Using Origami Boxes to Visualize Mathematical Concepts

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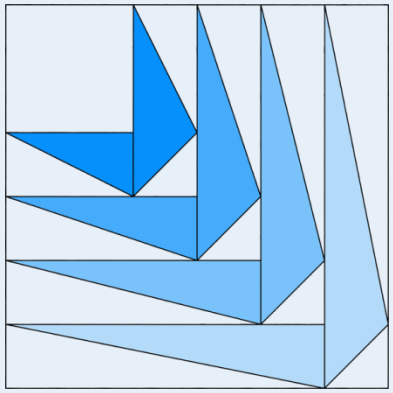


If people never did silly things nothing intelligent would ever get done.

Ludwig Wittgenstein

No longer is the purpose of education is simply to pick out those students who are intelligent, on one or another definition, and give them special access to higher education. Rather, the purpose of education now is to educate an entire population, for we cannot afford to waste any minds.

Howard Gardner
(1943-)
American Psychologist & Educator
(Gardner, 2006)

Once it has been demonstrated that algebra can be taught three or even thirty ways, it will be malpractice to declare “Johnny could not learn algebra my way- bring me another child.

Howard Gardner
(1943-)
American Psychologist & Educator
(Gardner, 2006)

Standards for Mathematical Practice

Make sense of problems and persevere in solving them
Reason abstractly and quantitatively
Construct viable arguments and critique the reasoning of others
Model with mathematics
Use appropriate tools strategically
Attend to precision
Look for and make use of structure
Look for and express regularity in repeated reasoning

Common Core State Standards Initiative (CCSSI). 2010. *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

NCTM Process Standards

Problem Solving
Reasoning and Proof
Communication
Connections
Representation

National Council for Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Origami

(from *ori* meaning *folding,* and *kami* meaning *paper*)

Origami is the Japanese art of paper folding.

It started in the 17th century AD and was popularized in the mid-1900s.

In 1930 Akira Yoshizawa, a Japanese origami artist/writer, comes up with a way of illustrating the steps. This revitalized origami throughout the world.

In origami the goal is to turn a flat piece of paper into a three dimensional sculpture.

Cutting and gluing are not acceptable.

Traditionally a square sheet of paper is used.

But it is okay to break this rule!

Boxes made from rectangular sheets can be very interesting because of two variables involved, length and width.

Rectangular sheets are more readily available.

Famous Names in Origami

Akira Yoshizawa
Japanese Origami Artist/Writer
(1911-2005)

Tomoko Fuse
Japanese Origami Artist/Writer
(1951-)

Robert J. Lang
American Physicist/Mathematician/Origami Artist
(1950-)

Erik Demaine
Canadian-American Computer Scientist/
Mathematician/Origami Artist
(1981-)

Thomas Hull
American Mathematician

Arnold Tubis
American Physicist

Kazuo Haga
Japanese Biologist

Toshikazu Kawasaki
Japanese Mathematician

Michael LaFosse
American Biologist

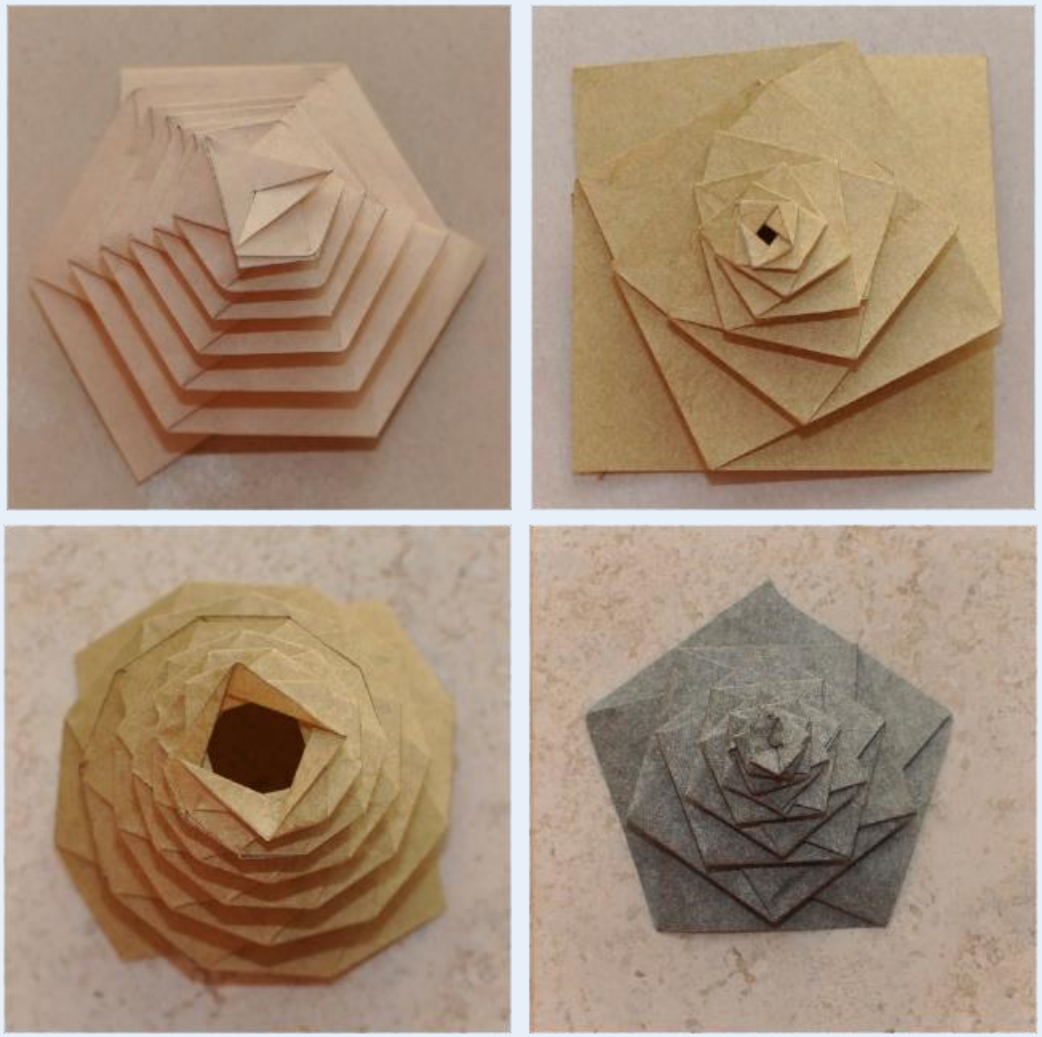
Carmen Sprung
German Origami Artist/Writer

Nick Robinson
British Origami Artist/Writer

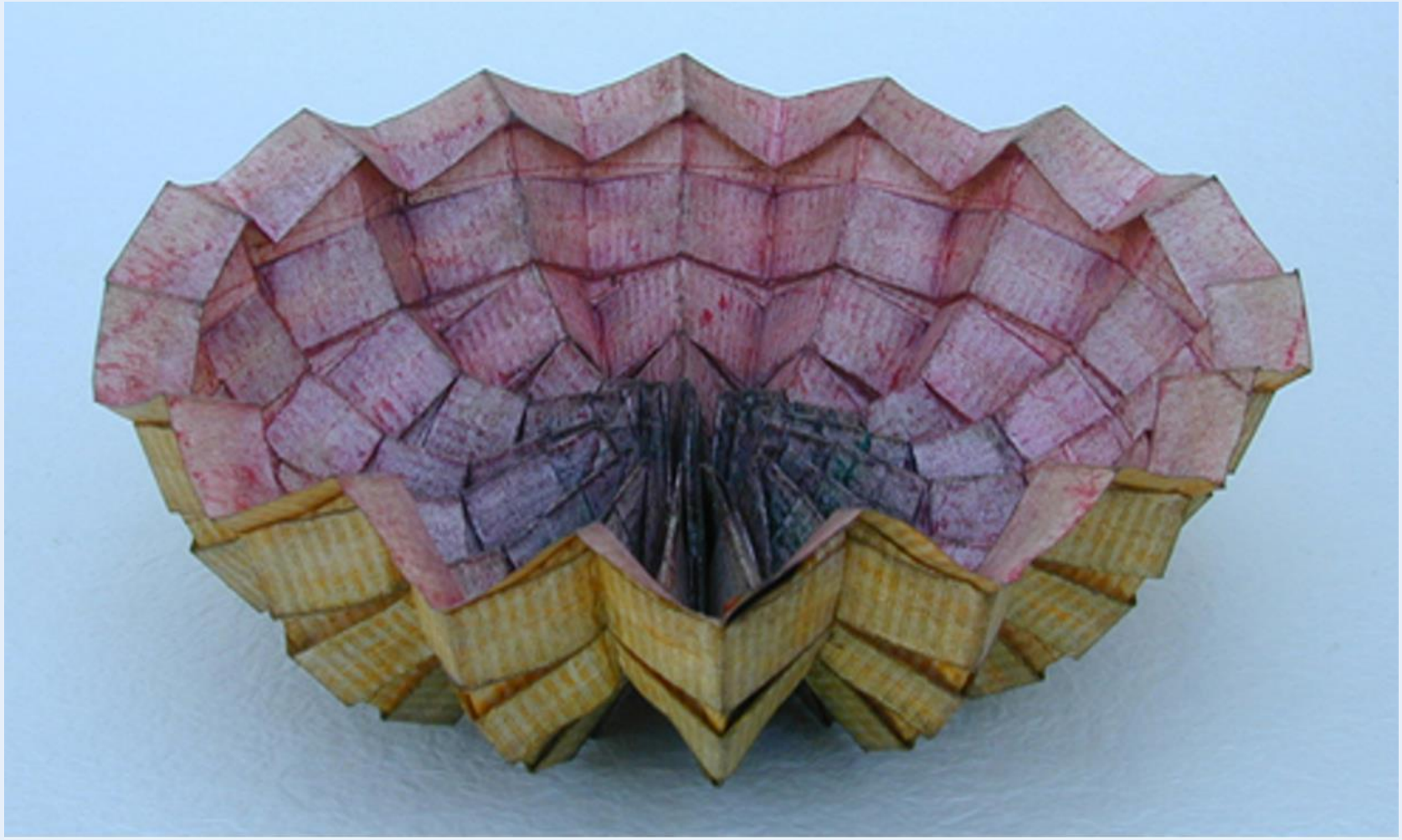
Paul Jackson
British-Israeli Origami Artist/Writer

Hugging Circles by Erik and Martin Demaine

Green Cycles by Erik and Martin Demaine



Whirlpools by Tomoko Fuse



Bowl by Paul Jackson

Can origami save someone’s life?

