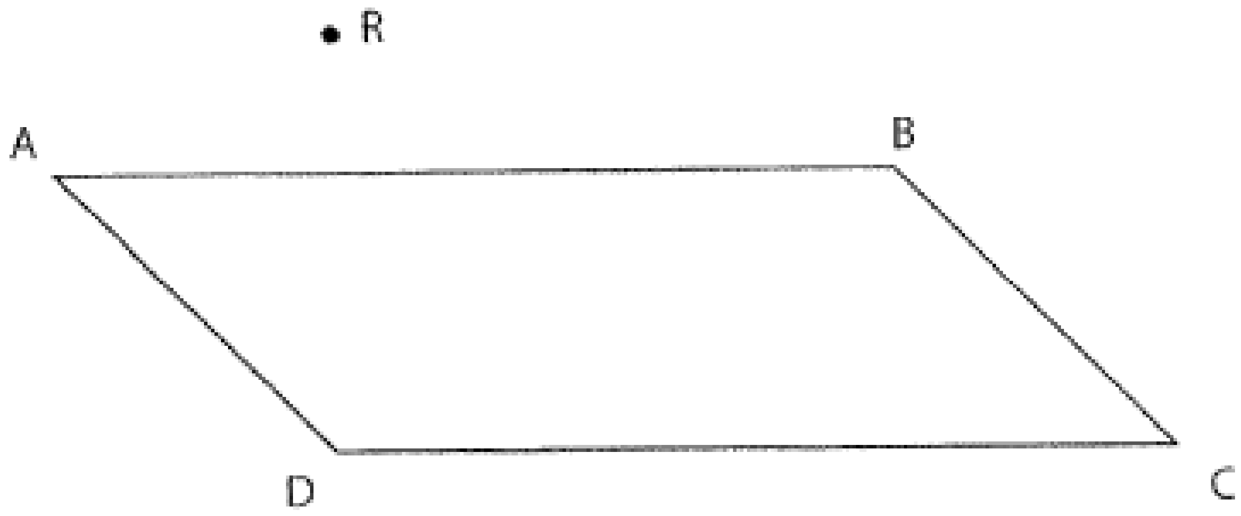
Rotating Square Problem
(NCTM 2010)

Two congruent squares (n units by n units) overlap as shown in the figure below. Vertex C of one square is at the center of the other square. If the square with vertex C is allowed to rotate about the center, C, of the other square, what is the largest possible value of the overlapping shaded area?

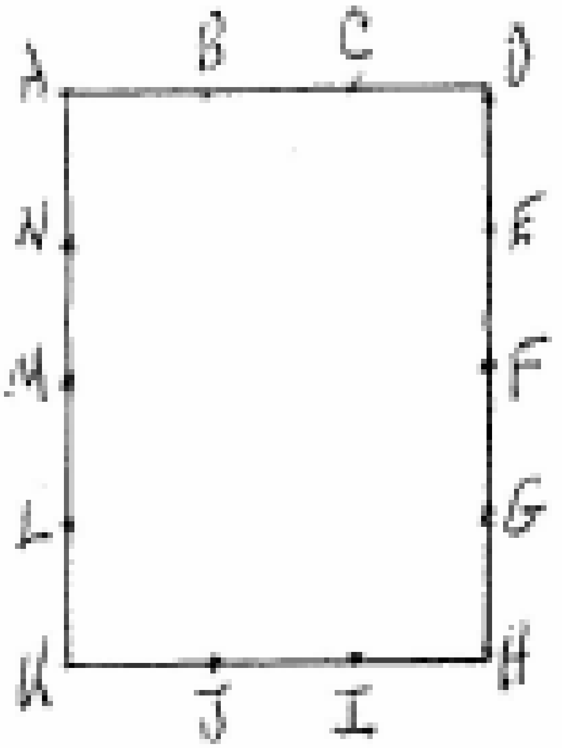


Congruence Challenge

In the figure below, AB∥ DC and AD∥ BC. Point R is anywhere in the same plane as figure ABCD. Draw a straight line that passes through point R and divides ABCD into two congruent parts. Justify your reasoning.



In Rectangle ADHK, points A, B, C, D, E, F, G, H, I, J, K, L, M and N are equally spaced.



What questions can we ask?

