FROGS, FRENCH FRIES, AND FAUCETS

EXAMINING PROPORTIONS THROUGH MULTIPLE LENSES

NCTM Annual Meeting
April 15, 2016
San Francisco, CA
Dr. Valerie V. Sharon and Dr. Mary B. Swarthout

SH | **Sam Houston** STATE UNIVERSITY

WHY ARE WE INTERESTED?

Increased emphasis on ratio and proportional relationships for students

CCSS for Grade 6 [6.RP],

Grade 7 [7.RP], and

Grade 8 [8.EE, 5 and 6]

CCSS – Mathematical Practices

NCTM –Process Standards

Doing mathematics involves problem solving, using representations, reasoning about quantities, choosing tools, modeling, noticing relationships, and communicating your thinking.

Principles to Actions
ENSURING MATHEMATICAL SUCCESS FOR ALL

NCTM
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Mathematics Teaching Practice: Use and connect mathematical representations

We will illustrate how the use of *multiple representations* can deepen students *conceptual understanding of proportional relationships*.

BELIEFS ABOUT TEACHING AND LEARNING MATHEMATICS

ROLE OF TEACHER

"**Engage students in tasks that promote reasoning and problem solving and facilitate discourse…**" **(p. 11, NCTM, 2014)**

ROLE OF STUDENT

"**be actively involved…making sense of mathematics tasks by using varied strategies and representations, justify solutions, making connection to prior knowledge … considering the reasoning of others**" **(p. 11, NCTM, 2014)**

5 CATEGORIES FOR INFORMAL ACTIVITIES

- Identifying Multiplicative Situations
- Equivalent–Ratio Selection
- Comparing Ratios
- Scaling with Ratio Tables
- Construction and Measurement Activities

Van de Walle & Lovin, 2006, *Teaching Student-Centered Mathematics Grades 5-8,* page 157 – 168

Essential Understanding #1

How does ratio reasoning differ from other types of reasoning?

Reasoning with ratios involves attending to and coordinating **two** quantities.
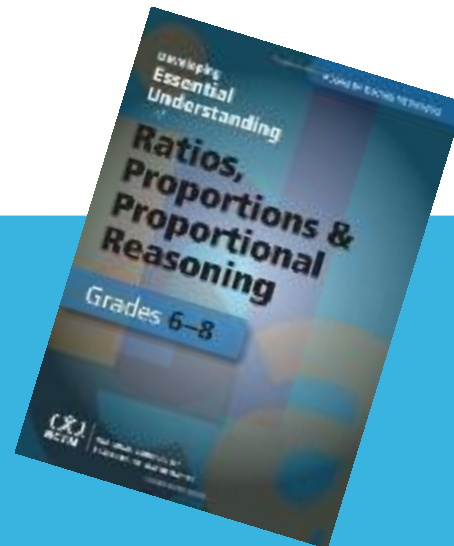
Mr. Tall and Mr. Short – Assessing Basic Understanding

