Learn Recursion Through Manipulatives and Pictures

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<u>Activity 1</u>: Recursive Sequences (Murdock, Kamischke, & Kamischke, 1996, p.21)

stage 1 stage 2 stage 3 stage 4

Establish u_1 and u_n for recursion and closed forms. Making a chart is beneficial.

A) Make a sequence suggested by the figure.
B) Make a sequence by looking at the outer perimeter.
C) Make a sequence by looking at the total number of segments.

<u>Activity 2</u>: Handshake Problem

How many handshakes are possible for a group of people if everyone shakes hands once with all the members of the group?

<u>Activity 3</u>: Role Play the Computation of 5! (Dale & Weems, 1991)

What is 5!	What is 4! 5! = 5 * 4! Then 5!=120	What is 3! 4! = 4 * 3! Then 4!=24	What is 2! 3! = 3 * 2! Then 3!=6	What is 1! 2! = 2 * 1! Then 2!=2	What is 0! 1! = 1 * 0! Then 1! = 1	0! = 1

<u>Activity 4</u>: Tower of Hanoi

Use a model of the Tower of Hanoi to keep track of the number of moves to get a "tower" of disks from one peg to another. Only one disk can be moved at a time while a larger disk cannot be placed on a smaller disk. A solution is the least number of disk moves to shift a "tower" of disks from one peg to another.

A) Fill in the following table:

Number of disks	1	2	3	4	5	6	7
Number of moves							

B) What is the initial term of the sequence? What is the general term of the sequence?

<u>Activity 5</u>: Zookeeper's Puzzle (Cornell & Siegfried, 1991)

A certain zookeeper has n cages in a row and two indistinguishable lions. The lions have to be in separate cages, and they may not be in adjacent cages. How many ways could the zookeeper assign the lions to cages?

A) How many ways can 2 lions be placed in 7 cages?
B) If the zookeeper wants to place the lions differently each day for a month, how many cages are needed? (p. 152)

<u>Activity 6</u>: Diagonals of a Polygon (NCTM, 2008-2009)

A polygon has d diagonals if a diagonal is defined as a segment connecting two nonadjacent points. If the polygon had one more side, it would have d + 10 diagonals. How many sides does the polygon have? (p. 360)

<u>Activity 7</u>: Jump & Slide

A puzzle common to family restaurants consists of a narrow piece of wood with seven holes in a row. Three blue golf tees are placed in the right side holes while three white golf tees are placed on the left side. The objective is to interchange the tees by sliding (moving one space) or jumping (jump over one opponent). Only forward moves are allowed.
What is a general expression for the minimum number of moves needed for a solution?

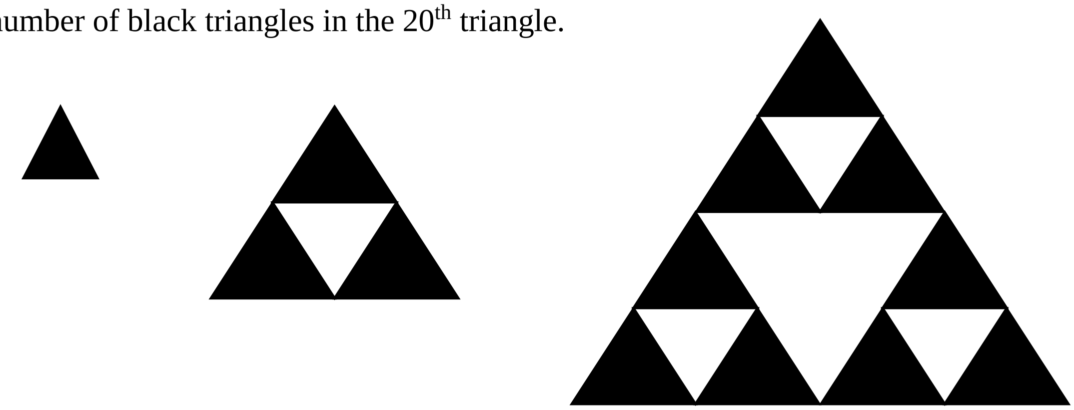
Pegs per side	1	2	3	4	5	…	n
moves							

Will counting the number of different moves give more insight into a solution?

Pegs per side	1	2	3	4	5	…	n
Slides							
Jumps							

<u>Activity 8</u>: Find the Number of Black Triangles (Sierpinski's Triangle)

Find the number of black triangles in the 20th triangle.