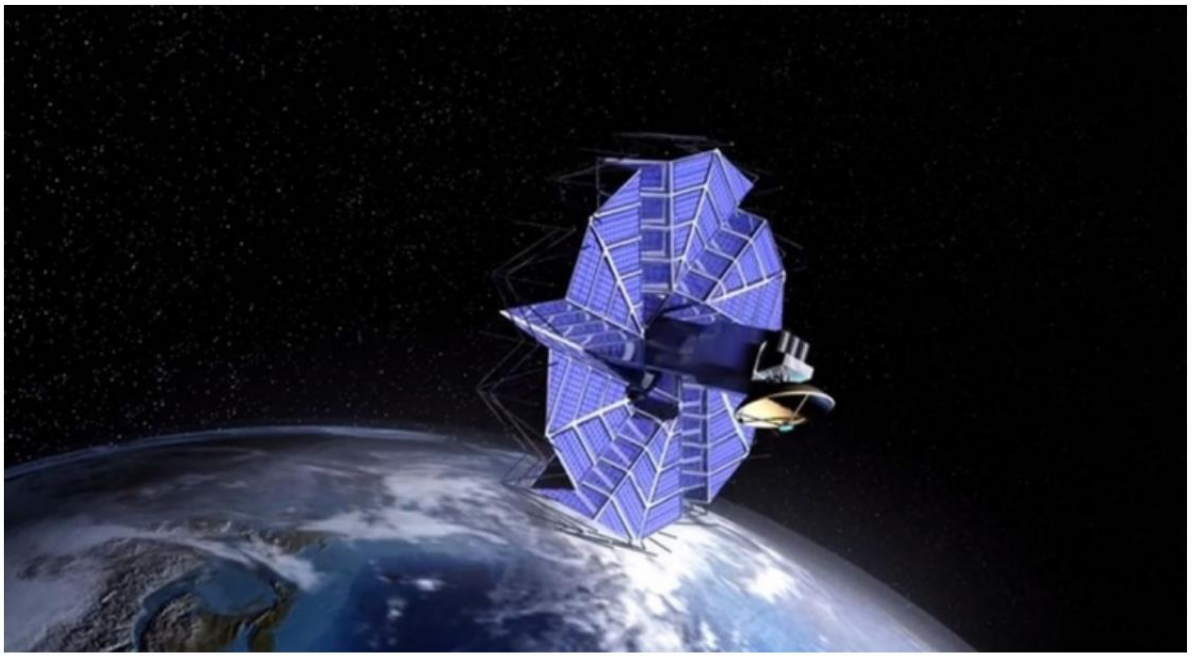
Zhong You and Kaori Kuribayashi

Department of Engineering Science
University of Oxford
Parks Road
Oxford, OX1 3PJ
UK

SPACE | SOLAR PANELS | ORIGAMI

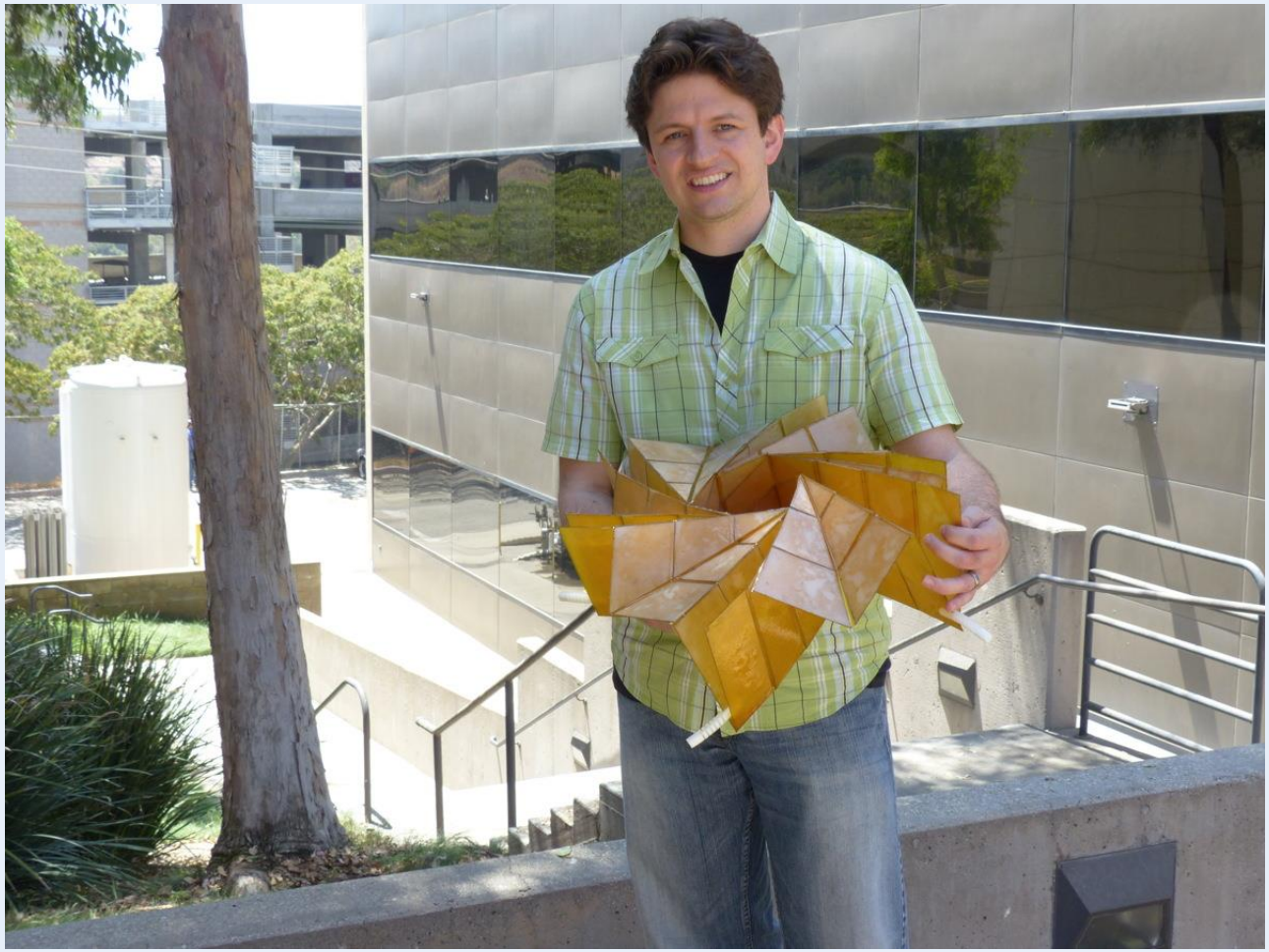
Origami-inspired solar panel unfurls in space like a paper flower

Michael Trei Monday, December 16, 2013 - 5:32pm



Launching anything into space is really, really, really expensive. So, the number one concern when building something that needs to be launched skyward is keeping it small, and keeping it light. Most of the solar panels used on spacecraft are designed to fold open once they reach their destination, but just *how* they're folded can have a big effect on how small they are at launch time. To get the maximum possible efficiency in packing, a group of spacecraft engineers at Brigham Young University have sought out the advice of **origami expert** and physicist Robert Lang.

The BYU team has been working with NASA's Jet Propulsion Laboratory to develop a solar array that folds open to nearly ten times its packed size, and which is capable of generating 250 kilowatts of power. When folded, the array is designed to wrap around the core of the spacecraft. While a specific application for the array has yet to be determined, NASA expects to continue developing the origami-inspired designs for the next several years.



NASA mechanical engineer Brian Trease holds the prototype of the origami-inspired solar panel arrays.

Source:
http://www.space.com/27485-origami-space-solar-panels-video.html

According to Robert Lang,

98% of the innovations in origami came in the last 2% of the art's existence (Lang, 2012).

Lang, R. (2012). *Origami design secrets*. Boca Raton, FL: CRC Press.

Gardner identified the following eight intelligences:

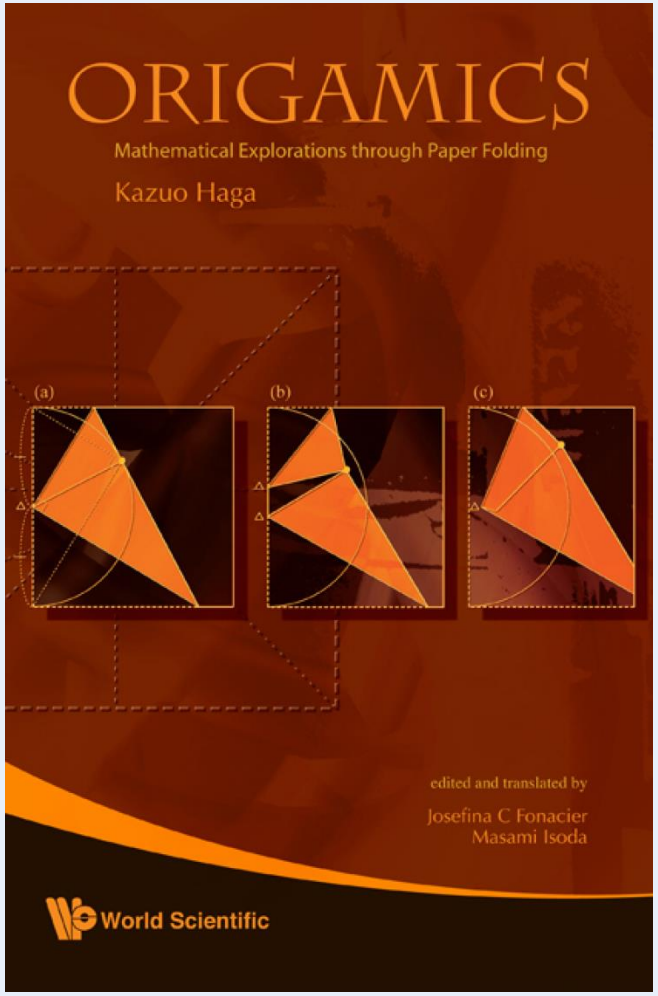
linguistic intelligence,
logical-mathematical intelligence,
bodily-kinesthetic intelligence,
spatial intelligence,
musical intelligence,
interpersonal intelligence,
i*ntrapersonal intelligence*, and
naturalist intelligence.