In the diagram, you can see the height of Mr. Short measured with paperclips. Mr. Short has a friend Mr. Tall. When we measured their heights with buttons, Mr. Short's height is 4 buttons and Mr. Tall's height is 6 buttons.
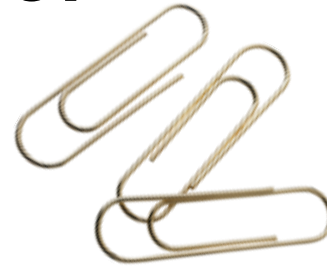
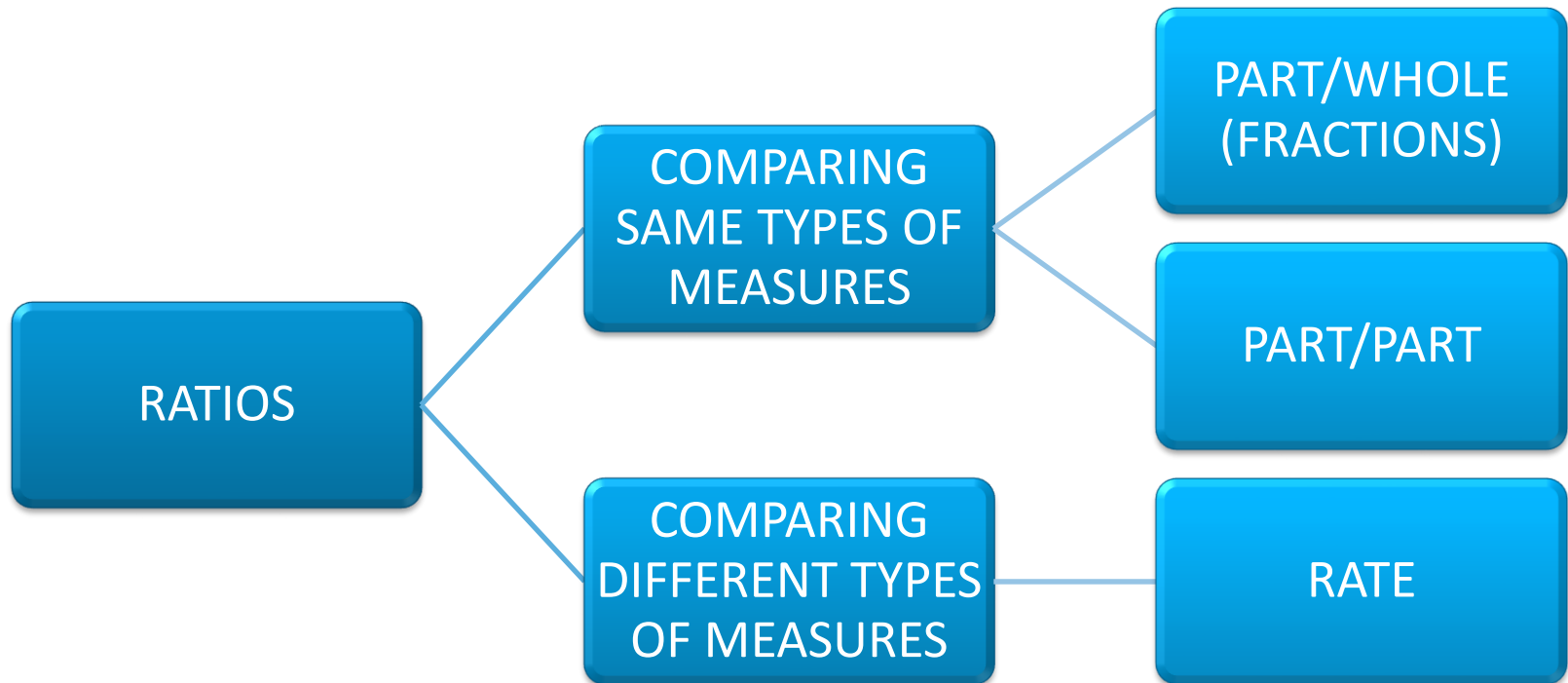6 paper clips

Mr. Short
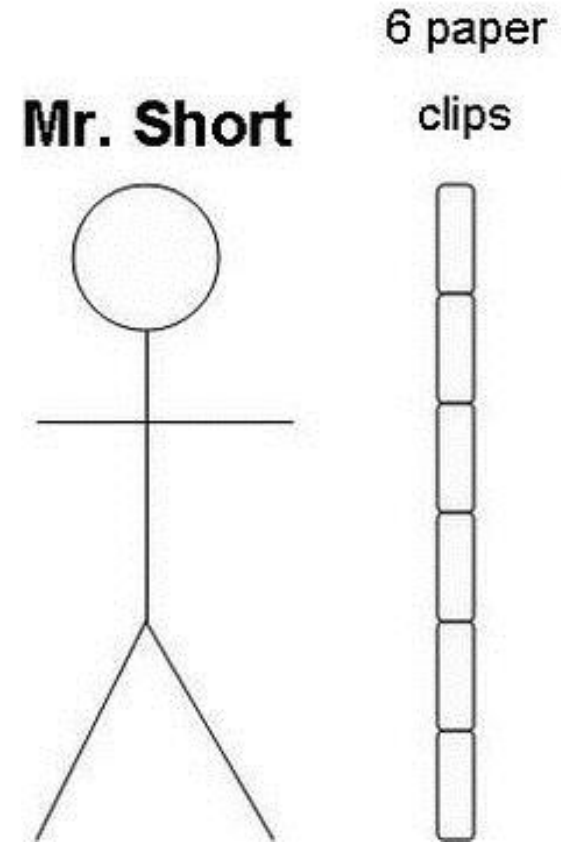
How many paperclips in height is Mr. Tall?

