“No, that’s a Rectangle” Activities to Combat Shape Misunderstandings

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Geometry is One of US Students' Weakest Subjects

- On the National Assessment of Educational Progress (NAEP) overall geometry scores tend to be lower than other content domains
  - For the 2013 Implementation Measurement was the lowest
- On the Trends in Mathematics and Science Studies (TIMSS) the geometry strand was the lowest strand.
- Students entering school (age 6) often have the same conceptions of shape in grade 6 (Clements & Samara, 2009)

Most experiences with shapes are rigid - Daren

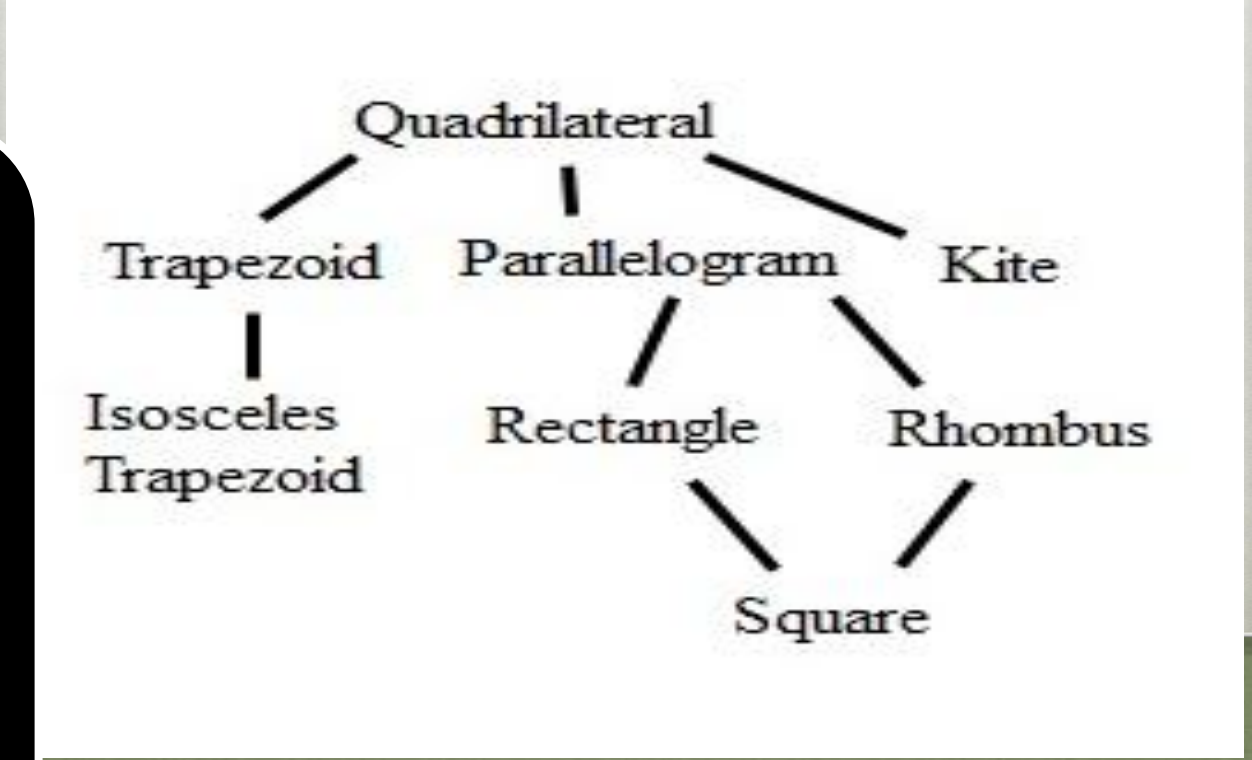
Elementary Common Core State Standards

Kindergarten	• Correctly name shapes regardless of their orientations or overall size. • Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
1st Grade	• Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2nd Grade	• Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
3rd Grade	• Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories

Inclusive vs. Exclusive Definitions

Discuss:

1) What is this image? What does it mean?
2) What do K-2 children know about the math in this image?