Gardner, H. (2006). *Multiple Intelligences: New Horizons*, New York, NY: Basic Books.

Math concepts and skills that can be fostered through origami (Tubis & Mills, 2006)

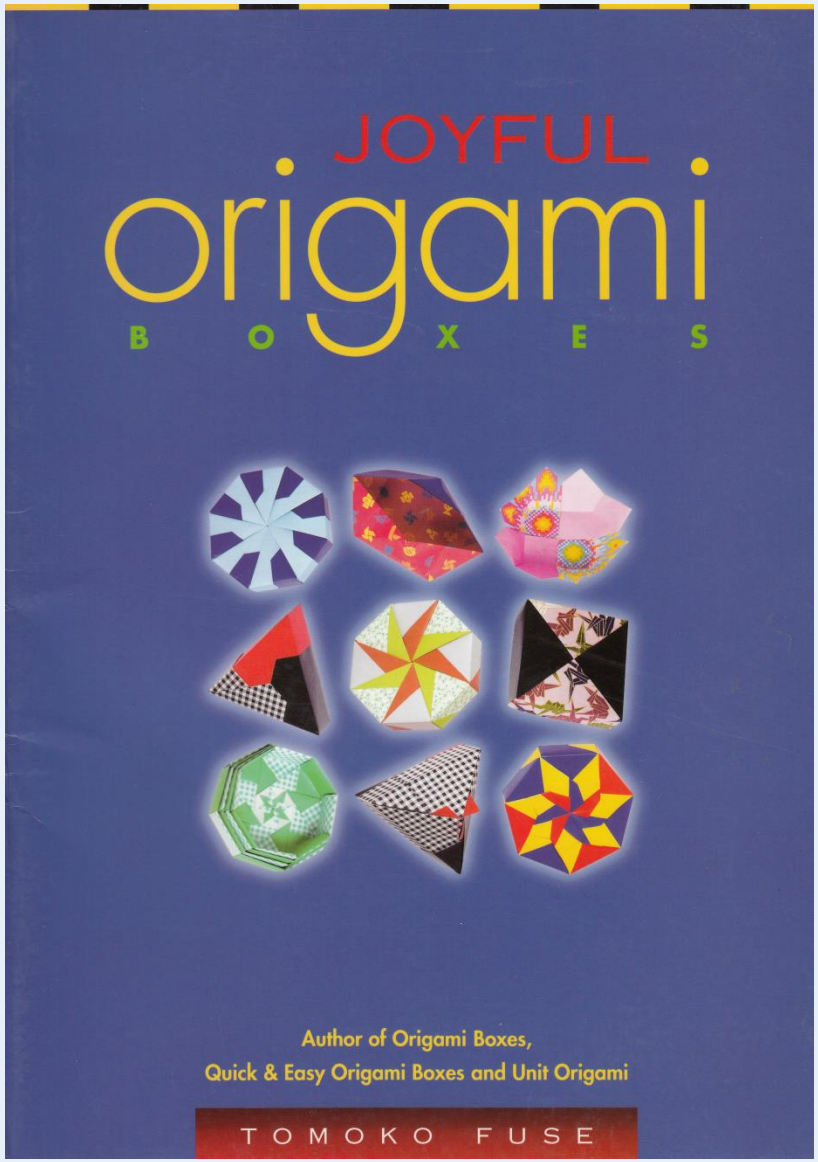
Algebraic equations
Angles
Area, volume, and surface area
Arithmetic
Bisection of lines and angles
Congruence
Fractions and ratios
Graphing
Inequalities
Maximization and minimization of parameters
Polygons
Pythagorean theorem
Spatial visualization
Symmetry

Tubis, A. & Mills, C. (2006). *Unfolding mathematics with origami boxes*. Emeryville, CA: Key Curriculum Press.



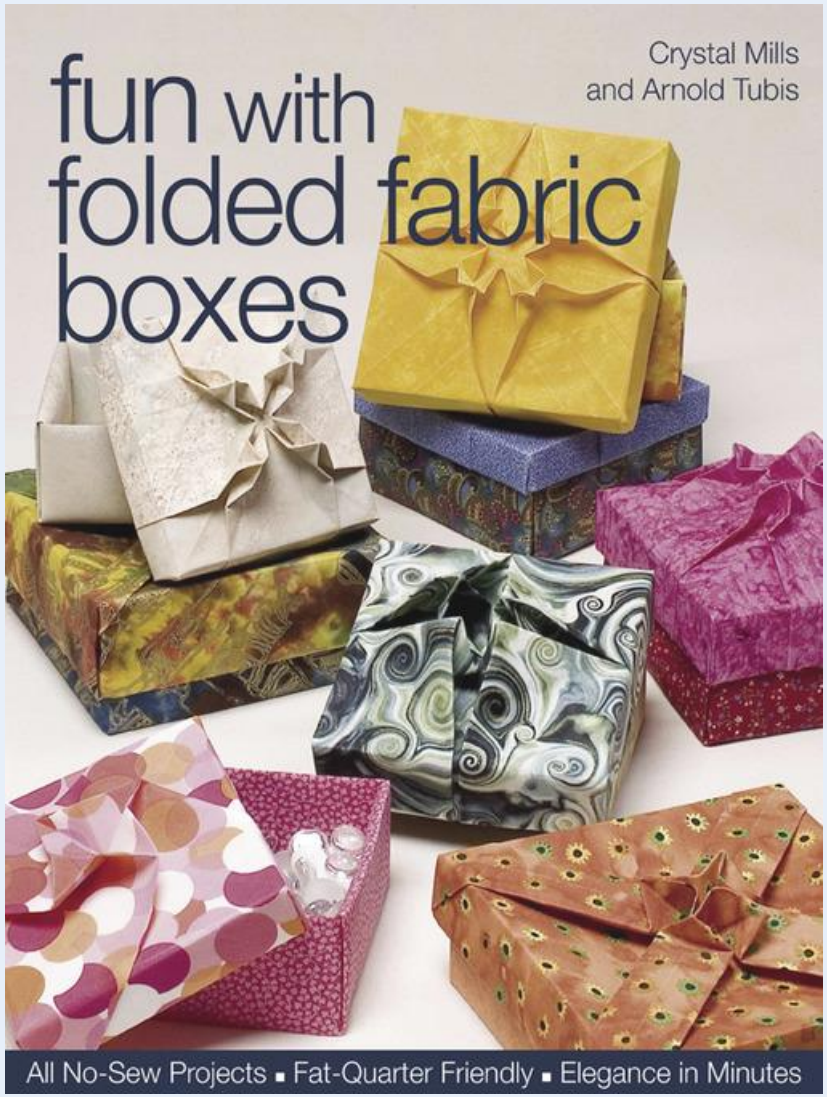
Kazuo Haga’s Book

Haga, K. (2008). *Origamics*. Hackensack, NJ: World Scientific.



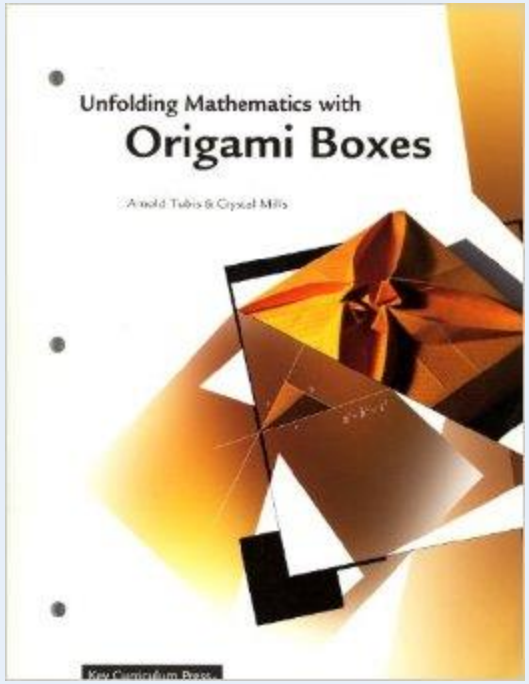
One of Fuse's many great books.

Fuse, T. (1995). *Joyful origami boxes*. Tokyo: Japan Publications.



A book by Crystal Mills and Arnold Tubis.

Mills, C. & Tubis, A. (2007). *Fun with folded fabric boxes*. Concord, CA: C & T Publishing.