What do you think students gave as their response?

8 paperclips

Why are students giving this answer? What is the misunderstanding?

RATIOS

COMPARING SAME TYPES OF MEASURES
- PART/WHOLE (FRACTIONS)
- PART/PART

COMPARING DIFFERENT TYPES OF MEASURES
- RATE

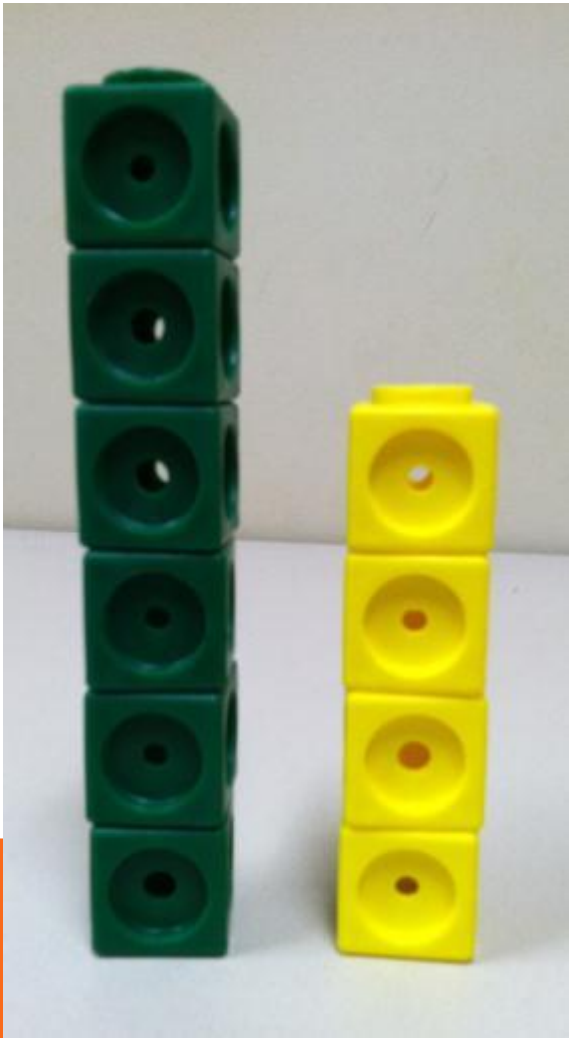Van de Walle & Lovin, 2006, *Teaching Student-Centered Mathematics Grades 5-8,* page 155

Essential Understanding #2
What is a ratio?

A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
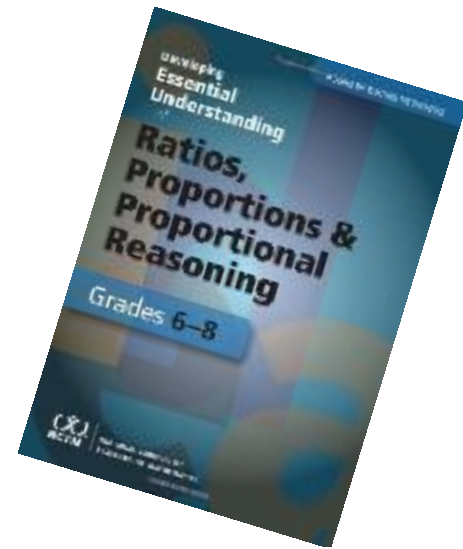
Ratio as a multiplicative comparison of TWO quantities

The GREEN tower is 1 ½ times the height of the YELLOW tower.

