

Nurturing Computational Fluency & Mathematical Thinking

Pia Hansen

Director of Professional Development

Math Learning Center

Fluency & Strategies

Solve the following problems mentally.

- As you come up with the answer, be aware of the strategy that you used to determine the answer.
- Most likely, your brain will make small adjustments quickly, and you will use an informal strategy to find the result.

$$9 + 7 = ?$$

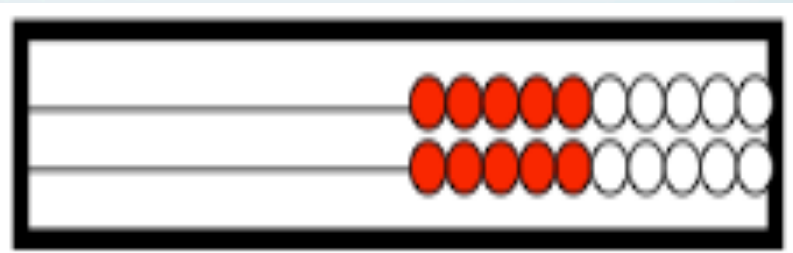
- What mental adjustments did you make as you solved this problem?

What is Computational Fluency?

- *Efficiency* implies that the student does not get bogged down in many steps or lose track of the logic of the strategy.
- *Accuracy* depends on several aspects of the problem-solving process; careful recording, the knowledge of basic number combinations and relationships.
- *Flexibility* requires the knowledge of more than one approach. Students need to be able to choose an appropriate strategy for the problem. They can use one and another method to double-check the results.

Question...

- We use these strategies as adults. How can we make them concrete and accessible for our students?
- The **Rekenrek/Number Rack** is a powerful tool that helps children “subitize”, develop cardinality, 1-1 correspondence, and work flexibly with strategies.



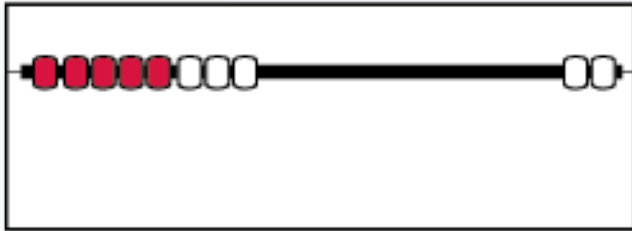
What is the Number Rack?

- Two strings of 10 beads each, strategically broken into groups of 5, entices students to think in groups of 5 and 10.
- Students “see” the structure and begin to compose and decompose numbers... to see groups of 2, 5 & 10.

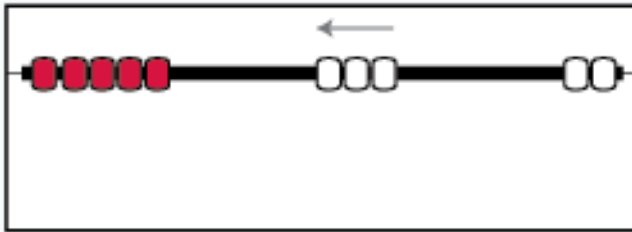
Let's use a Number Rack!

- Starting Position
- Do it in one push-"subitize"
- Show me
- Make 5, 10, and sums to 20
- Doubles and doubles plus and minus 1
- Ten and some more
- Flashing & screening

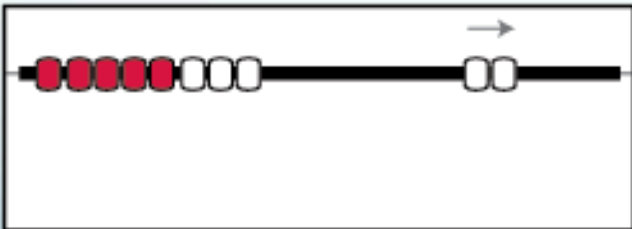
Differentiating our instruction




“1, 2, 3, 4, 5, 6, 7, 8. That’s 8.”



“Slide 5, and then 3 more makes 8.”



