Wearing Math Bifocals: Seeing Mathematical Writing as Product and Process

A Presentation for the NCTM Regional Conference
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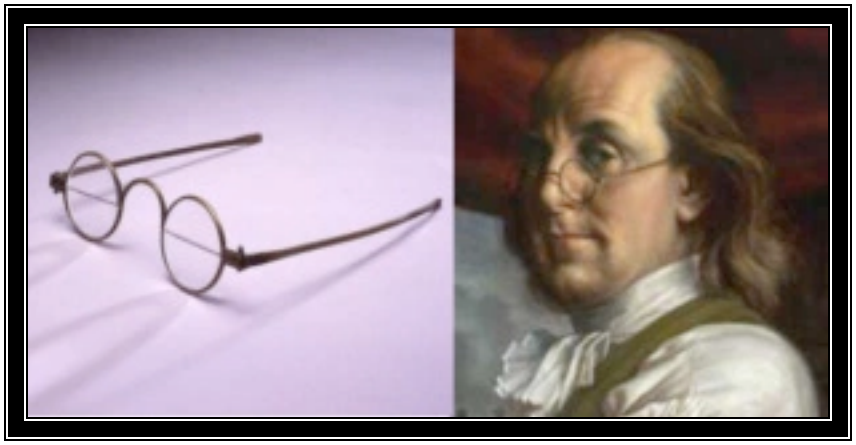

https://thedayintech.files.wordpress.com/2013/04/franklinbifocals.jpg

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A Knight's Tour

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

These are some of the things I notice about what I see above:

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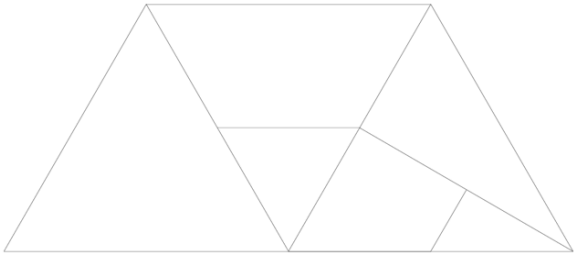
A Few More "Things I Notice" Examples

$$2x + 3y = 9$$

These are some of the things I know about the given equation:

$$24, 36, 48, 60, 72$$

These are the things I notice about these numbers:



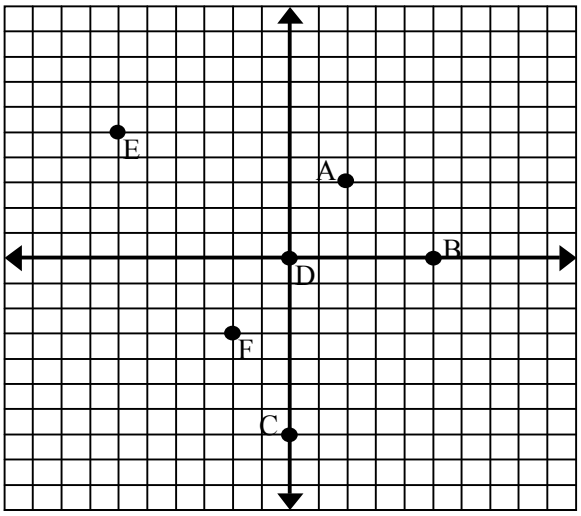
These are the things I notice about what I see above:

$$20r + 34c = 504$$

$$15r + 17c = 327$$

These are some of the things I notice about this system of equations:

A Plain Plane



Examine the coordinate plane shown above. Record some of the things you notice.

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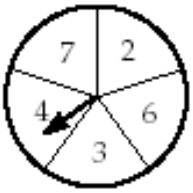
The two spinners below are used for a math game. If both arrows land on even numbers, one point is given. Otherwise, no points are given.



These are some of the things I know about these spinners and this game:

3, 4, 5 6, 8, 10 5, 12, 13

Things I notice:

These are some of the things that I notice about the figure drawn above:

TRANSITIONING TO dialogue:
Write the equations of two lines.

Swap papers with a partner.
Find out everything you can about the lines your partner gives you. Use as many words from our Word Wall as you can.

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My Name:

My Partner's Name

Dealing in Horses

A man bought a horse for fifty dollars and sold it for sixty dollars. He then bought the horse back for seventy dollars and sold it again for eighty dollars. What do you think was the financial outcome of these transactions?

- ❑ Lost $20
- ❑ Lost $10
- ❑ Came out even
- ❑ Other (describe)
- ❑ Earned $10
- ❑ Earned $20
- ❑ Earned $30

Once you commit to an answer, in the area below, convince your partner that your answer is correct. Although you may wish to show your work as support for your argument, your argument should tell why you know you're right!