Compare the height of the YELLOW tower to that of the GREEN tower.

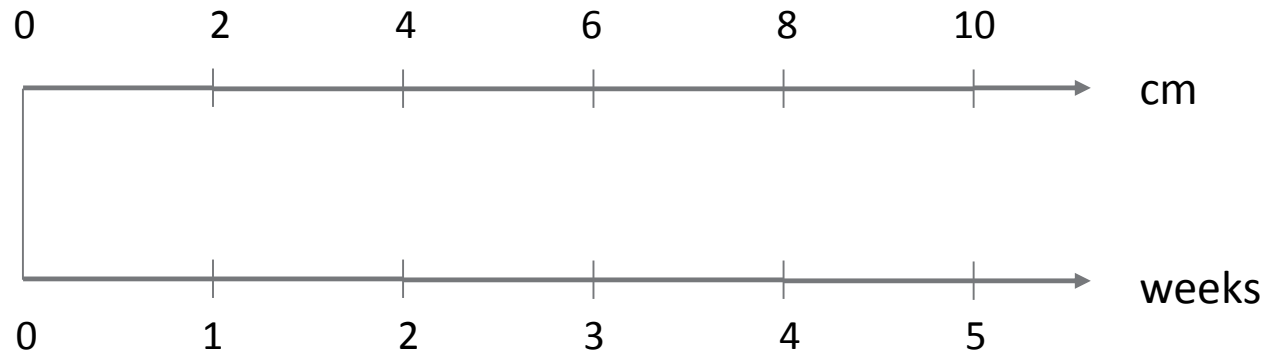Ratio as a **Composed Unit**

➔ <u>*pre-ratio*</u> reasoning

(Lesh, Post and Behr, 1988)

➔ <u>not</u> sophisticated

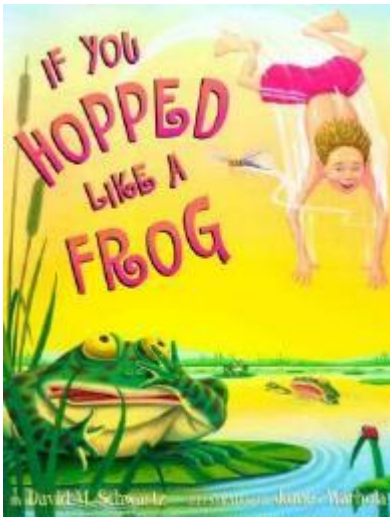**Form a ratio by joining two quantities to create a new unit**

EXAMPLE:

During the growing season, the diameter of
a lily pad increases 2 cm each week.