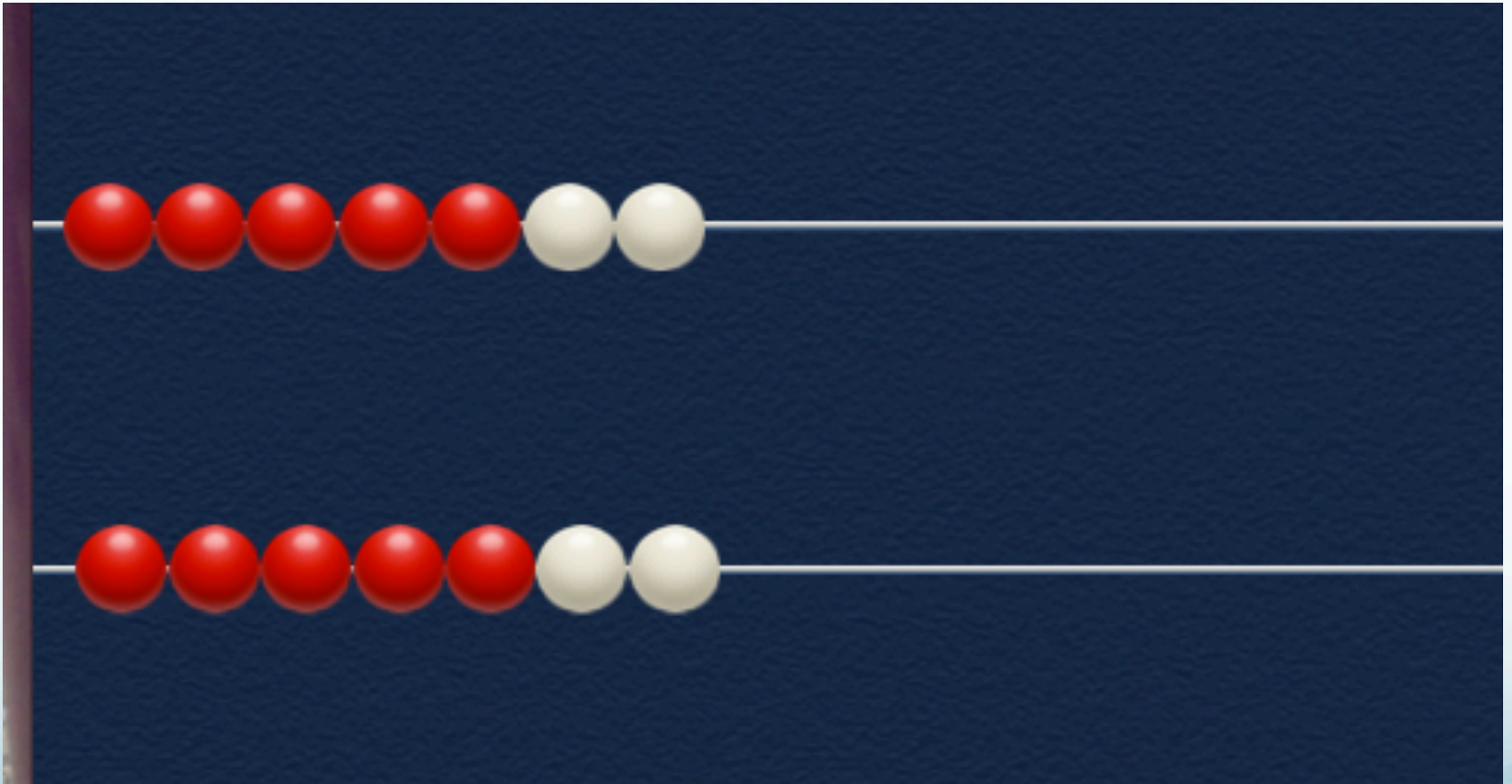
“I know it’s 8 because it’s 2 less than 10.”