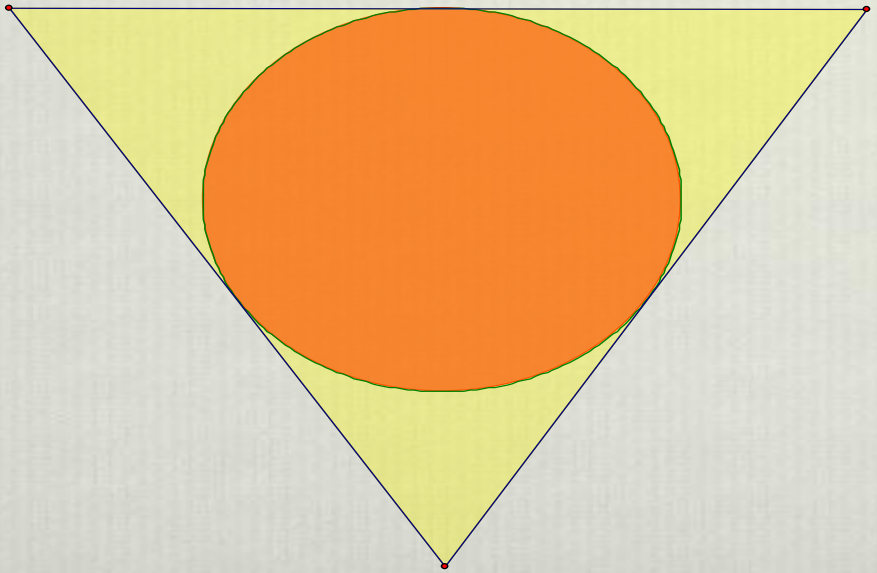
"A Square is a rectangle, but a rectangle is not necessarily a square".

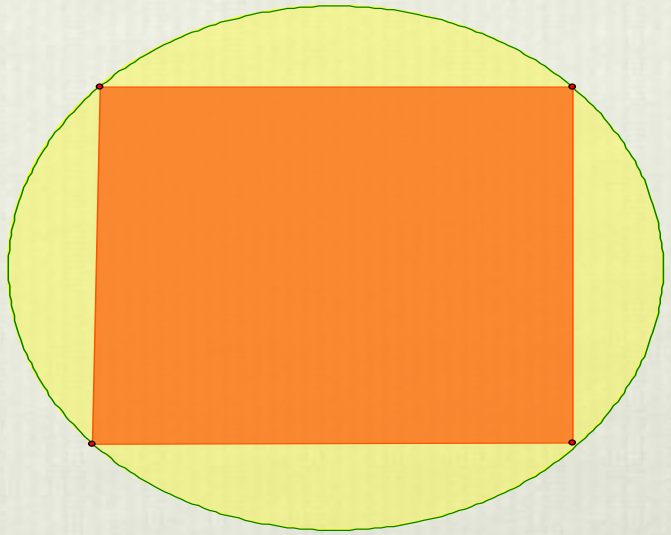
Van Hiele Levels

A Framework for Assessing Geometric Reasoning

Levels

- Visualization (Level 1)
- Analysis (Level 2)
- Informal Deduction (Level 3)

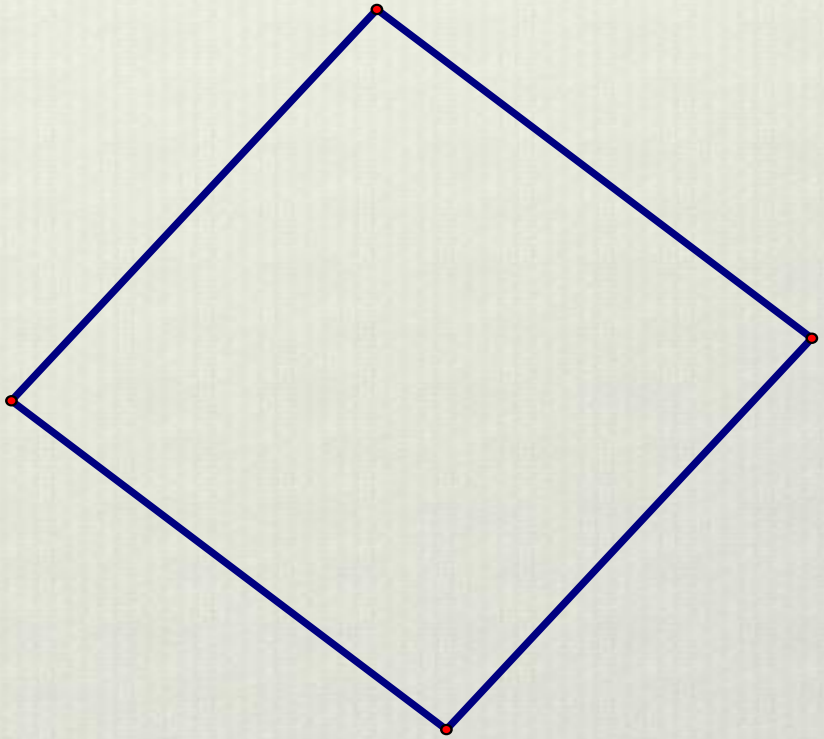




- Levels are sequential.
- Levels are not age dependent.
- The learner must be engaged in appropriate experiences to advance to a higher level.