0	2	4	6	8	10

cm

weeks

0	1	2	3	4	5

ACTIVITY – *ITERATING A COMPOSED UNIT*
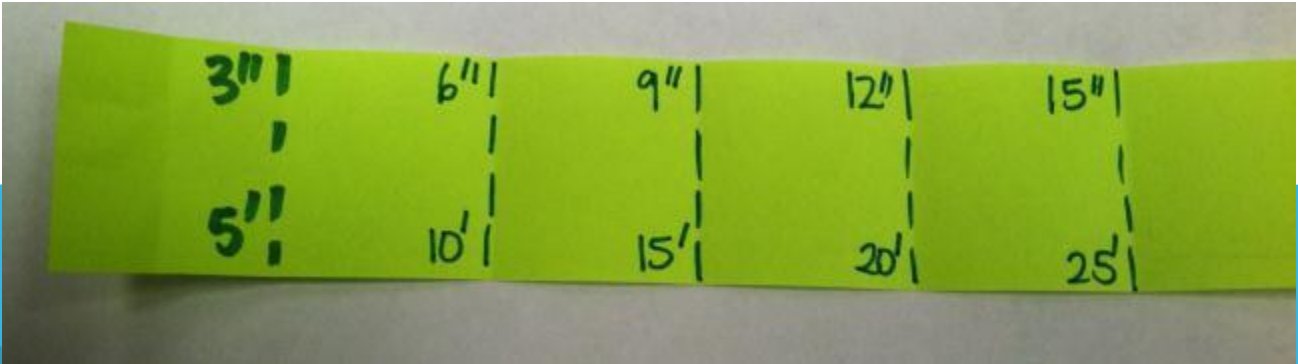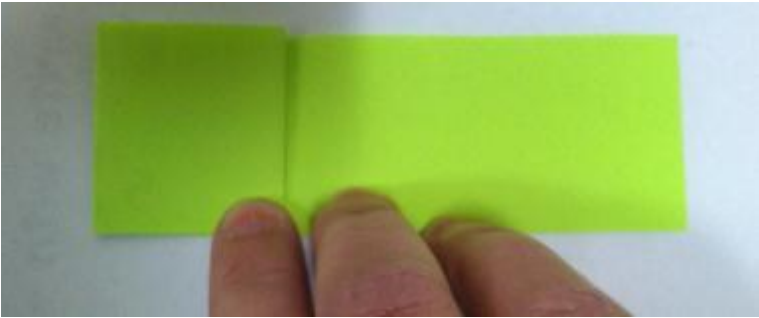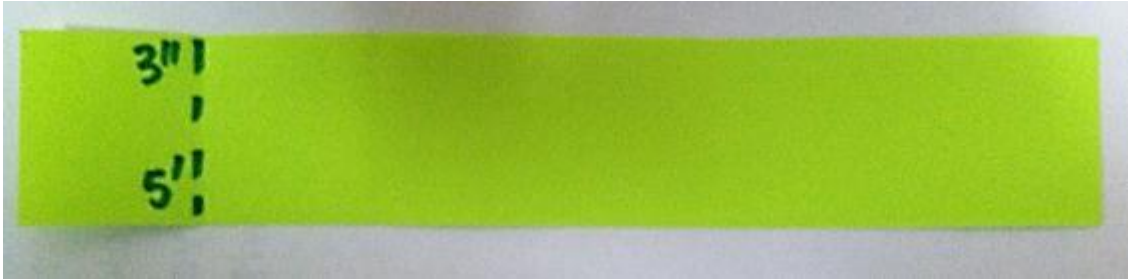
If You Hopped Like a Frog

Problem

A 3" frog can jump 20 times its body length. How far could you jump if you hopped like a frog?

Using paper strips:

About how far could a **newborn baby** leap if she could jump like a frog?
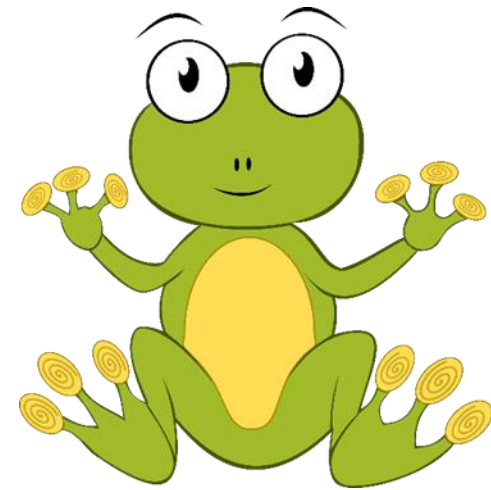
If you were **_15 times longer_** than a frog, then how far could you leap if you could jump like a frog?

Use your strip and multiplicative reasoning to complete the table.

Body Length	1 ½ ”	3”	60”	120”	180’’		300 ”	
Leaping Distance		5’				400’		5280’

Explain what strategies you used to fill in the table.

THE BIG IDEA
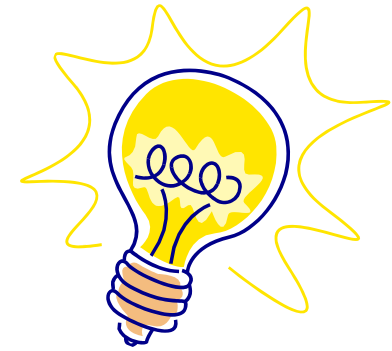
When two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor.