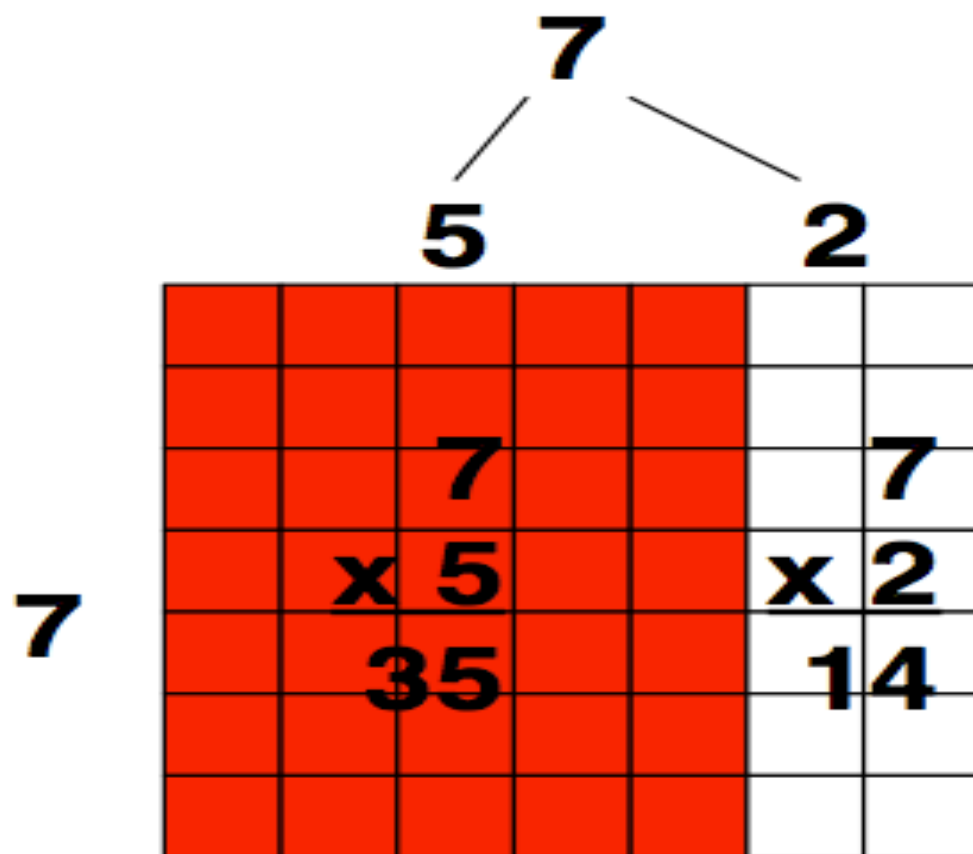


Using the Rekenrek as a Visual Model for Strategic Reasoning in Mathematics

<http://catalog.mathlearningcenter.org/apps/number-rack>

Arrays to 100





$$\begin{aligned}
 7 \times 7 &= 7 \times (5 + 2) \\
 &= 7 \times 5 + 7 \times 2 \\
 &= 35 + 14 \\
 &= 49
 \end{aligned}$$

Drill & Practice

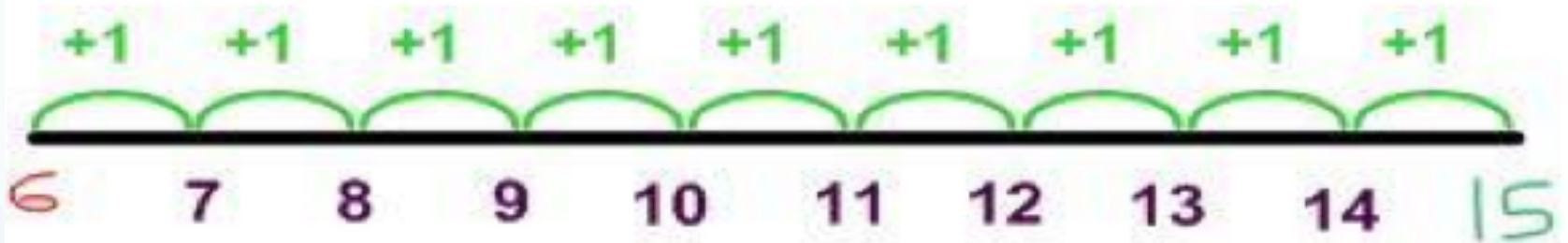
- Provide untimed practice to develop efficient and flexible strategies using a variety of models.
- Probe student thinking frequently.
- Notate student thinking on anchor charts for others to see.
- Once strategies are developed, drill for individual fact strategies. Progress monitor often.
- Finally, provide mixed fact practice to develop automaticity and fluency.

Thinking strategies...