(problem from Burns, M. (1992). About Teaching Mathematics: A K-8 Resource. Sausalito, CA: Math Solutions Publications; dialogue journal format developed by Jill Perry)

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My Name:

My Partner's Name

The 216 Cubes Problem

I am holding a 6 cm by 6 cm by 6 cm cube made up of 216 small cubes. If I were to remove one layer of small cubes from each of the faces, how many cubes would I remove?

In the area below, convince your partner that your answer is correct. Although you may wish to show your work as support for your argument, your argument should tell why you know you're right!

(problem from From Atkins, S. (1999). Listening to students. *Teaching Children Mathematics, 5* (5), 289 – 295.; dialogue journal format developed by Jill Perry)

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My Name:

My Partner's Name

This Is How We Operate

While Lulu and Jim were working together on their math homework (as great study buddies *do*), they ran into a bit of a glitch. Though they were both working on simplifying the expression $4 \times 4 \div 4 \times 4$, they got two different answers (as you can see below).

Lulu's Work

$4 \times 4 \div 4 \times 4 =$

$16 \div 16 =$

1

Jim's Work

$4 \times 4 \div 4 \times 4 =$

$16 \div 4 \times 4 =$

$4 \times 4 =$

16

Here's a question for you: Who is correct?

A. Lulu
B. Jim
C. Both of them
D. Neither of them

Once you commit to an answer, in the area below, write an argument that you think will convince your partner that you are correct.

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My Name:

My Partner's Name

It's Gnatural

Each year the Yellowbellied Gnat causes problems for the residents in Bloomville. Cats and dogs in the area are attacked voraciously by these insects. In early March, the Yellowbellied Gnat population is estimated to be at 1000. All of these gnats are at the newborn stage in their development. Gnats begin reproducing at two months old and continue reproducing monthly until they die. Two-month-old gnats reproduce at a rate of 200%; three-month-old gnats reproduce at a rate of 250%; and four-month-old gnats reproduce at a rate of 80%. At five months of age all gnats die and (thankfully) the entire population will die with the first hard freeze (no earlier than mid-October). How many gnats will there be in the Bloomville area in early October?

Solve the problem on a separate sheet of paper.
If you need to ask your partner questions, write them in the area below.
Once you have solved the problem, use the area below to **convince** your partner that your answer is correct. Although you may wish to show your work as support for your argument, your argument should **tell why you know you're right!**

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Dialogue Journals in Mathematics and Science

4 Types (and **Prompt**(s) for Each)

Convincing

- *Convince your partner that your answer/solution is correct. Although you may wish to show your work as support for your argument, your argument should **tell why you know you're right**!*
- *Read the information given below and make a conjecture/hypothesis about what you think will happen when you run the experiment/lab. Convince your partner that your conjecture/hypothesis is reasonable.*

Clarifying/Questioning

- *These are some of the questions I need answers to before I start solving this problem:*
- *These are some of the questions I need answers to before I begin this lab:*

Explaining

- *Explain to your partner how you solved the problem. Although you may wish to show your work to illustrate your explanation, go beneath the surface and explain your thinking!*
- *The statements that you have been given are incorrect. Explain to your partner why each statement is incorrect.*

Strategizing

- *Do not solve the problem! Instead, suggest at least two strategies that you could use to solve this problem. For each strategy, explain why your strategy would be reasonable.*
- *Do not begin the lab/activity. Instead, after you read it, suggest at least two approaches you could take to proving or disproving your conjecture/hypothesis.*

Instructions for the Dialogue Journal Process

1. Review and clarify the instructions. You might say to students:
   - "Fold your papers in half along the line segment on the paper.
   - "Write your name at the top of the left-hand column and your partner's at the top of the right-hand column.
   - "In the left-hand column, solve the problem and then write an argument that convinces your partner that your solution is correct." [See the horse problem for how I stated this part.] [Modify this to fit each topic listed above.]
   - "Once you have written a convincing argument, quietly tell your partner that you are ready -- then wait for him or her to finish his or her thought. If you are not finished when your partner is, give him/her an estimate of how much time you think you will need to complete your thought. Once you are both willing, swap papers.
   - "Read your partner's solution and argument. Respond to the argument in the right-hand column.
   - "You may not talk to each other -- any questions or comments should be written on your papers!
   - "You are to continue writing back and forth to each other until you reach consensus, an impasse, or I call 'time.'"
2. Give students 4-6 minutes of silent time depending on the problem and journal type. Just monitor the class to see if they need more time.
3. Give students 2-3 minutes to talk with their partners to clarify/ask questions/discuss/debate.
4. Facilitate a whole-class discussion about the problem/task. Open by asking for the answers they got (or questions they have). Then ask for solutions and arguments (or answers, explanations, etc.). Facilitate the discussion/debate (keeping them focused on the big ideas).