ACTIVITY – *REASONING UP AND DOWN*

PROBLEM

A small order of McDonald's french fries weighs about 75 grams. Complete the ratio table to determine the number of calories in a small order of fries if each fry weighs about 2 grams and contains 6.4 calories.

Talk with a tablemate – What strategies did you use to complete each table? What additional strategies might we expect students to use?

What key ideas about ratio and proportion are promoted by these two activities?

Serving size, g	2							
Total Calories	6.4							

Explain the strategies you used to complete the table.

Essential Understanding #7
What are the key aspects of proportional reasoning?

➔ Equivalent ratios can be created by iterating and/or partitioning a composed unit.

➔ If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

➔ The two types of ratios – composed units and multiplicative comparisons – are related.

Pages 36 - 41

# ACTIVITY – *Making Connections*

PROBLEM

A faucet is dripping in the bathroom. Jason placed a measuring cup under the faucet to capture the water as it leaked. After 8 minutes, he noticed 6 fluid ounces of water had accumulated in the cup. Knowing that the cup was empty when he began his experiment, and that the water was dripping at a constant rate, he created the following graph.

Leaky Faucet
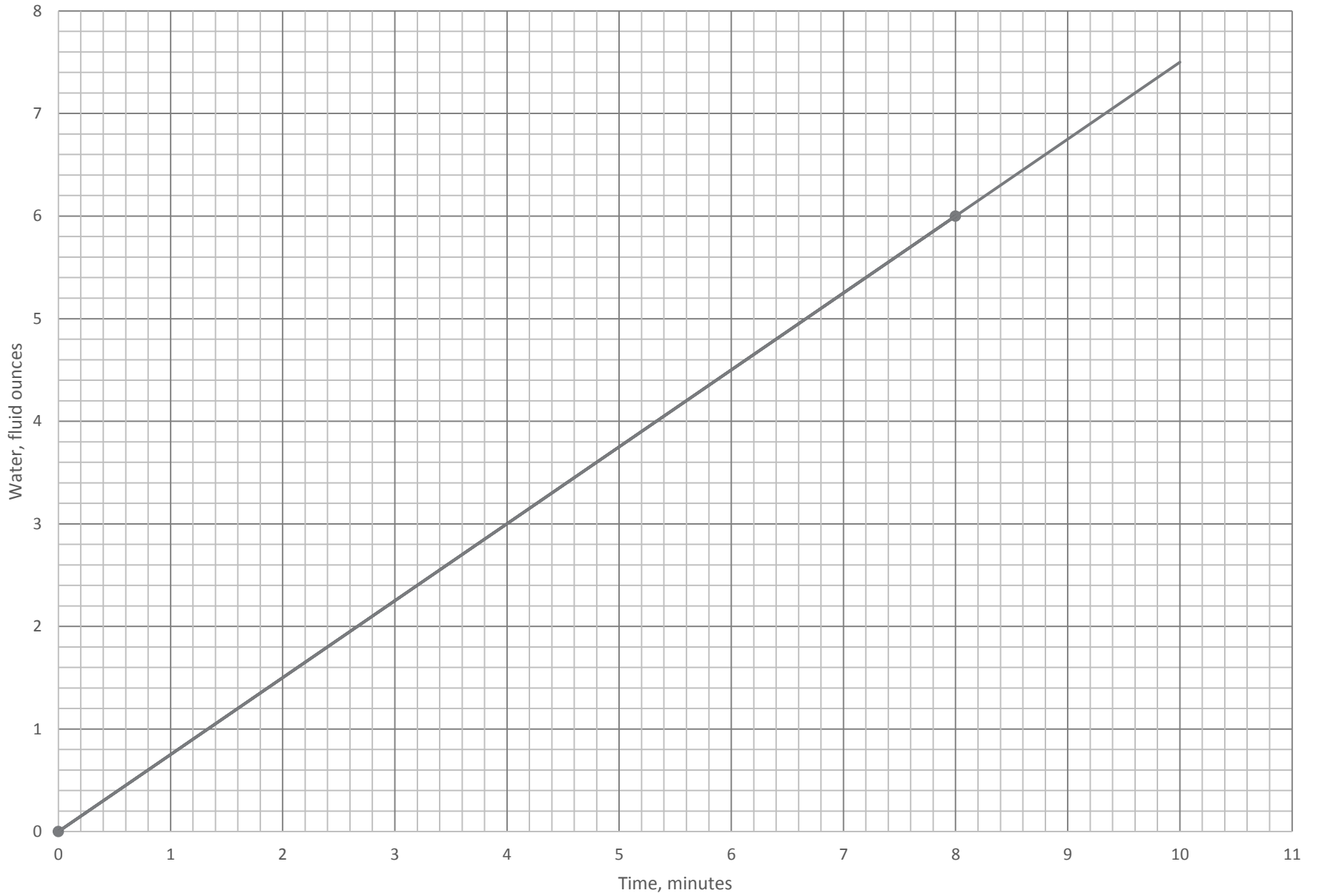
Use the graph to complete the table of values. You may have to use reasoning to fill in some of the values.

