Increasing Student Engagement in Calculus

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Calculus I at CSU

- 350-450 students/ semester
- Average Section Size: 30-40 students
- Also 1-2 larger sections
- Taught primarily by [new] GTAs
- Meets 4 days a week

Majors:

- Mathematics
- Engineering
- Physics
- Computer Science
- Chemistry
- Statistics

Background & Motivation

High DFW Rates

↓

Administration: Increase STEM major retention rates

↓

Educators: Improve student learning

↓

MAA National Calculus Study: 7 characteristics of successful calculus programs[1]

Pilot Study – Spring 2015

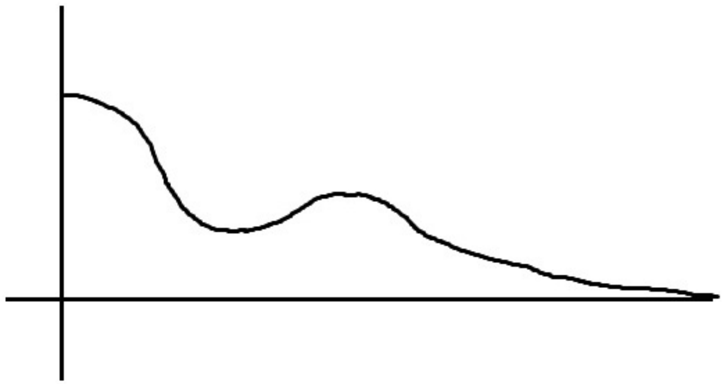
Experimental Section (N=36):

- Minimal lecture
- Carefully constructed group activities
- Strategically formulated groups
- Writing & Discussion
- Oral assessments

Control Section (N=33):

- Primarily lecture
- Some group work and writing

Writing



- Where does the object have the largest position? On the graph, mark that point with a capital P.
- Where does the object have the largest velocity? On the graph, mark that point with a capital V.
- Write a story to match the graph.
- Walk the graph!

Writing Task Discussion

- *How could this activity be modified?*
- *Are there multiple entry points for such a task?*
- *What types of responses do you think you would see from your students?*
- *Would you have your students walk the graph before writing a story or after?*

Student Responses

- Graph = Position
  - A ball is rolling down a hill, then it goes up a smaller hill then it rolls down some more.
  - A skydiver jumps out of a plane and opens the parachute.
  - I drive home but hit road construction so have to turn back and take an alternate route then can continue home.

- Graph = Velocity
  - A ball is rolling down a hill, then it goes up a smaller hill then it rolls down some more.
  - I was running away from zombies, and I was getting tired so I was running slower and slower. Then I noticed that the zombies were getting closer, so I started running faster. Then a zombie caught me and bit my leg off, and I fell down and crawled until I could crawl no longer.

Principles to Action

1. Establish Mathematical Goals to Focus Learning
2. Implement Tasks That Promote Reasoning and Problem Solving
3. Use and Connect Mathematical Representations
4. Facilitate Meaningful Mathematical Discourse
5. Pose Purposeful Questions
6. Build Procedural Fluency from Conceptual Understanding
7. Support Productive Struggle in Learning Mathematics
8. Elicit and Use Evidence of Student Thinking

Continuity Activity

You have a handout with 6 different graphs.

Follow the directions as a new calculus student would.

The Definition of Continuity

- How do you define *continuity* in your class?
- What challenges do your students have with understanding continuity?
- What are the common misconceptions?