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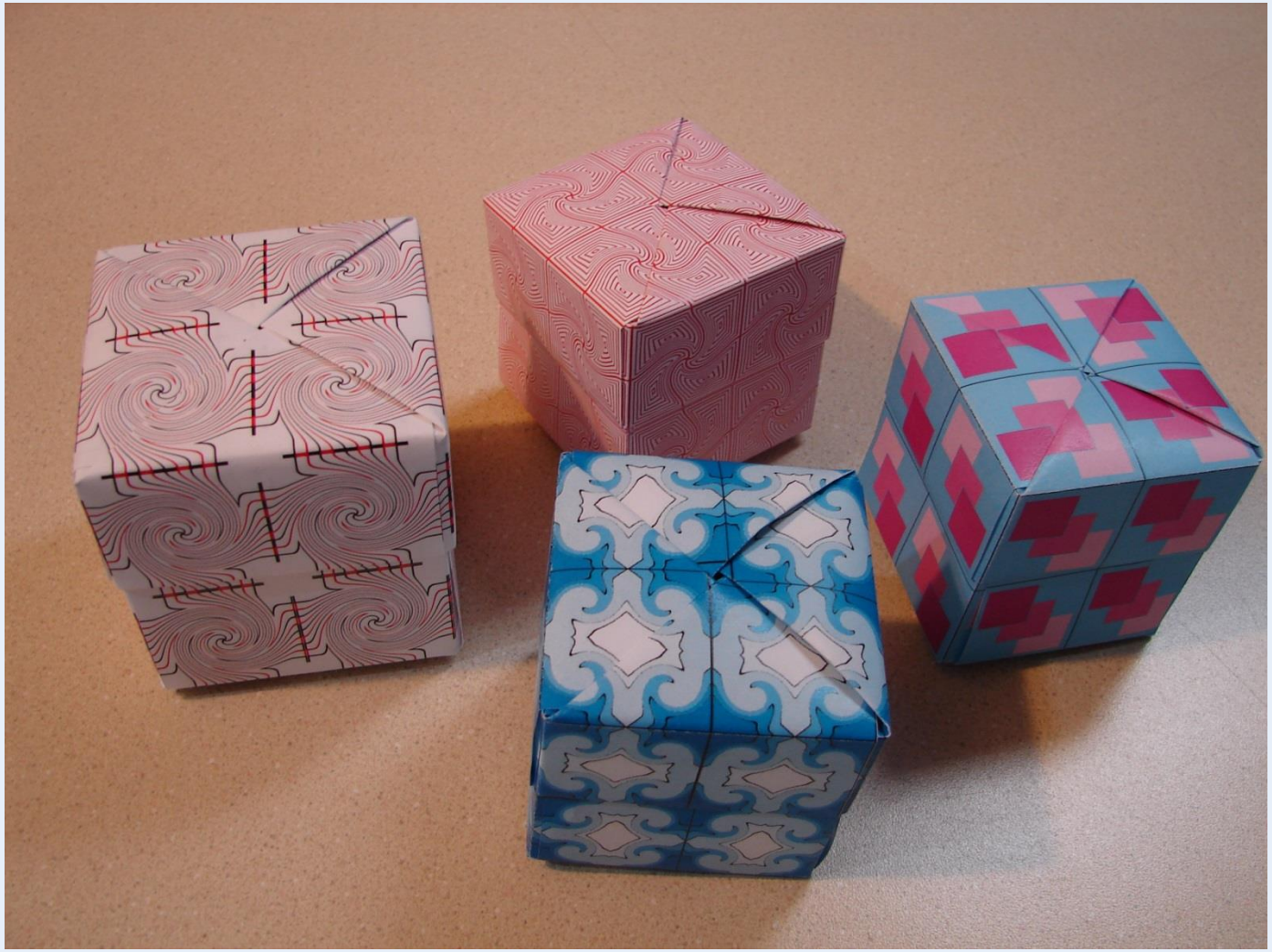
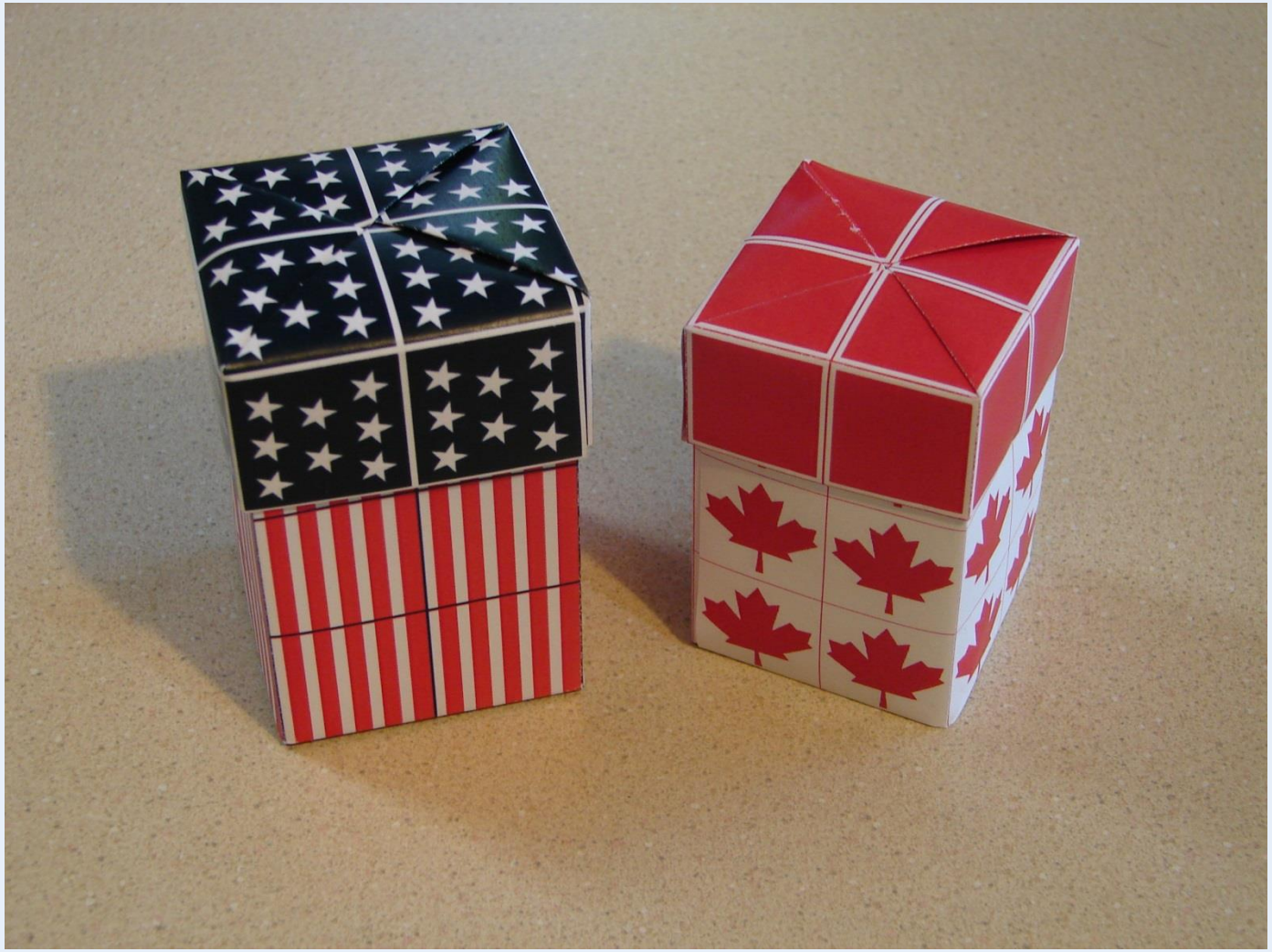
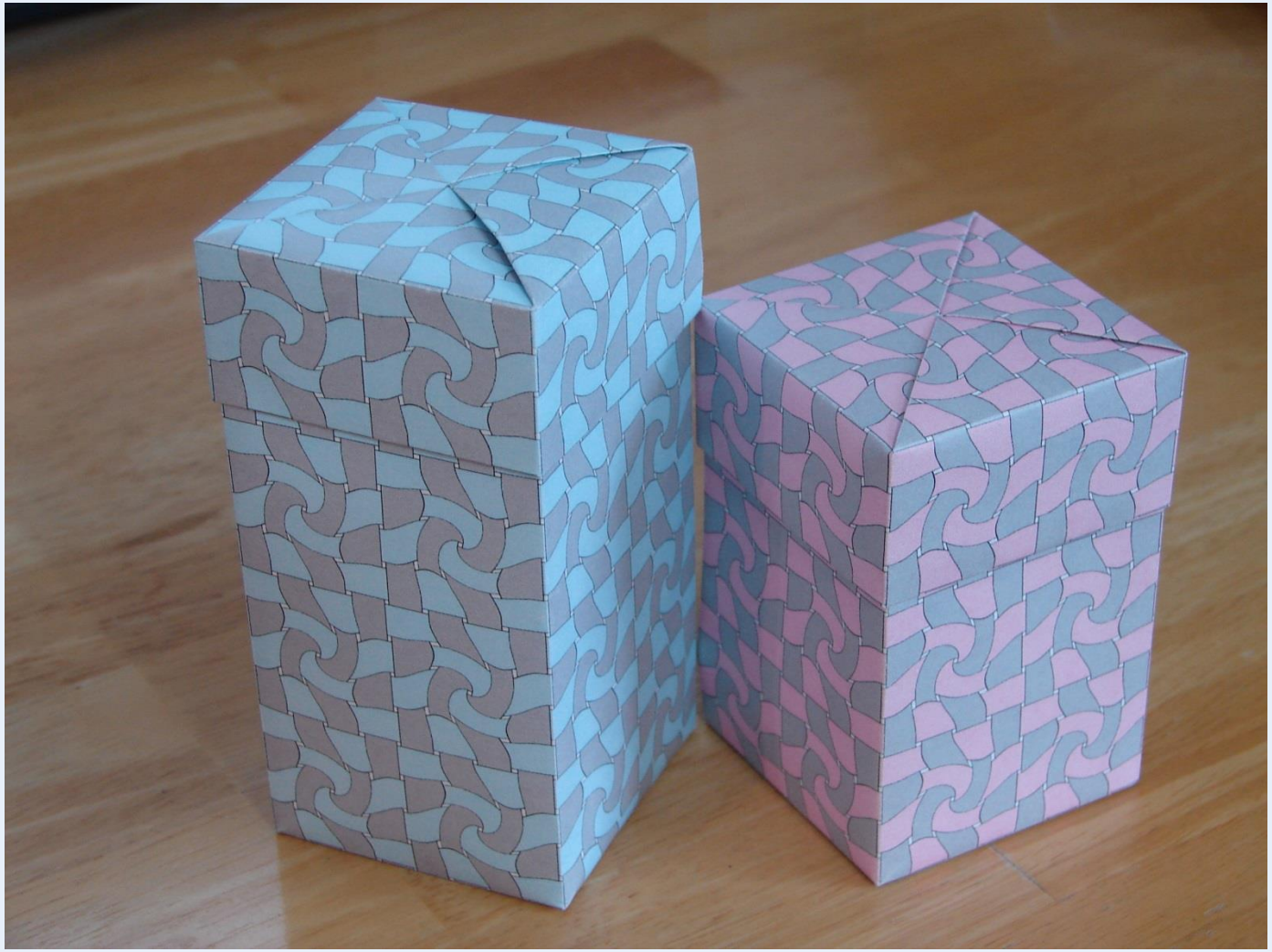
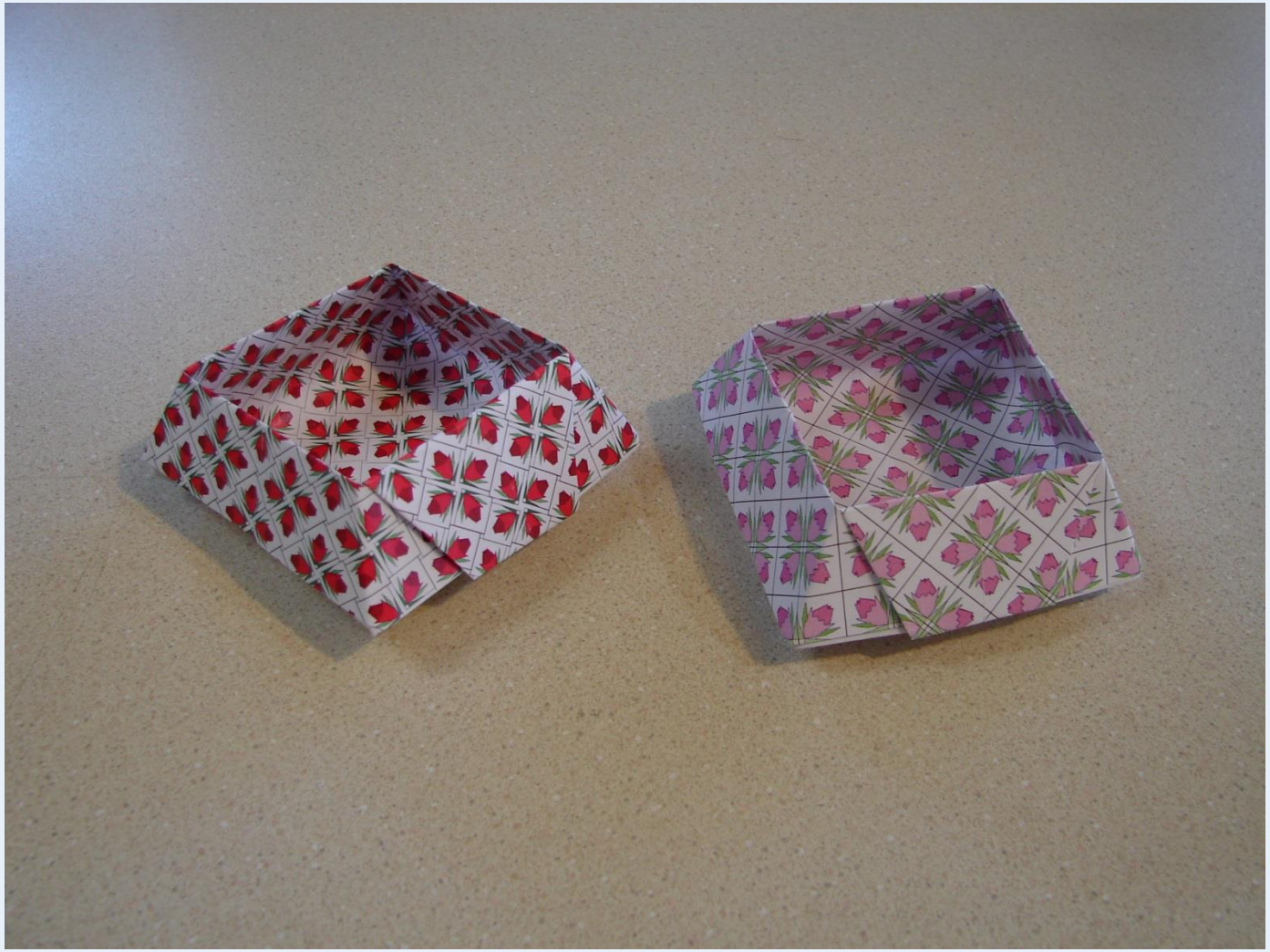
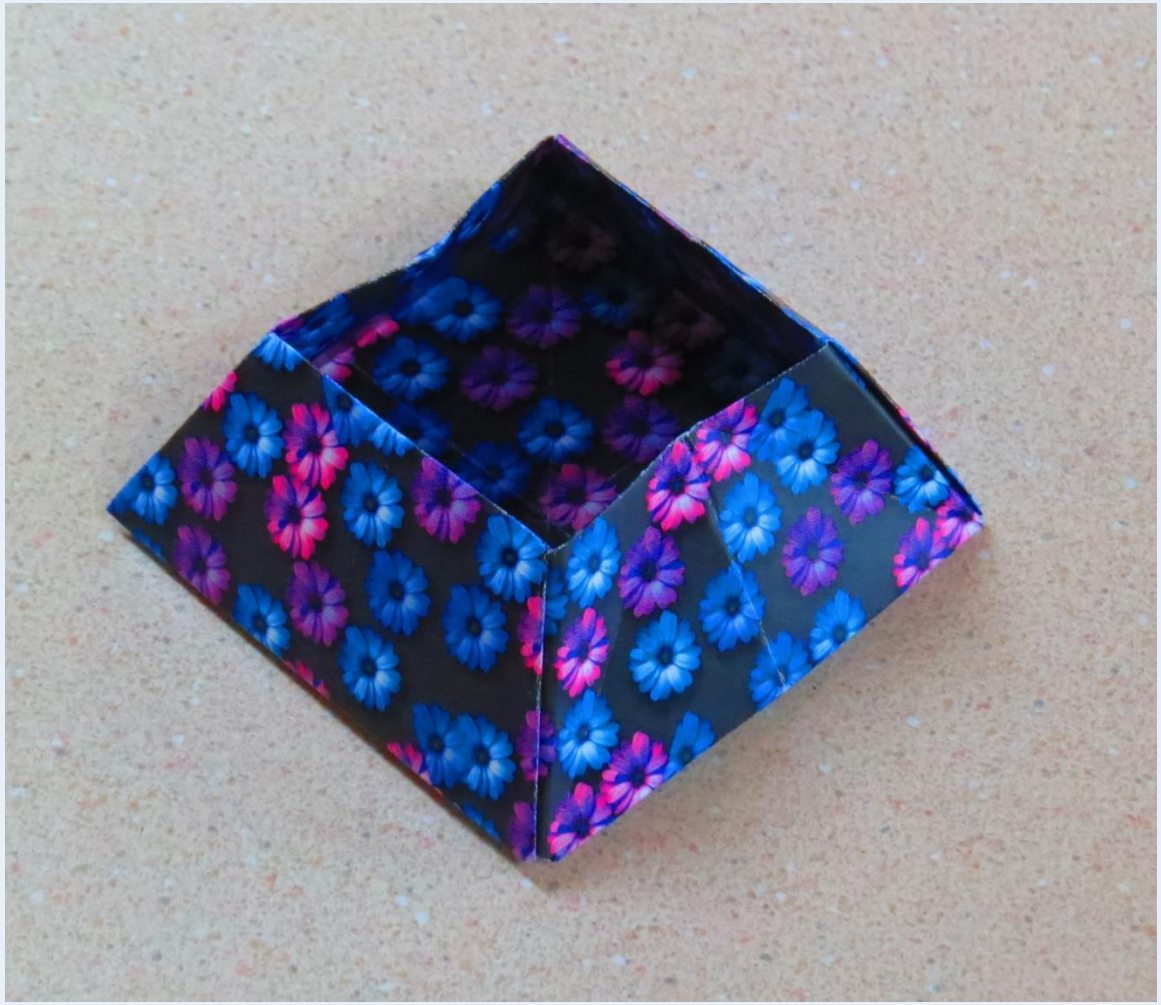