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Tips for Analyzing Mathematical Writing

Jill A. Perry and Sandra Atkins, 2004

First Things First . . .

1. Identify the *mathematical understanding* that the task is intended to reveal. (You might consider mathematical concepts, vocabulary, strategies, and/or skills.)
2. Identify what that understanding might look like (and acknowledge that it might reveal itself in different ways, as well).

Keep in Mind . . .

- ? Is this a "new" idea in your classroom? In other words, are you giving the task so that you can find out what your students already know about a concept (or topic) that you soon will be studying together?

OR

- ? Is this a concept or skill that you have been studying together? In other words, do you want to find out the sense they are making of the things that you have been doing in class?

As You Analyze Their Writing and Work, Answer Statements Like These. . .

- ◎ The student knows _____. Here's the evidence:
- ◎ The student understands _____. Here's the evidence:
- ◎ The student is able to _____. Here's the evidence:
- ◎ The student does not know _____. Here's the evidence:
- ◎ The student does not understand _____. Here's the evidence:
- ◎ The student is not able to _____. Here's the evidence:

Then Decide . . .

Where should we go next?

Some Additional Categories for Analysis (and a few of the things that you might look for)

Writing/Articulating a Convincing Argument	Reading Strategies
Addressing the given prompt Supporting conclusions/argument Considering audience	Rereading Underlining key details/ideas/words Using the conventions of print Using text supports Questioning Having background knowledge related to context

Confidence	Questioning	Problem Solving Strategies

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"Academic" Mathematics Journals

DAY 1: Why would I keep a MATH JOURNAL?

BRAINSTORM: Given a topic, students write, draw, construct, and calculate everything they can, related to that topic.

COPY: Students copy their solutions to specific homework problems into their journals. They then write an explanation of the method(s) they used to solve the problem.

EXPLAIN: Given a specific topic, students write as though they are explaining it to a given audience (a younger person, a peer, a mathematician, etc.)

PREDICT: Students make predictions as to the content of new units, topics, etc., the outcomes of upcoming tests and projects, success in new groups

REFLECT: "Reflect" is a heading used when students have completed a section or unit, taken a test or quiz, or worked on a group or individual project. It is also effective at the close of a class. "What is one new concept or skill you learned in class today? How does it connect to what you already knew? How will you use your new knowledge or skill?"

TRY: Students are given a problem – usually with a twist – related to the current content. They are to show all work and try anything (use any strategies and/or tools they think might help them solve the problem).

VENT: Students are given the opportunity to express their thoughts, feelings, frustrations, . . . with the current course content, course structure, groups, etc.

VOMIT: Given a specific topic, students are to write WITHOUT STOPPING. (3-5 minutes)

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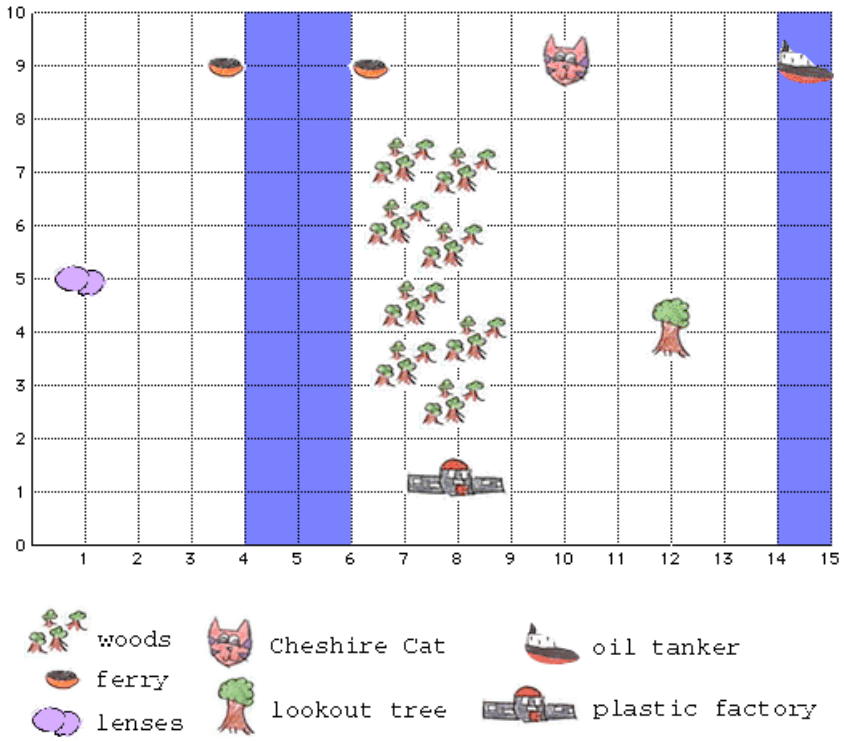
The Mad Hatter's Gone!