Visualization

- Students recognize and name shapes by appearance.
- Students are often not able to recognize properties.
- Students may not recognize the shape in a different orientation (e.g., shape at right may not be recognized as square).

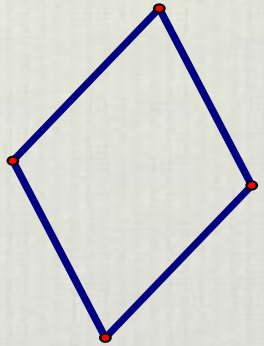


Analysis

- Students can identify some properties of shapes.
- Students at this level may have difficulty explaining the relationship between shape and properties.

Informal Deduction

- Students can see relationships of properties within shapes.
- Can follow informal proofs (e.g., every square is a rhombus because all sides are congruent).
- Not able to construct a formal proof.



K-2 Education Focuses on Supporting students to move from Visualization to Analysis

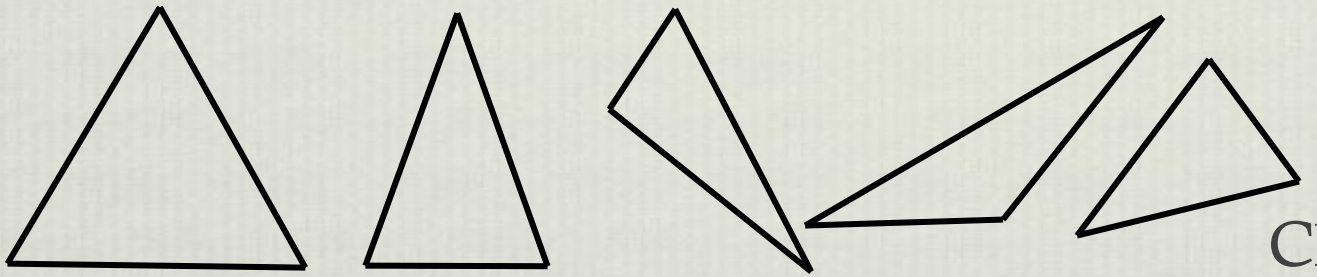
- Challenge students to test ideas about shapes using a variety of examples for a category
  - “Let’s see if that is true for other rectangles”
  - “Can you draw a triangle that does not have a right angle?”
- Provide ample to compose and decompose shapes around characteristics/properties
- Focus on properties of figures rather than identification
- Apply ideas to entire classes:
  - “All triangles have 3 sides”

So how do we design instruction based on students being at different van Hiele levels?

Guiding Features for Effective Instruction

1. Use varied examples and non-examples

- Exemplars – typical forms (equilateral triangle, square, regular hexagon, circle, etc.)
- Variants – all other cases.
- Exemplars may be a necessary 1st step and can help bootstrap learning



Exemplars

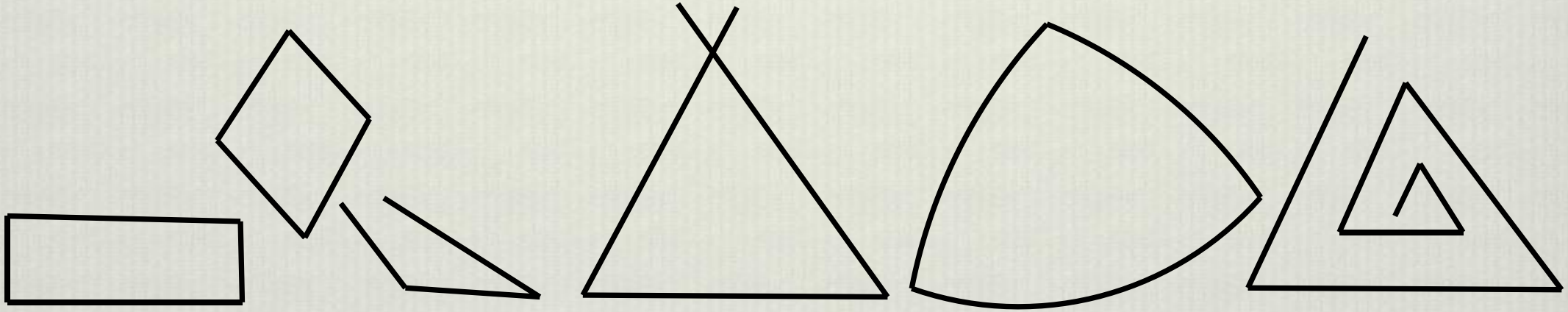
Variants

Clements & Samara (2009)