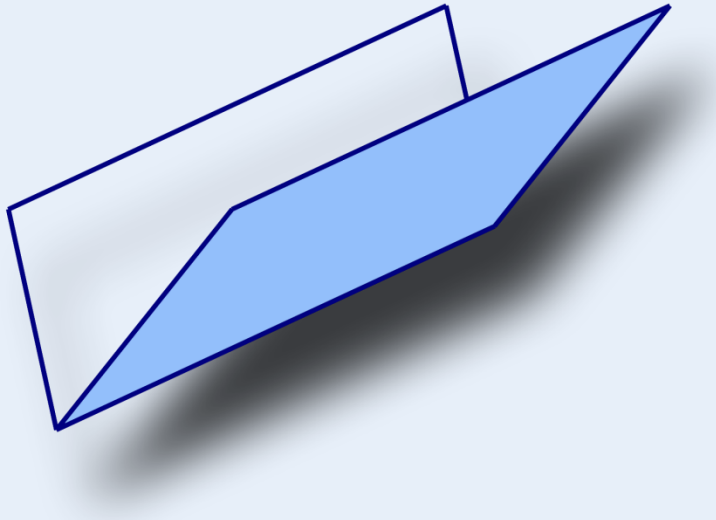




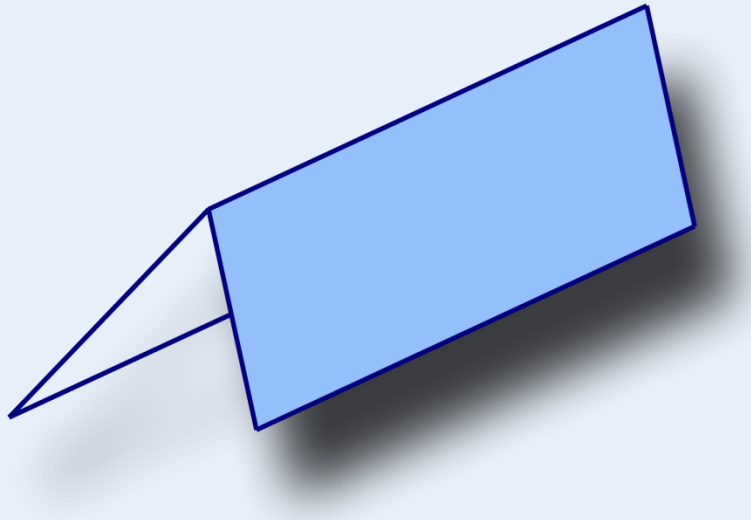


Let us make a box.