A Geometry to Algebra 2 Example

The area of a circle with diameter 10 inches is greater than the combined area of a circle with diameter 8 inches and a circle with diameter 2 inches by approximately how many square inches?

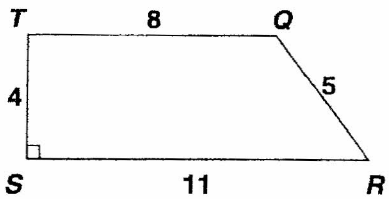
A Better Question

Is there a pattern in the difference between the area of a circle with diameter 10 and two smaller circles with diameters that sum to 10?

Summary

- Technology Makes Mathematics
  - Dynamic, Visual
  - Exploration driven
  - Open for discussion and discovery
- Technology Drives Changes in Curriculum
  - More conceptual
  - Focus on connections, big ideas
- Results in a Change in
  Student Engagement/Interaction with the Mathematics

77 What is the area of trapezoid *QRST* in square units? $\left(A = \frac{1}{2}h(b_1 + b_2)\right)$



A 22

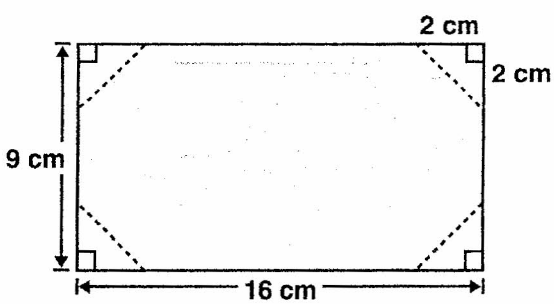
B 27

C 38

D 48

CSM10225

78 Cherie cut four congruent triangles off the corners of a rectangle to make an octagon, as shown below.



What is the area of the shaded octagon?

A 128 cm^2

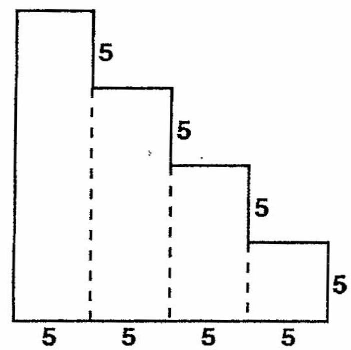
B 136 cm^2

C 140 cm^2

D 152 cm^2

CSM00308

79 Elisa divided the staircase figure below into rectangles to help determine its area. All measurements are in millimeters.



What is the total area of the figure?

A 150 mm^2

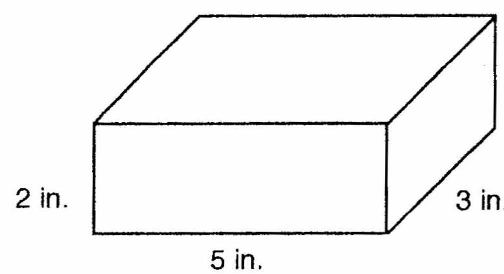
B 200 mm^2

C 250 mm^2

D 325 mm^2

CSM21056

80 What is the volume of the rectangular solid shown below?



A 10 cubic inches

B 25 cubic inches

C 30 cubic inches

D 62 cubic inches

CSM01901

Who Am I?

Investigation 1:

- Draw three non-collinear points A, B and D.
- Draw a vector from A to B.
- Translate point D by the vector AB to obtain point C.
- Hide the vector.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 2:

- Draw segment AC.
- Construct the perpendicular bisector of segment AC.
- Draw point B on the perpendicular bisector of segment AC.
- Reflect point B in Segment AC to obtain point D.
- Hide the segment and the perpendicular bisector.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 3:

- Draw segment AC
- Construct the perpendicular bisector of segment AC.
- Draw point B on the perpendicular bisector of segment AC and another point D on the perpendicular bisector on the other side of AC from B.
- Hide the segment and the perpendicular bisector.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 4:

- Draw triangle ABD.
- Construct the midpoint M of side BD.
- Rotate the triangle 180^{o} around the point M.
- Label the unnamed point C.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 5:

- Draw triangle ABD.
- Reflect the triangle in side BD of the triangle.
- Label the unnamed point C.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 6:

- Draw a line.
- Draw segment AD on one side of the line.
- Reflect the segment AD in the line.
- Label the new segment BC where points B and C are the reflections of A and D respectively.
- Hide the line.
- Construct the polygon ABCD.