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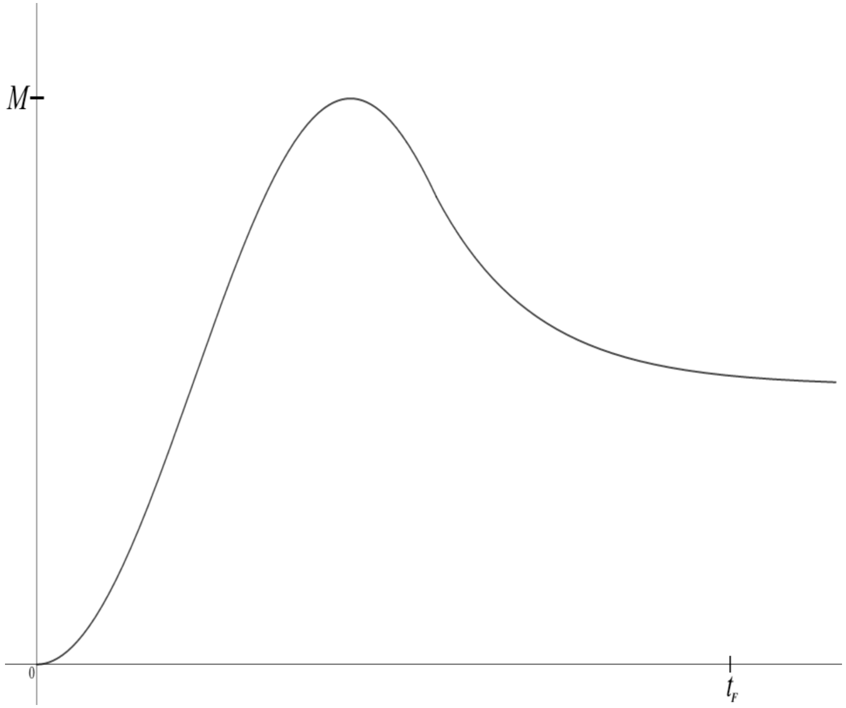
Oral Assessments^2

Below is the graph of a function that changes with respect to time (could be position, velocity, or acceleration). Which of the following statements are accurately modeled by the graph?



- Olga climbs to the top of a mountain, but then quickly descends halfway down.
- Tatiana is jumping on a trampoline until her foot slips and she falls to the ground.
- Alexie accelerates from a stop sign before reaching a school zone and then slows down to a legal speed.
- Maria's plane accelerates for take-off, reaches cruising altitude, slows down to a constant speed, and stays there for the rest of the flight.
- Anastasia maintained a constant pace while running a marathon. She was proud of her final time being 3 hours.

How did oral assessments go?

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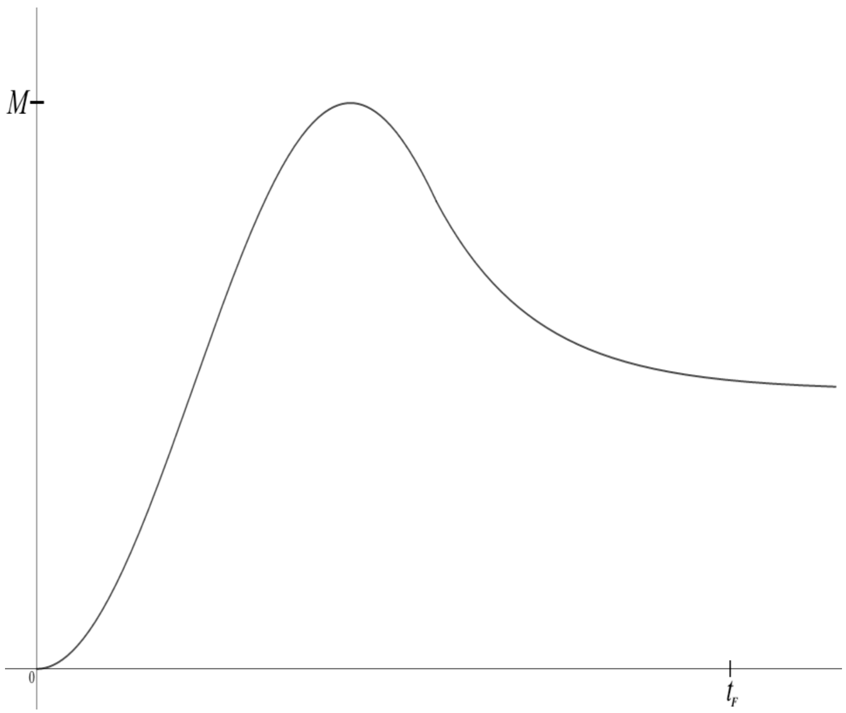
Oral Assessments[2]



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Audio Clip Highlights

Does the graph represent position?