When you are folding make sure the creases are **ACCURATE** and **SHARP**.



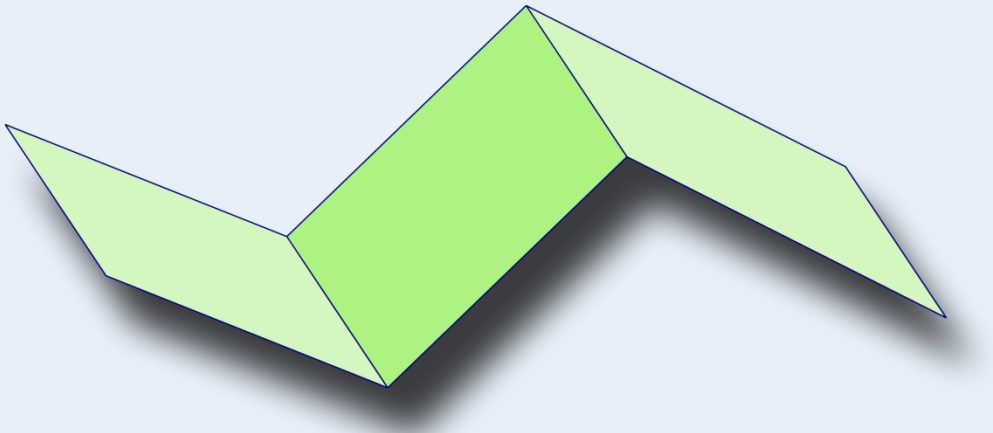
Valley Crease



Mountain Crease

LOOKING FROM ABOVE

CONVEX
CREASE



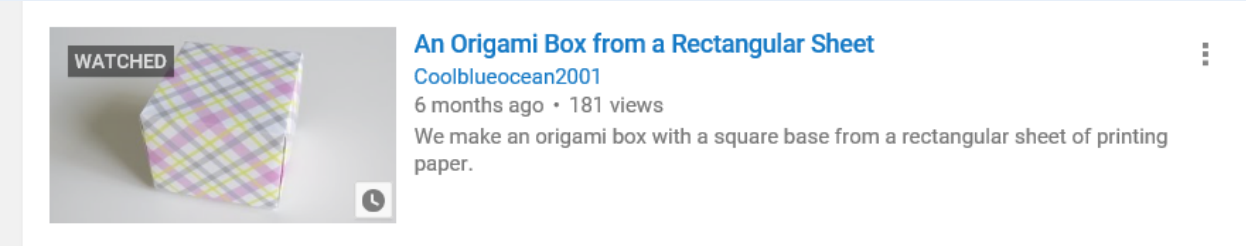
CONCAVE
CREASE

Here is the link to the video that we will be using to fold the box:

https://www.youtube.com/watch?v=vjCzf0hese0

Keyword search on youtube

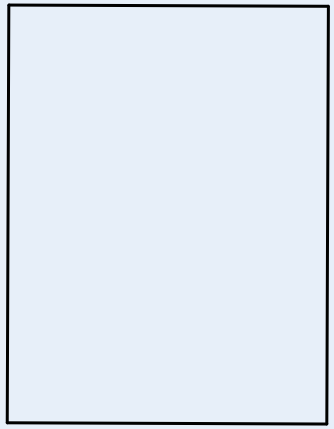
“An Origami Box from a Rectangular Sheet coolblueocean2001”



This is the box we will be making.

Let’s make the box.

Purple represents the fancy side, and white represents the plain side.



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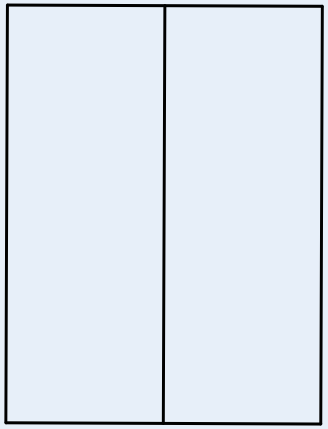

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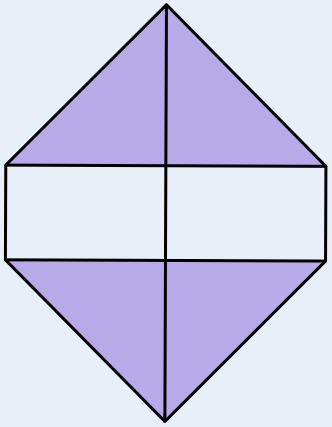

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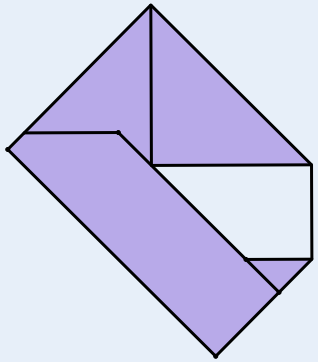

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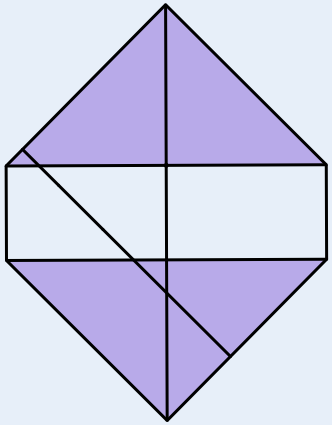

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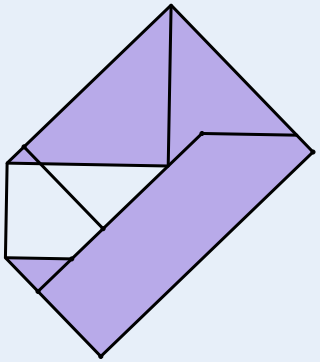

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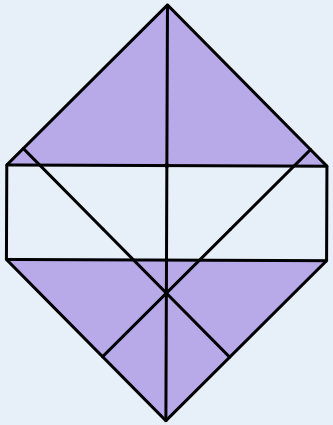

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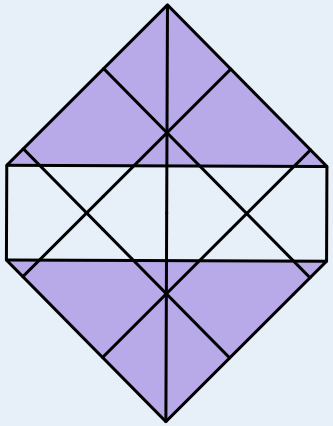

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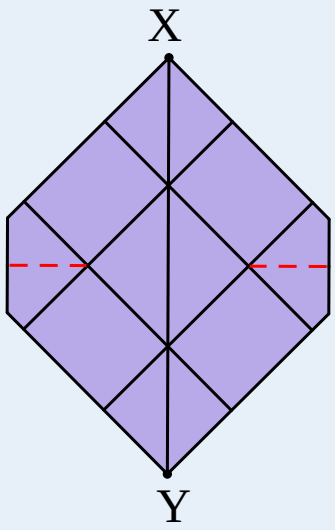

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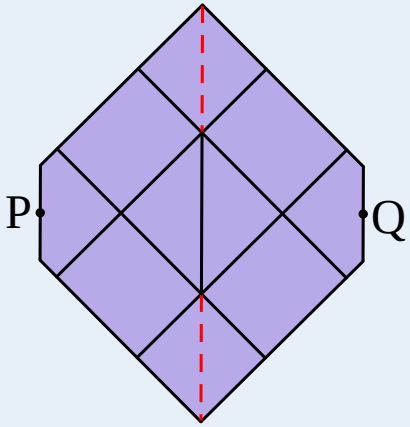

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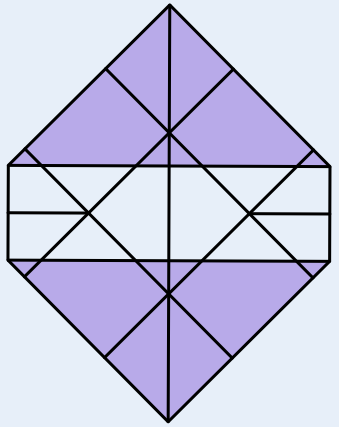

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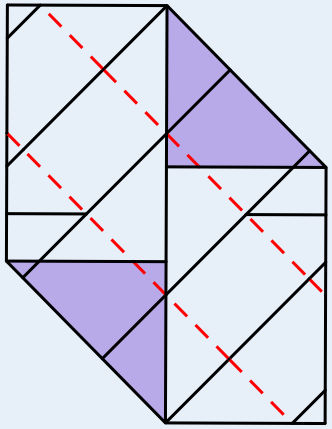

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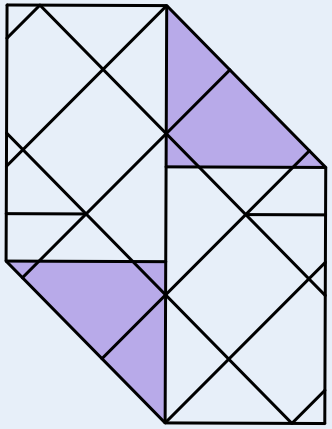