The Cheshire Cat loved Cheshire cheese. One day when he went to the cupboard to have a snack, he discovered it was missing! Tweedle Dum and Tweedle Dee told him that the Mad Hatter had run off with it, mumbling something about a tea party with Alice.

The Cheshire Cat was really hungry so he decided to join the party. To find its location, he needs to create some special x-ray binoculars designed to locate cheese. This is where the Cheshire Cat needs your help. To make the binoculars he needs you to collect the lenses and oil, and then head to the front entrance at the plastic factory. We all know that the Cheshire Cat can appear when and where he wishes. However, you must travel by land, using the ferry to cross the river and avoiding the woods (or you'll become hopelessly lost). Once you have the binoculars made he needs you to meet him at the lookout tree.

Can you find the shortest route to collect the materials, make the binoculars, and get to the lookout tree? Meet the Cheshire Cat on the map below and begin your planning. Your directions should include the whole number coordinates of each of the stops you make as well as the total distance you would travel.



Questions I need answered before I start solving this problem:

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Counting Chicken Wings

At Annie's Home-Cooked Chicken Wings Restaurant, chicken wings are served by the bucket. The Biggest Bucket O' Wings is really big! Let's figure out how many wings are in it. If they're removed two at a time, one wing will be left. If they're removed three at a time, two wings will remain. If they're removed four, five, or six at a time, then three, four, and five wings, respectively, will remain. If they're taken out seven at a time, no wings will be left over. What's the smallest possible number of wings that could be in the bucket? How do you know?

Before I solve this problem, I need to know the answers to these questions:

These are the answers to my questions:

Problem available online:
http://mathforum.org/midpow/solutions/solution.ehtml?puzzle=136
Dialogue journal format developed by Jill Perry

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Rabbits

Two rabbits each weigh the same. There are two boxes that each weigh the same. If the total weight of the two rabbits and two boxes is 18 pounds, and the weight of one box is 3 pounds, what does one rabbit weigh?

Do not solve the problem! Instead, in the space below, suggest at least two strategies that you could use to solve this problem. For each strategy, explain why your strategy would be reasonable.

Problem available online:
http://www.olemiss.edu/mathed/brain/rabbits.htm
Dialogue journal format developed by Jill Perry"

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But What Does It MEAN?

Ben Dare took a road trip to see his old college friend, Dawn Thatt. Because he was stuck in traffic for most of the way to Dawn`s house, Ben`s average speed there was 30mph. His average speed on the way back home was 20mph. (The traffic was much worse!) What was his average speed for the roundtrip?

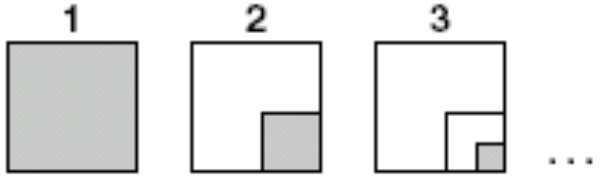
My answer:

In the area below, convince your partner that your answer is correct. Although you may wish to show your work as support for your argument, your argument should tell why you know you're right!

(problem from Eastaway, Rob & Wyndham, Jeremy. (1998). Why Do Buses Come in Threes? The Hidden Mathematics of Everyday Life. New York, NY: John Wiley & Sons; format developed by Jill Perry)

What a *Square*!

A sequence of shaded squares is displayed below. One vertex of each shaded square, after the first, is at the center of the preceding shaded square.



Find the ratio of the area of the 10th shaded square to the area of the 12th shaded square.

Do not solve the problem! Instead, in the space below, suggest at least two strategies that you could use to solve this problem. For each strategy, explain why your strategy would be reasonable.

Problem online:
http://www.state.nj.us/njded/stass/assessment/hspa_prep.pdf
Format developed by Jill Perry

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