Who am I? Why?

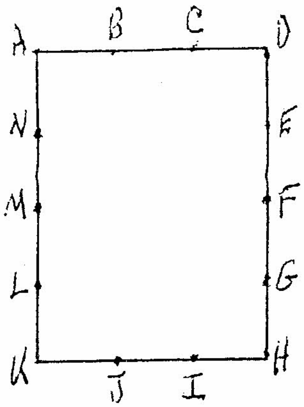
Investigation 7:

- Draw a line segment AB.
- Rotate segment AB 90^{o} around the point A.
- Label the unnamed point D.
- Rotate segment AD 90^{o} around the point D.
- Label the unnamed point C.
- Rotate segment DC 90^{o} around the point C.
- Construct the polygon ABCD.

Who am I? Why?

Investigation 8:

Create a ‘Who Am I?’ so that the resulting polygon ABCD is a rectangle.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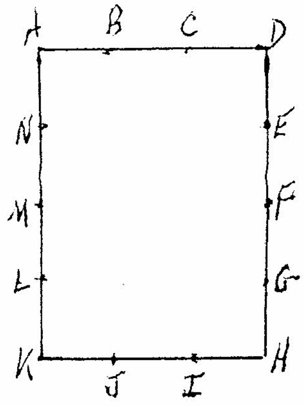
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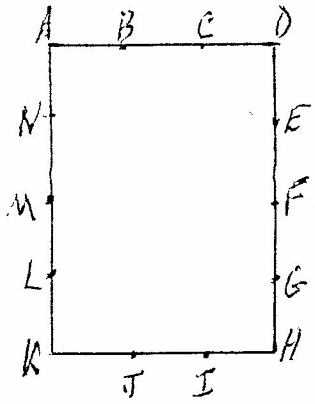
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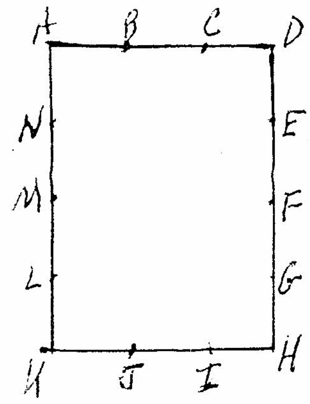
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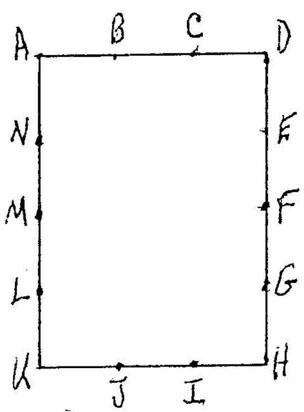
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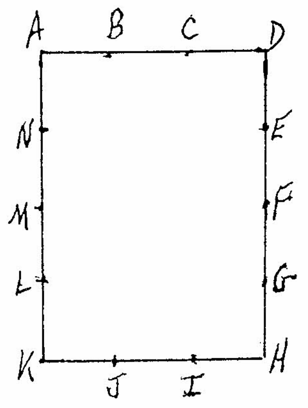
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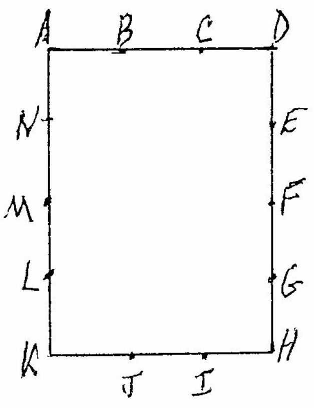
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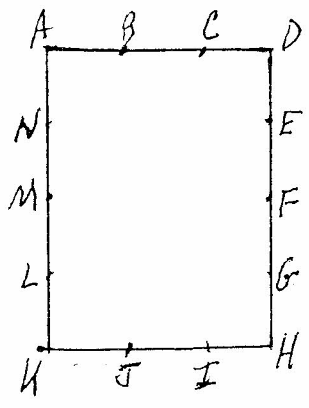
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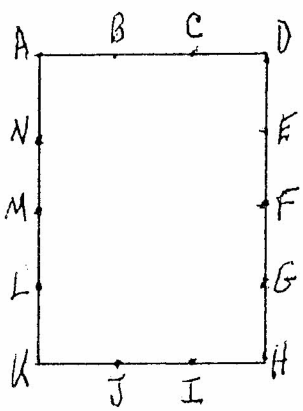
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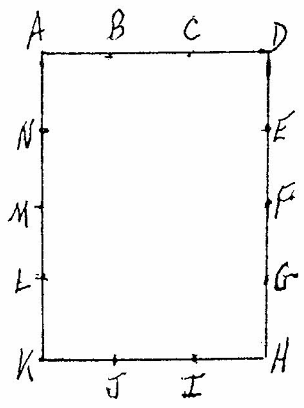
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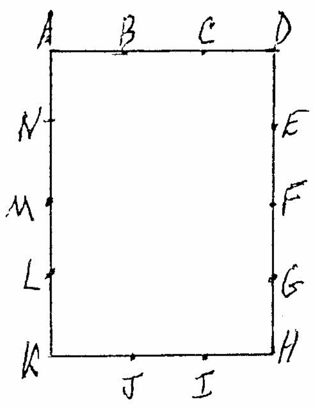