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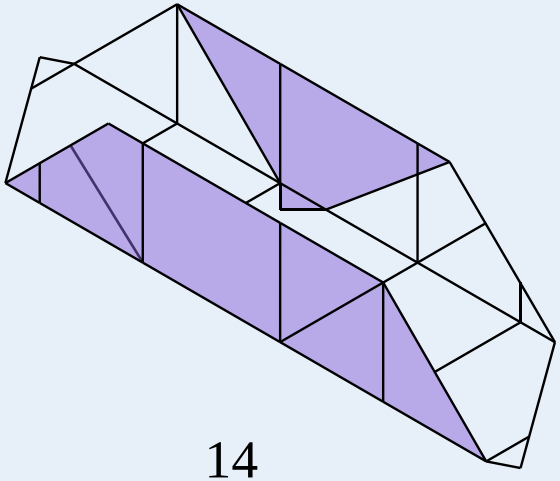

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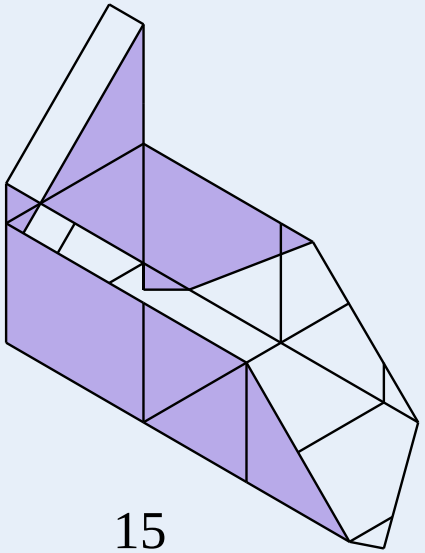

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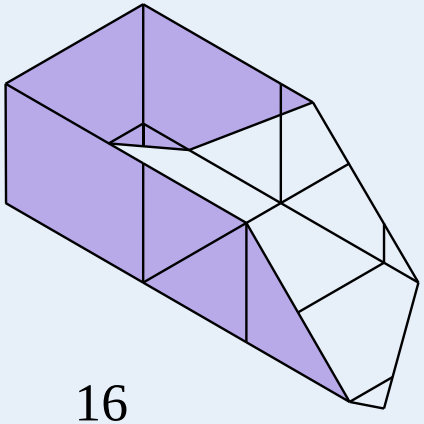

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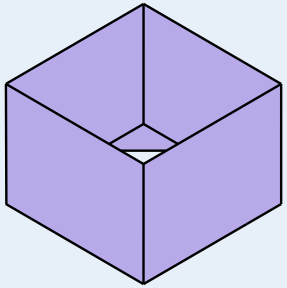

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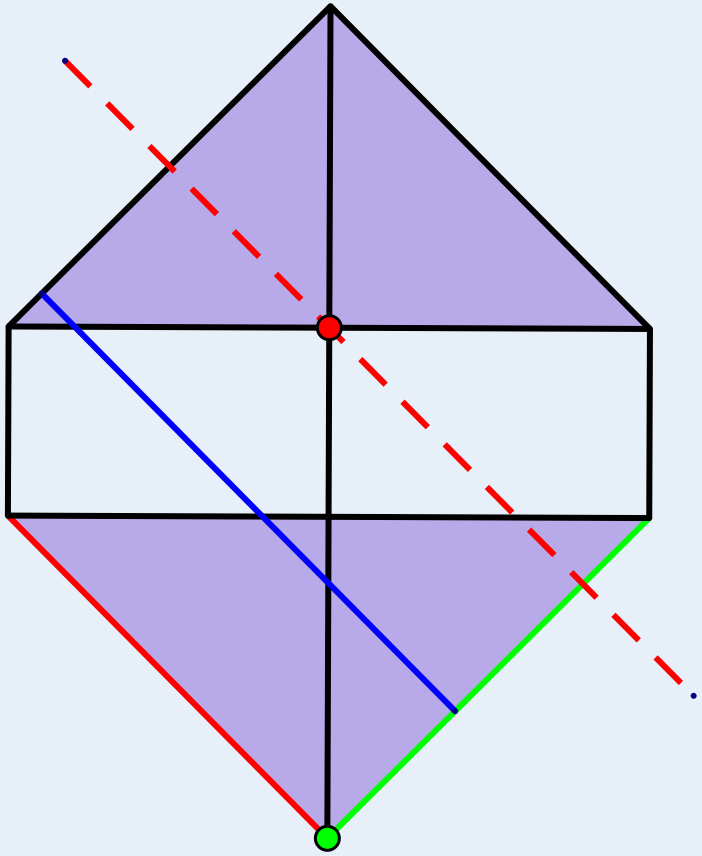
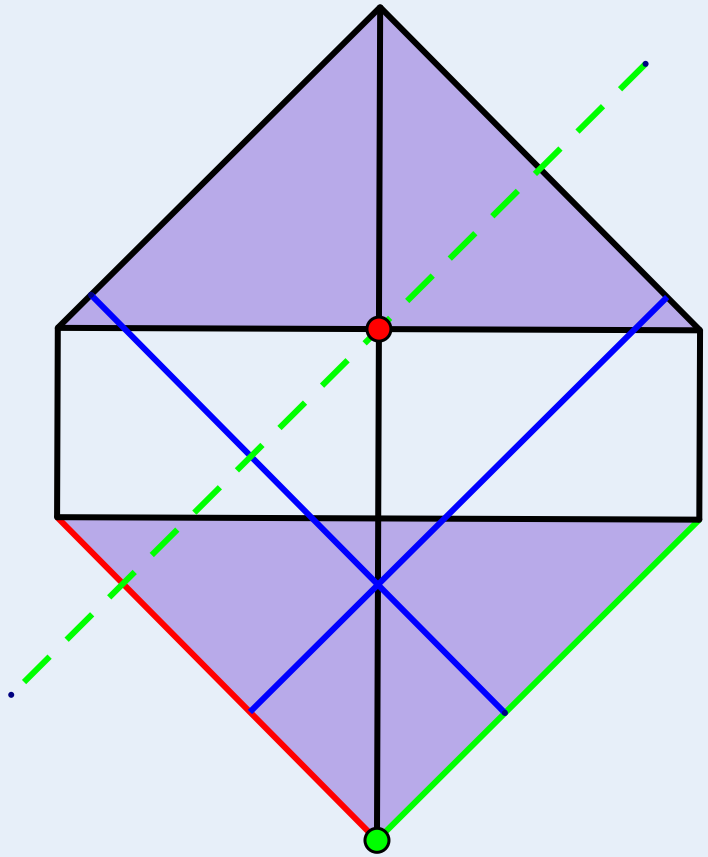
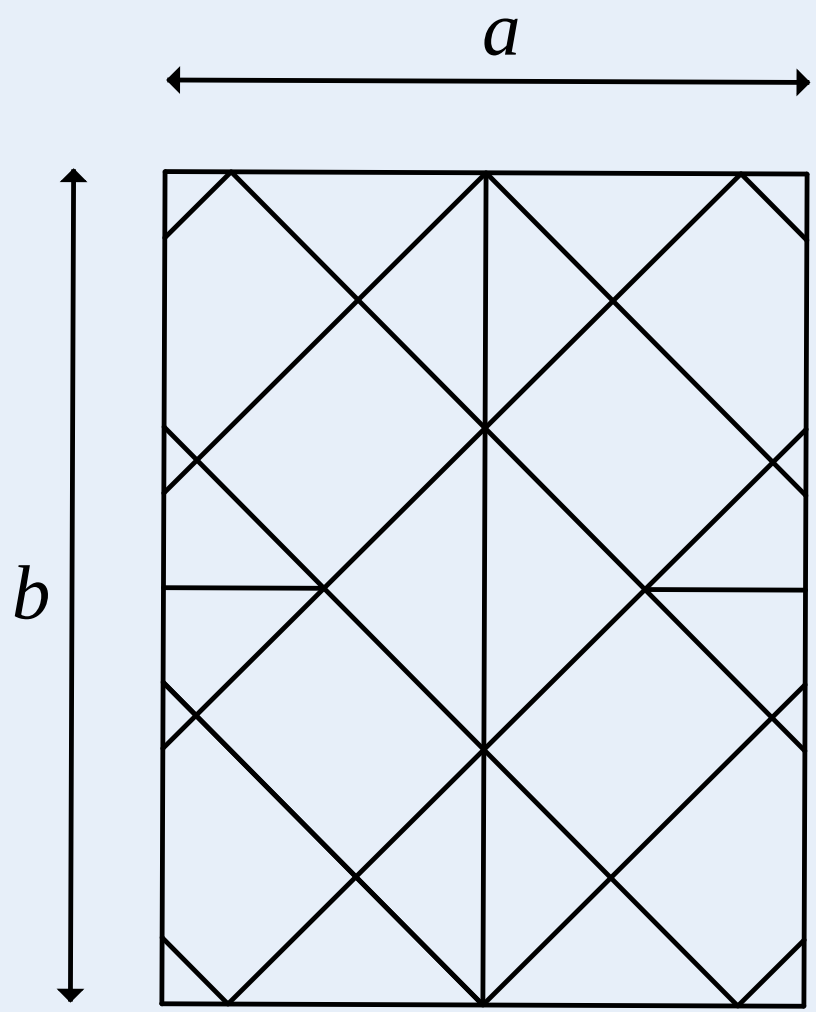
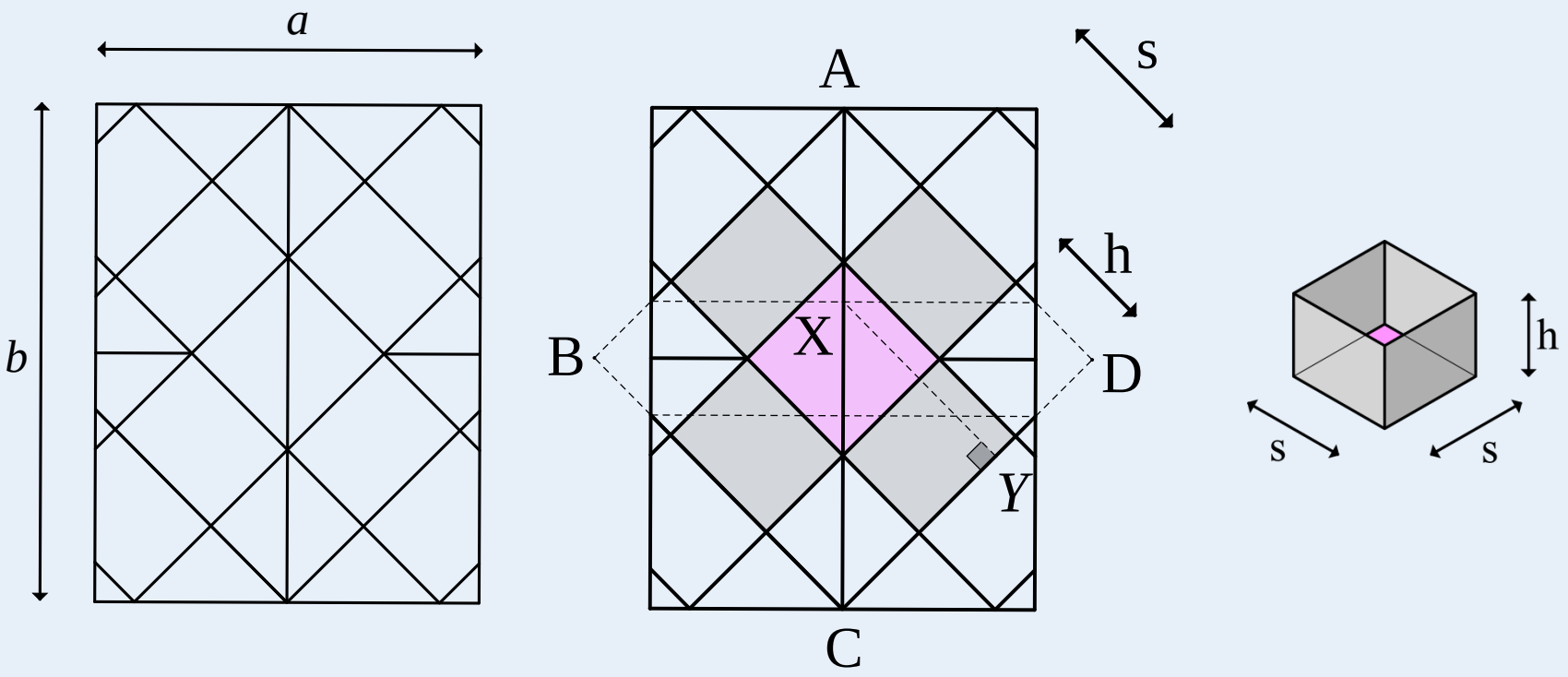