Activity 9: Divide the Rectangle into Squares

Begin with a 34 x 21 rectangle, and cut off the largest possible square. Take the remaining 21 x 13 rectangle, and cut off the largest possible square. Continue this process until no rectangle remains. Arrange the squares from smallest to largest, and determine a sequence using the length of the sides.

References

Cornell, R. & Siegfried, E. (1991). Incorporating recursion and functions in the secondary school mathematics curriculum. In M. Kenney & C. Hirsch (Eds.), *Discrete Mathematics Across the Curriculum, K-12, 1991 Yearbook of the National Council of Teachers of Mathematics* (149-157). Reston, VA: National Council of Teachers of Mathematics.

Dale, N. & Weems, C. (1991). *Introduction to Pascal and Structured Design (3rd ed.).* Sudbury, MA: Jones and Bartlett.

Murdock, J., Kamischke, E., & Kamischke, E. (1996). *Advanced Algebra Through Data Exploration: A Graphing Calculator Approach*. Berkeley, California: Key Curriculum Press.

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Activity Answers

Activity One – Recursive Sequences

A) The first figure has one square, $u_1 = 1$. Each succeeding figure has two squares added. Recursion: $u_n = u_{n-1} + 2$, closed form: $u_n = 2n – 1$

B) The first figure has a perimeter of four, $u_1 = 4$. Each succeeding figure adds four units to the perimeter. Recursion: $u_n = u_{n-1} + 4$, closed form: $u_n = 4n$

C) The first figure has four segments, $u_1 = 4$. Each succeeding figure adds six segments to the figure. Recursion: $u_n = u_{n-1} + 6$, closed form: $u_n = 6n – 2$

Activity Two – Handshake Problem

People	1	2	3	4	5
Number of handshakes	0	1	3	6	10

Recursion: $u_2 = 1$, $u_n = u_{n-1} + n – 1$ closed form: $u_n = (n(n-1))/2$ triangular numbers

Activity Four – Tower of Hanoi

A)

Number of disks	1	2	3	4	5	6	7
Number of moves	1	3	7	15	31	63	127

B) recursion: $u_1 = 1$ $u_n = u_{n-1} + 1 + u_{n-1} = 2u_{n-1} + 1$
closed form: $2^n – 1$

Activity Five - Zookeeper's Puzzle

A) 15
B) 10
Recursion: $u_3 = 1$, $u_n = n-2 + u_{n-1}$ closed form: $(n-1)(n-2)/2$ (triangular numbers)

Activity Six – Diagonals of a Polygon

sides	4	5	6	7		11	12		n	n + 1
diagonals	2	5	9	14		44	54		d	d + 10

initial term: $u_4 = 2$

closed form: $d = (n – 2)(n – 1)/2 – 1 = n(n – 3)/2$

(1) $2d = n^2 – 3n$

$d + 10 = (n + 1)(n – 2)/2$

(2) $2d + 20 = n^2 – n - 2$

subtracting (1) from (2)

$20 = 2n – 2$

$22 = 2n$

$11 = n$

recursion: $u_n = u_{n-1} + n – 2$

$d + 10 = d + (n + 1) - 2$

$11 = n$

Activity Seven – Jump and Slide

Pegs per side	1	2	3	4	5	…	n
Moves	3	8	15	24	35		

Pegs per side	1	2	3	4	5	…	n
Slides	2	4	6	8	10		
Jumps	1	4	9	16	25		

Closed form: moves = (pegs on side + 1)^2 - 1

Recursion: moves$_n$ = moves$_{n-1}$ + holes = moves$_{n-1}$ + 2(pegs per side) + 1

Formulas using the slides and jumps data:

Closed form: moves = jumps + slides = (pegs per side)^2 + 2(pegs per side) =
(pegs per side)(pegs per side + 2)

Recursion: moves$_n$ = slides$_{n-1}$ + 2 + (peg per side)^2

Activity Eight – Number of Black Triangles

Recursion: $u_1 = 1$ $u_n = 3u_{n-1}$ closed form: $u_n = 3^{n-1}$

20th triangle contains 1 162 261 467 smaller triangles.

Activity Nine – Divide the Rectangle into Squares

1, 1, 2, 3, 5, 8, 13, 21, 34 Fibonacci sequence

recursion: $u_1 = 1$, $u_2 = 1$, $u_n = u_{n-1} + u_{n-2}$ closed form not easily found

calculator note: nMin = 1, u(nMin) = {1,1}