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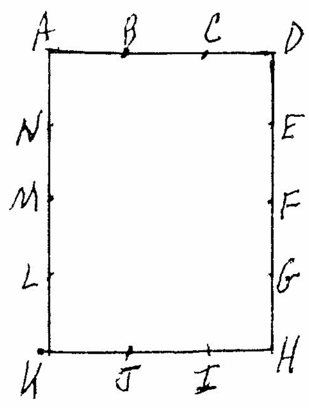
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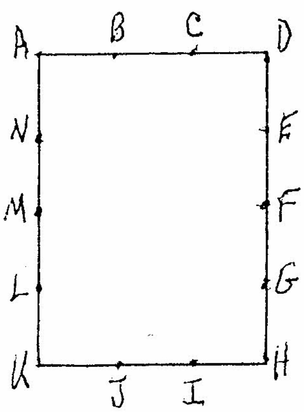
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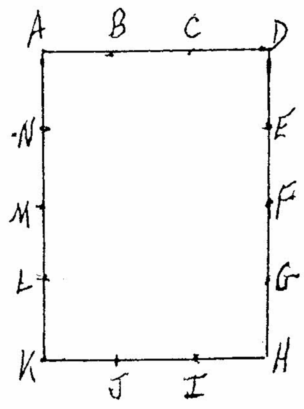
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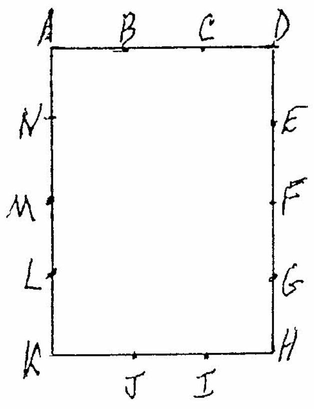
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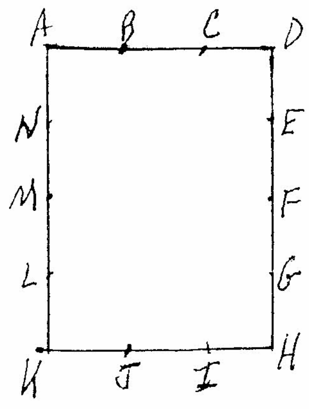
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Congruence Challenge

A-E Strand(s): Geometry. Sample Courses: Middle School Course 1, Middle School One-Year Advanced Course, Integrated 1, Integrated 3, and Geometry.

Topic/Expectation

G.B.6 Similarity and Congruence

- d. Explain why congruence is a special case of similarity; determine and apply conditions that guarantee congruence of triangles.
- e. Apply the definition and characteristics of congruence to make constructions, solve problems, verify basic properties of angles and triangles.

G.B.1 Angles in the plane

- a. Know and distinguish among the definitions and properties of vertical, adjacent, corresponding, and alternate interior angles.

Other Topic/Expectation(s)

G.A.1 Angles and triangles

- a. Know the definitions and basic properties of angles and triangles in the plane and use them to solve problems.