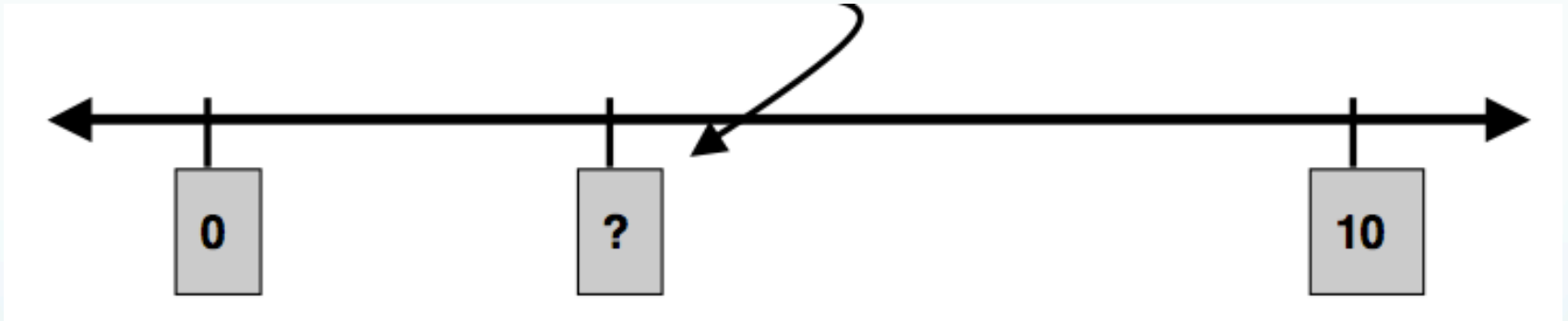
- Provide engaging opportunities for practice through the use of games and activities.
- Encourage strategic mathematical thinking
- Encourage efficiency in computation with groups, properties and place value
- Develop familiarity with the number system and compatible numbers (landmark).

Number lines for thinking

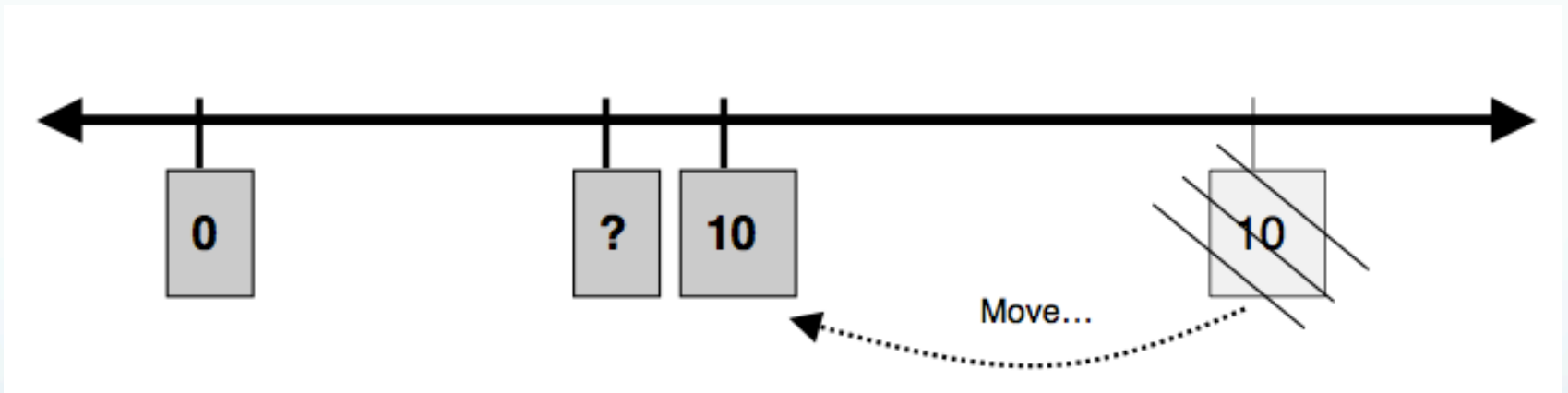
Counting by 1's



What goes in the empty box?



And now? What goes in the box?