- "It's not position because she doesn't hit zero when she falls down."

Could it be velocity?

- "No. Just because she breaks her foot and immediately stops."
- "It'd be oscillating."

For velocity, should the graph drop below the x-axis?

- "Yeah, it should come back down the same way."
- "Well, you can never get negative velocity."
- "Yeah you can. When you start to fall down."
- "Ohhhhh. Yeah."

Promoting Dialogue

The Extreme Value Theorem: If *f* is continuous on a closed interval [a,b], then *f* attains both an absolute maximum value and an absolute minimum value on [a,b].

True or False? If False, give a counterexample.

If *f(x)* is defined on [−3,5], then *f(x)* will have both an absolute maximum value and absolute minimum value on [−3,5].

If *f(x)* = 24, then *f(x)* will have both an absolute maximum value and absolute minimum value on [4,10].

If *f(x)* is continuous on (−3,5), then *f(x)* will have both an absolute maximum value and absolute minimum on (-3,5).

Promoting Dialogue

Definition: A point of the domain of a function f where f' is zero or undefined is called a critical point of f.

Now you generate some True/False questions!

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Reflective Writing

Daily Reflection (to be filled out before class ends)

What was the muddiest part of today's class and how do you feel about the content?

Stuck in the mud ———————————————— Squeaky clean

What ideas from today were clear to you? What sections were unclear to you?

In 2-3 sentences, write about what infinity means to you.

What excites you most about this course? What scares or concerns you the most?

Tell me something else you think I should know.

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Stuck in the mud ———————————————— Squeaky clean

What ideas from today were clear to you? What sections were unclear to you?

The next test is a week from today. Do you know what topics/sections will be on the exam? If so, list them.

Have you begun studying for the exam? YES / NO

What exam topics do you understand well? What exam topics do you need more practice on?

Discussion

- What is beneficial about having students write reflectively?

- What questions would you ask?

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Secondary Level

- Which activities or tasks would you implement? What would you modify? How would you implement?

Thank you!

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References

1. Bressoud, D., & Rasmussen, C. (2015). Seven Characteristics of Successful Calculus Programs. Notices of the AMS, 62(2).

2. Nelson, M. A. (2010). Oral assessments: Improving retention, grades, and understanding. PRIMUS, 21(1), 47-61.

Quantitative results

	Experimental Section N=36	Control Section N=33
Exam 1	82.8*	76.1
Exam 2	67.9	63.3
Exam 3	64.2	60.8
Final	71.6*	62.1

Exam 2 Snapshot

	Experimental Section N=36	Control Section N=33
Conceptual Writing Question	4.44* (p=0.002)	2.82
Procedural Question	13	11.9

Qualitative Analysis

- **<u>Coding</u>**:
  - Correct vs. Incorrect: Ideas, Vocabulary, Notation
  - Use of pronouns

	Correct	Incorrect
Ideas	"The equation [for] f'(a) is the slope of the tangent line at a given point."	"The equation is for tangent lines and secant lines at any given point on the graph."
Vocabulary	"The equation describes the slope of the tangent line at point a."	"A secant line is the slope between a and b."
Notation	(a,f(a)) *(in reference to an ordered pair)*	f(a) *(in reference to an ordered pair)*

Qualitative Analysis

- Experimental section had a **higher** percentage of
  - Correct ideas
  - Use of correct vocabulary
  - Use of correct notation
- Experimental section had a **lower** percentage of
  - Incorrect ideas
  - Use of incorrect vocabulary
- Use of incorrect notation: the same
- Experimental section had a **lower** percentage of use of pronouns