a
b


a
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A
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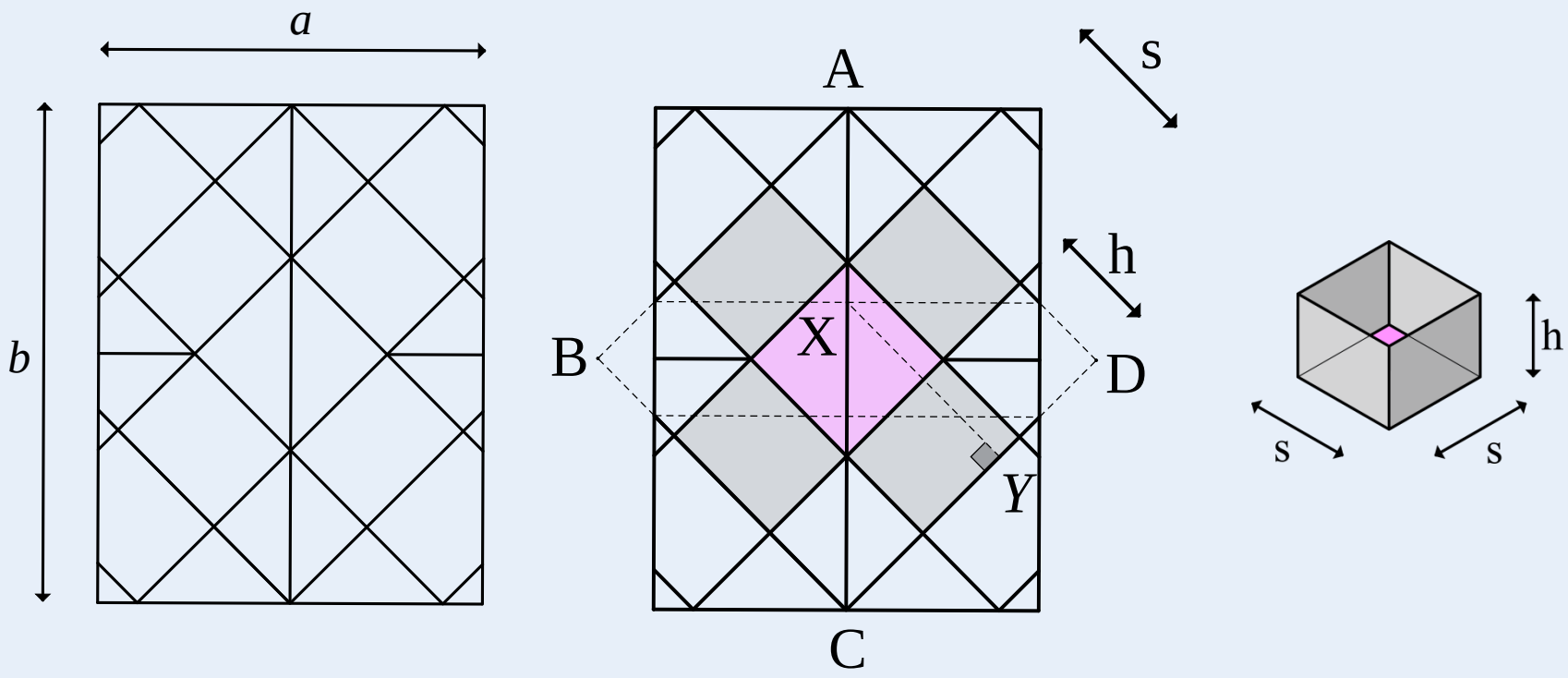
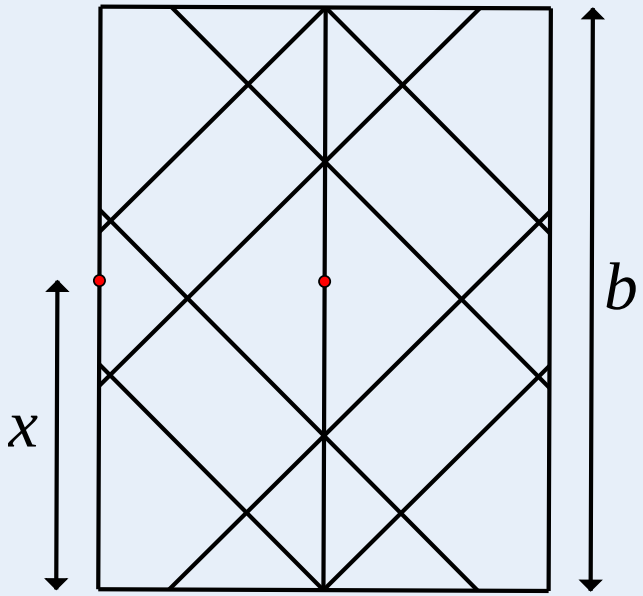
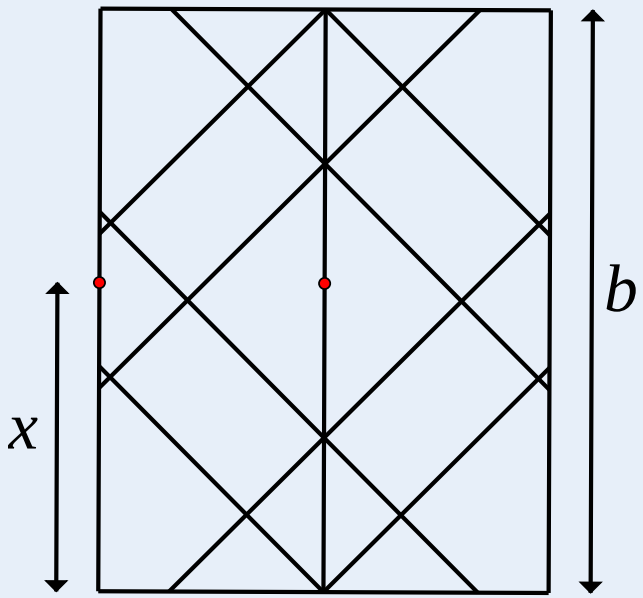


$$h = \frac{b}{2\sqrt{2}} - \frac{a}{4\sqrt{2}}, \text{ and } s = \frac{a}{2\sqrt{2}}.$$



Height of the box: $\frac{x}{2\sqrt{2}}$.

Length of the base of the box: $\frac{b}{\sqrt{2}} - \frac{x}{\sqrt{2}} = \frac{b-x}{\sqrt{2}}$



The volume of the box =

$$V(x)=\frac{x}{2\sqrt{2}}\left(\frac{b-x}{\sqrt{2}}\right)^2=\frac{x(b^2-2bx+x^2)}{4\sqrt{2}}=\frac{b^2x-2bx^2+x^3}{4\sqrt{2}}$$

$$V'(x) = \frac{b^2 - 4bx + 3x^2}{4\sqrt{2}}$$

$$V'(x) = 0$$

$$\Rightarrow x = b, \text{ or } x = b/3$$

$$V''(x) = \frac{4b + 6x}{4\sqrt{2}}$$

$$V''\left(\frac{b}{3}\right) = \frac{4b + 2b}{4\sqrt{2}} = \frac{6b}{4\sqrt{2}} > 0.$$

Therefore, when $x = b/3$, the box has a maximum volume.