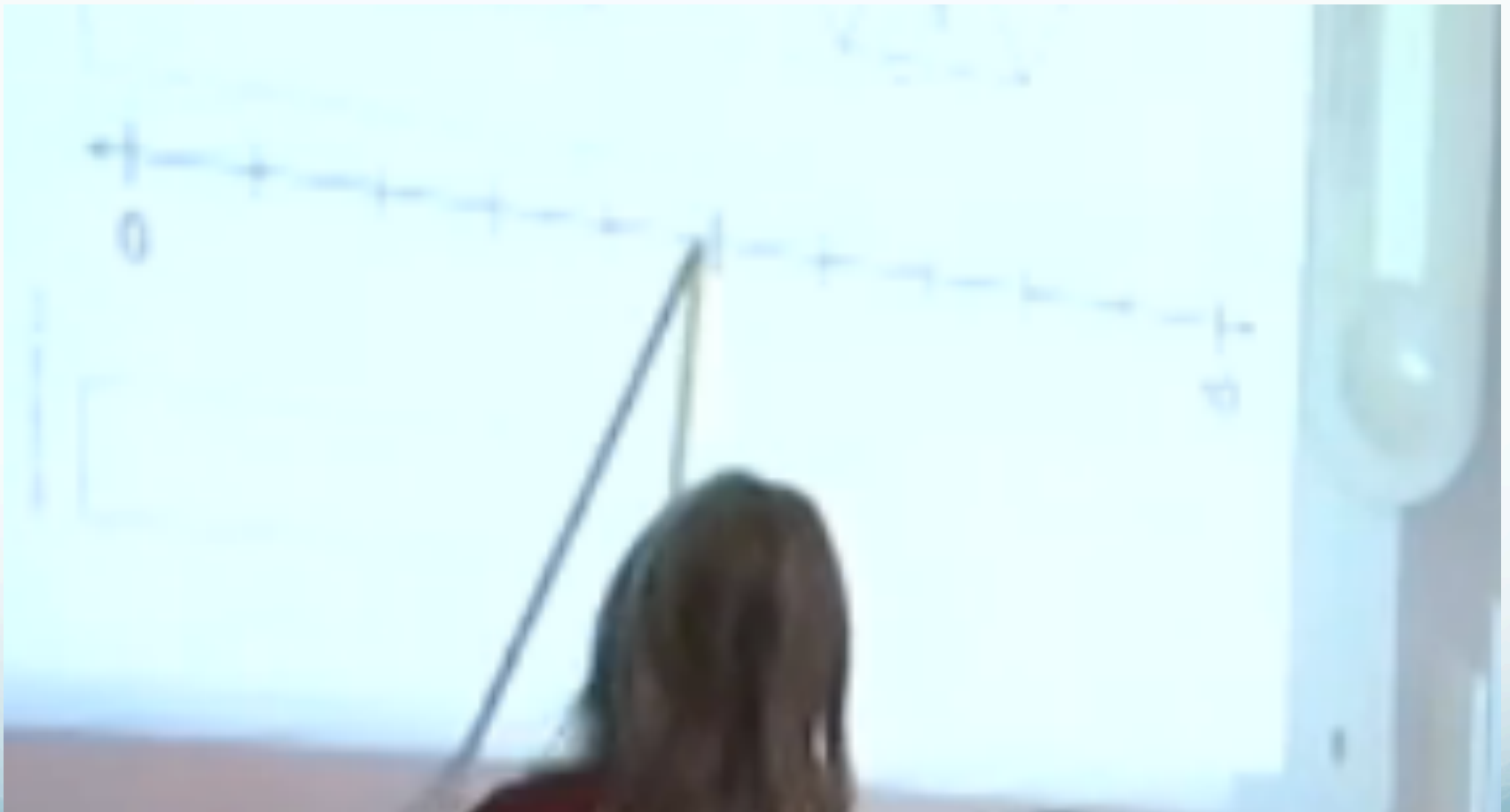
Kinders & Number Lines

Kindergarten Students

Use the Number Line

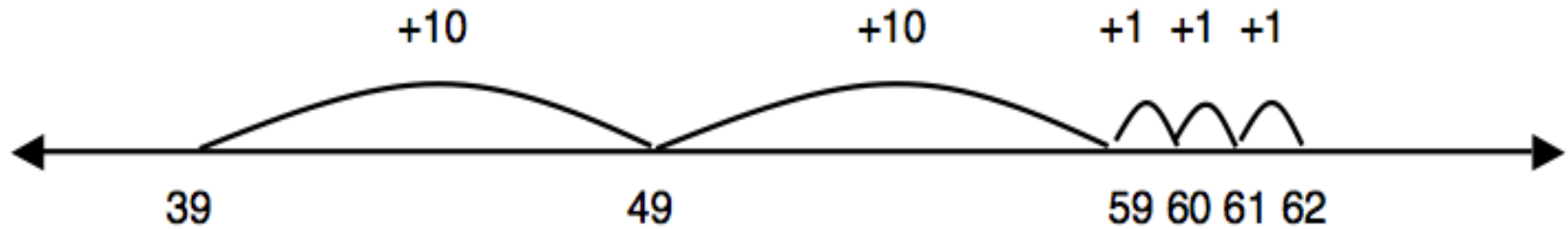
for the First Time

Interval and Discrete models