Time, minutes	1		2	4	6	8	12			40
Water, fl oz		1				6		12	18	

What rate was the water dripping from the faucet? Explain how that rate is represented in the table.

Let's find the connections!

What rate was the water dripping from the faucet? Explain how that rate is represented in the table.

Interpret the slope in the context of the problem.

A faucet in the science lab was also dripping. Jason placed a measuring cup under the faucet and recorded the total amount of water in the cup over time. Fill in any missing information in his table and then graph the data on the same axis provided above (Leaky Faucet).

Time, minutes	1		3		10	12	15		30	
Water, fl oz		1		5			10	12		40

Which faucet is dripping at a faster rate? Use both tables and graphs to justify your response.

Leaky Faucet: Part 2

EXTENSION: LOOKING AHEAD

Suppose the cup Jason used to collect water from the faucet in the science lab already contained 1 fluid ounce of water.

- How will this change the graph you just drew?

- Will the relationship between the amount of water collected in the cup and elapsed time still be directly proportional?

Looking Back, Looking Forward in Support of Student Learning

Bridge to Procedural Understanding – Cross–Multiplication Algorithm

Bridge to Algebraic Concepts

What bridges do you see?

References

Cohen, J.S. (2013). Strip Diagrams: Illuminating Proportions. *Mathematics Teaching in the Middle School*, 18(9), 536 – 542.

Harris, P. W. (2011). Building Powerful Numeracy for Middle and High School Students. Portsmouth, NH: Heinemann.

NCTM. (2010). *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics in Grades 6 – 8*.

NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA. Author.

Rathouz, M., Cengiz, N., Krebs, A., and Rubenstein, R. N. (2014). Tasks to Develop Language for Ratio Relationships. *Mathematics Teaching in the Middle School*, 20(1), 38 – 44.

Riehl, S.M. and Steinthorsdottir, O. B. (2014). Revisiting Mr. Tall and Mr. Short. *Mathematics Teaching in the Middle School*, 20(4), 220 – 228.

Van de Walle, J.A. and Lovin, L.H. (2006). *Teaching Student-Centered Mathematics: Grades 5 – 8*. Boston, MA: Pearson Education, Inc.
[Chapter 6 – Developing Concepts of Ratio and Proportion]

Link to McDonald's nutrition information:
http://www.mcdonalds.com/us/en/food/food_quality/nutrition_choices.html

Questions?

Contact us at:

Valerie Sharon   vvs001@shsu.edu

Mary Swarthout   swarthout@shsu.edu

Tips for a great conference!

Rate this presentation on the conference app
www.nctm.org/confapp

Download available presentation handouts from the Online Planner!
www.nctm.org/planner