G.B.1 Angles in the plane

- b. Identify pairs of vertical angles and explain why they are congruent.
- c. Identify pairs of corresponding, alternate interior, and alternate external angles in a diagram where two parallel lines are cut by a transversal and show that they are congruent.
- d. Explain why, if two lines are intersected by a third line in such a way as to make corresponding angles, alternate interior angles, or alternate exterior angles congruent, then the two original lines must be parallel.

G.C.2 Axioms, theorems, proofs in geometry

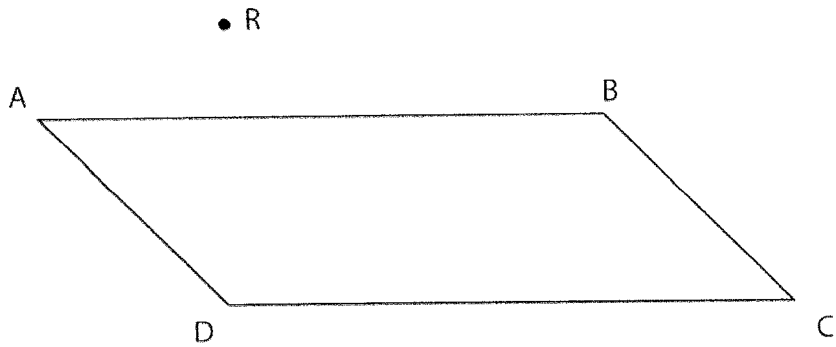
- a. Use geometric examples to illustrate the relationships among undefined terms, axioms/postulates, definitions, theorems and various methods of reasoning.

Rationale

This task challenges students to apply what they know about parallel and intersecting lines, parallelograms, and the angles and segments related to these figures to solve a geometric problem in a purely mathematical context. It requires students to think about how to approach the problem and to justify their work.

Instructional Task

In figure ABCD, AB||CD and AD||BC. Point R is in the same plane as ABCD. (Point R can be placed anywhere in the plane.)



Draw a straight line that passes through point R and divides ABCD into two congruent parts. Justify your reasoning that the two parts are congruent.

Discussion/Further Questions/Extensions

Depending on the teacher's goals, this task can be adapted to require any level of reasoning from an informal justification to a formal proof.

Some students may have difficulty realizing that they must connect point R to the point at the intersection of the parallelogram's diagonals. While it may be tempting for the teacher to guide students toward connecting point R to this point of intersection, students should first have adequate opportunity to constructively struggle with the task as presented, perhaps working with partners or in small groups.

If additional scaffolding becomes necessary, it might be helpful for students to list all the attributes of parallelograms and the angles formed by parallel and intersecting lines that they know. If necessary, encourage students to place point R at different locations and sketch where they think the line might bisect the parallelogram.

As an extension, ask students to determine whether the method they used to prove that both halves of the bisected parallelogram are congruent would work if point R were located anywhere on the same plane as ABCD—inside, outside, or on the parallelogram itself.

Sample Solution

Students might approach this problem using properties of parallelograms, parallel lines, intersecting lines, and the angles and segments related to these figures. Other solution methods are possible, including the use of transformations, if students can provide adequate justification.

This sample solution uses the following information:

- *The diagonals of a parallelogram bisect each other.*
- *Alternate interior angles formed by two parallel lines and a transversal are congruent.*
- *Vertical angles formed by two intersecting lines are congruent.*

- *Opposite sides of a parallelogram are congruent.*
- *Two triangles can be proven to be congruent using a variety of triangle congruence theorems or postulates (e.g., side-angle-side (SAS), side-side-side (SSS), angle-angle-side (AAS), angle-side-angle(ASA)).*

1. *Draw the two diagonals of the parallelogram.*
2. *Label the intersection of the two diagonals, O.*
3. *The line through R and O intersects the line AB at N and the line CD at M.*
4. *The triangles AON and COM are congruent (by ASA), and triangles MOD and NOB are congruent (also by ASA). Triangles AOD and COB are congruent (by SSS or SAS). Quadrilateral ANMD is composed of triangles AON, MOD, and AOD, and quadrilateral BNMC is composed of triangles COM, NOB, and COB. Therefore, quadrilateral ANMD is congruent to quadrilateral BNMC.*