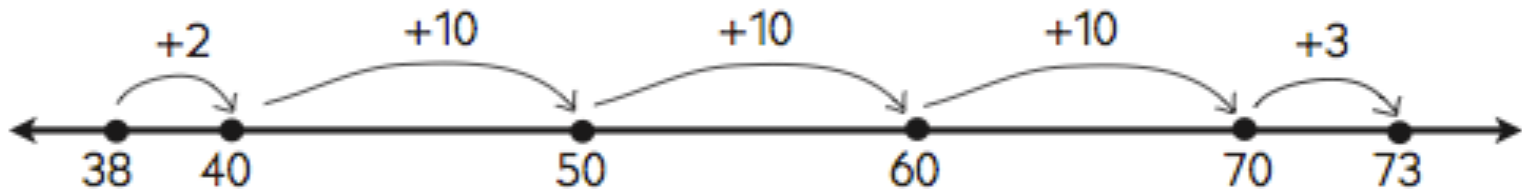
Addition with the line

$$39 + 23 = 62$$

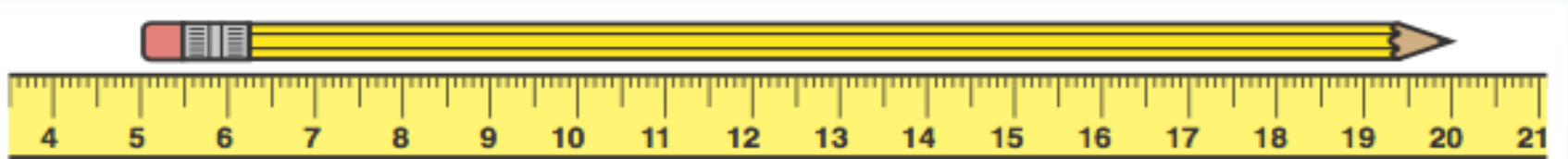


Finding the difference, or
distance between two numbers

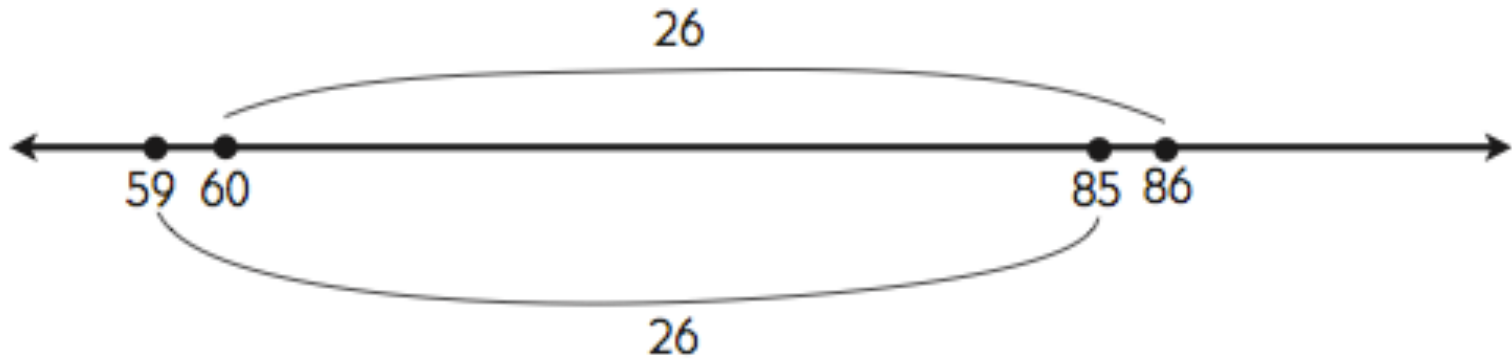
$$73 - 38 = ?$$



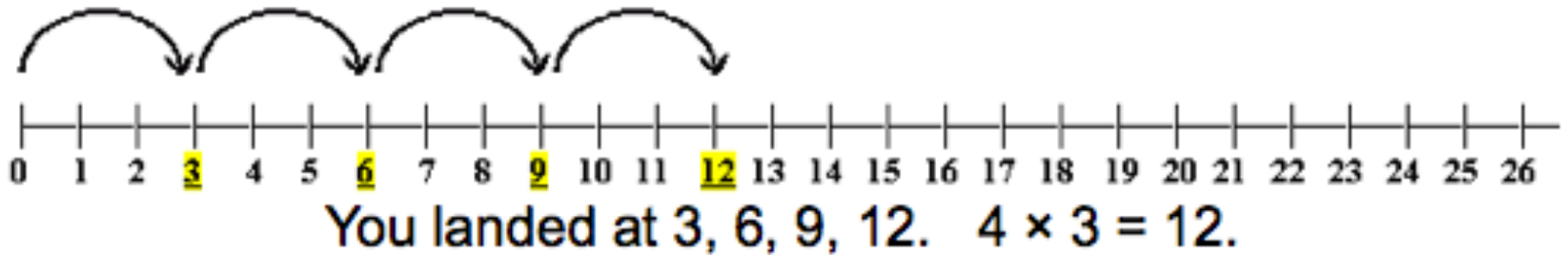
Measure with the number line



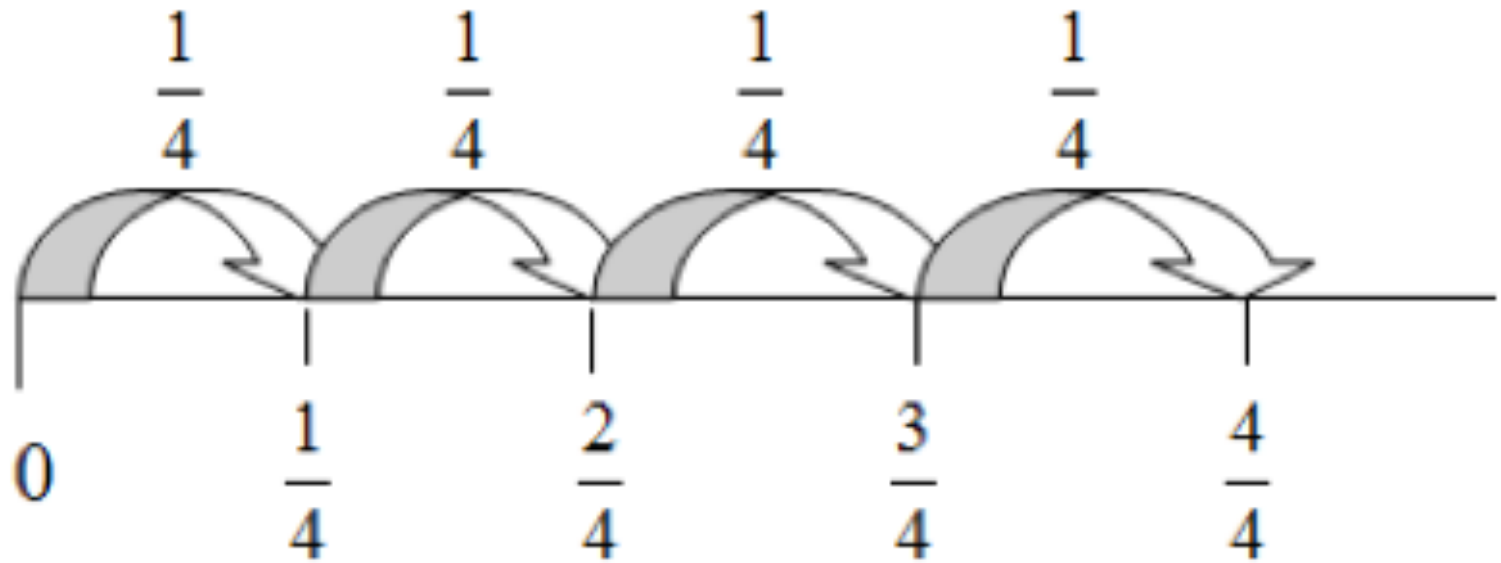
Adjust the numbers,
make the problem simpler!



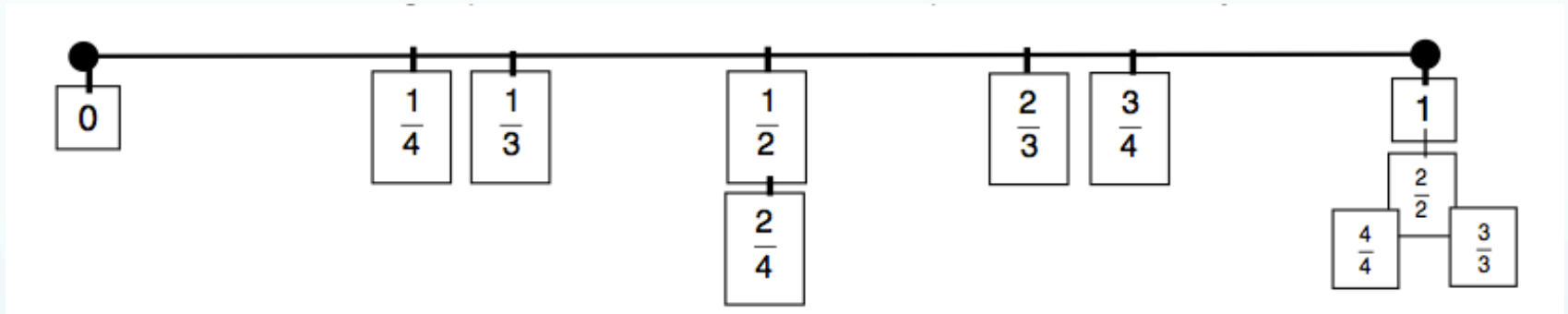
Multiplication on the number line



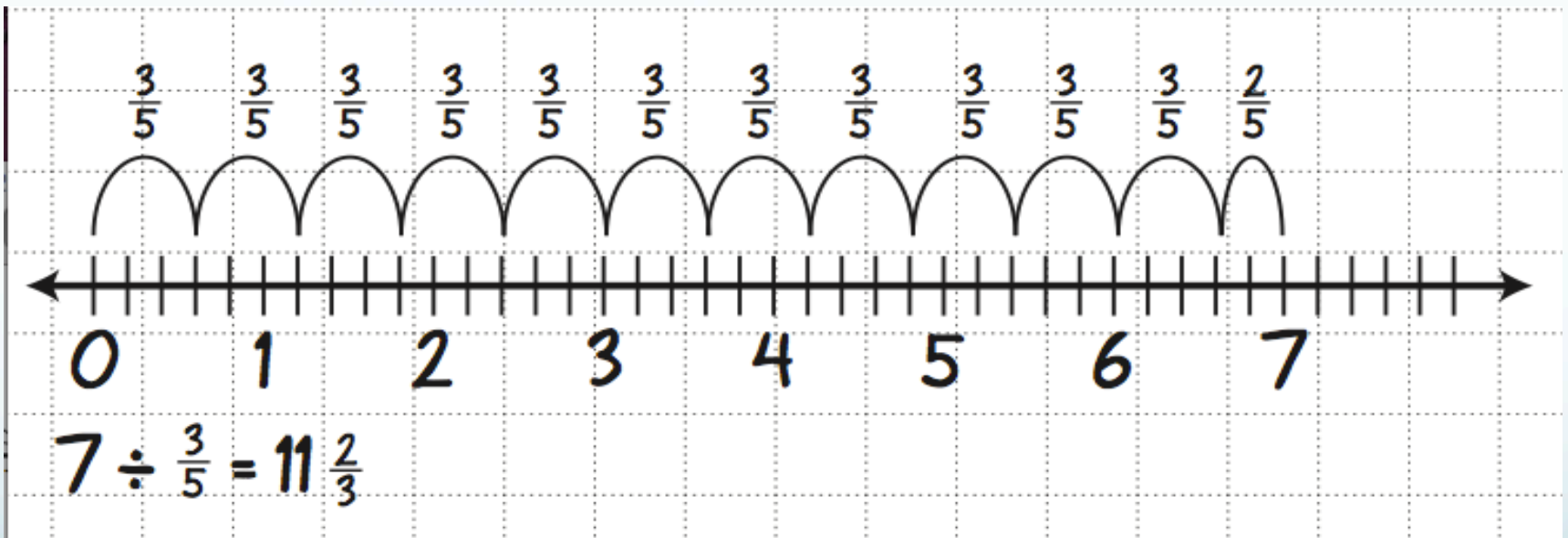
Unit Fractions on the line



Equivalence



Error or not?



Thinking Strategies & Fluency

- What are some examples of thinking strategies that you engaged in?
- How can you nurture fluency in your classroom?