Guiding Features for Effective Instruction

1. (Continued) Use varied examples and non-examples

- Easy Distractors – little to no overall resemblance to exemplars
- Difficult Distractors – highly visually similar



Easy Distractors

Difficult Distractors

Guiding Features for Effective Instruction

2. Have discussions about shapes and their attributes

- Have students lead
- Expect and accept children's visual reasoning but encourage attribute and property based responses
- Take caution with your language

Clements & Samara (2009)

Guiding Features for Effective Instruction

3. Use a wider variety of shape classes

- Learning trajectories for number are not the same as learning trajectories for shape.
- Children's rigid conceptions of shape can become engrained as early as age 8

Clements & Samara (2009)

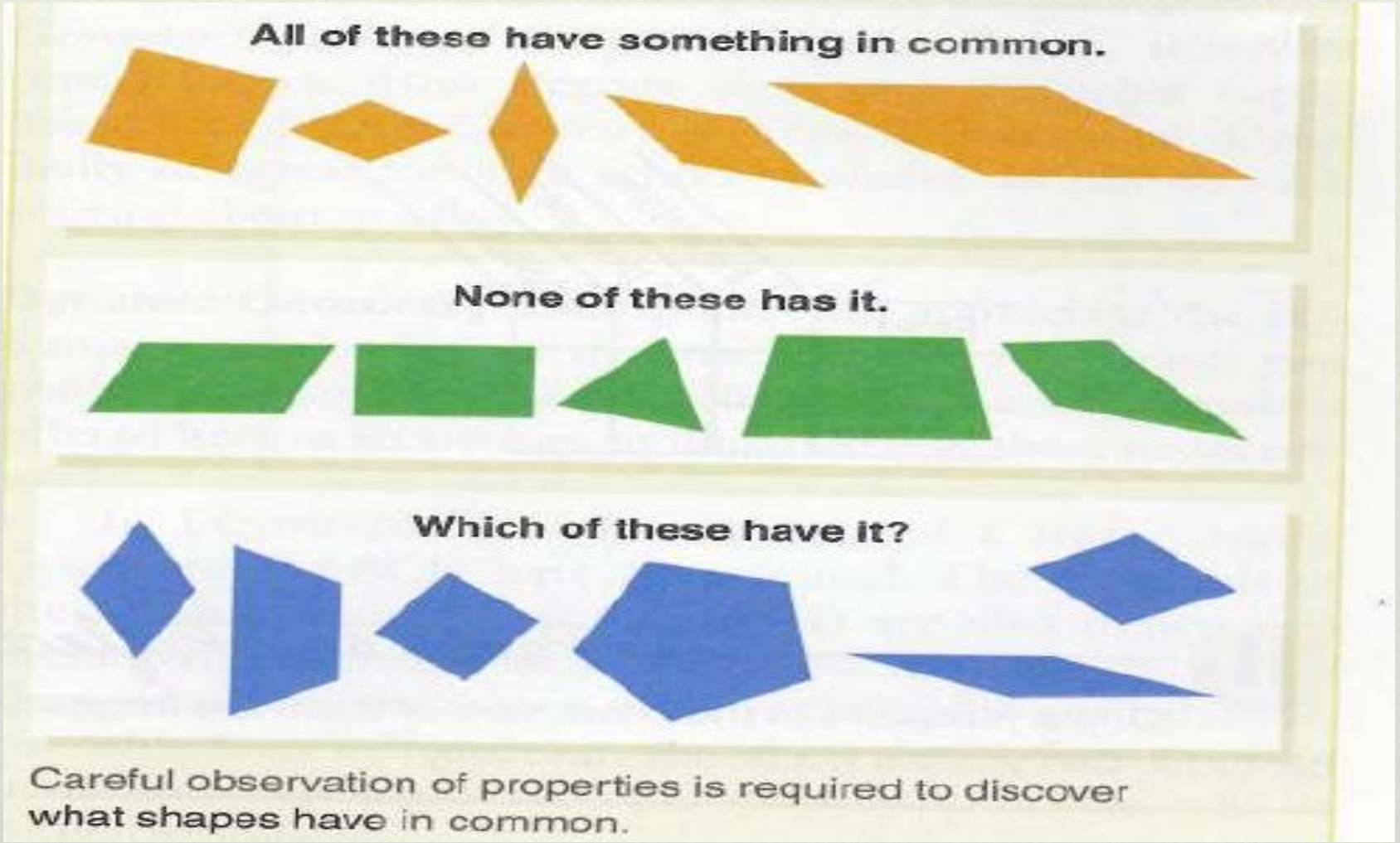
Guiding Features for Effective Instruction

4. Use a broad array of geometric tasks

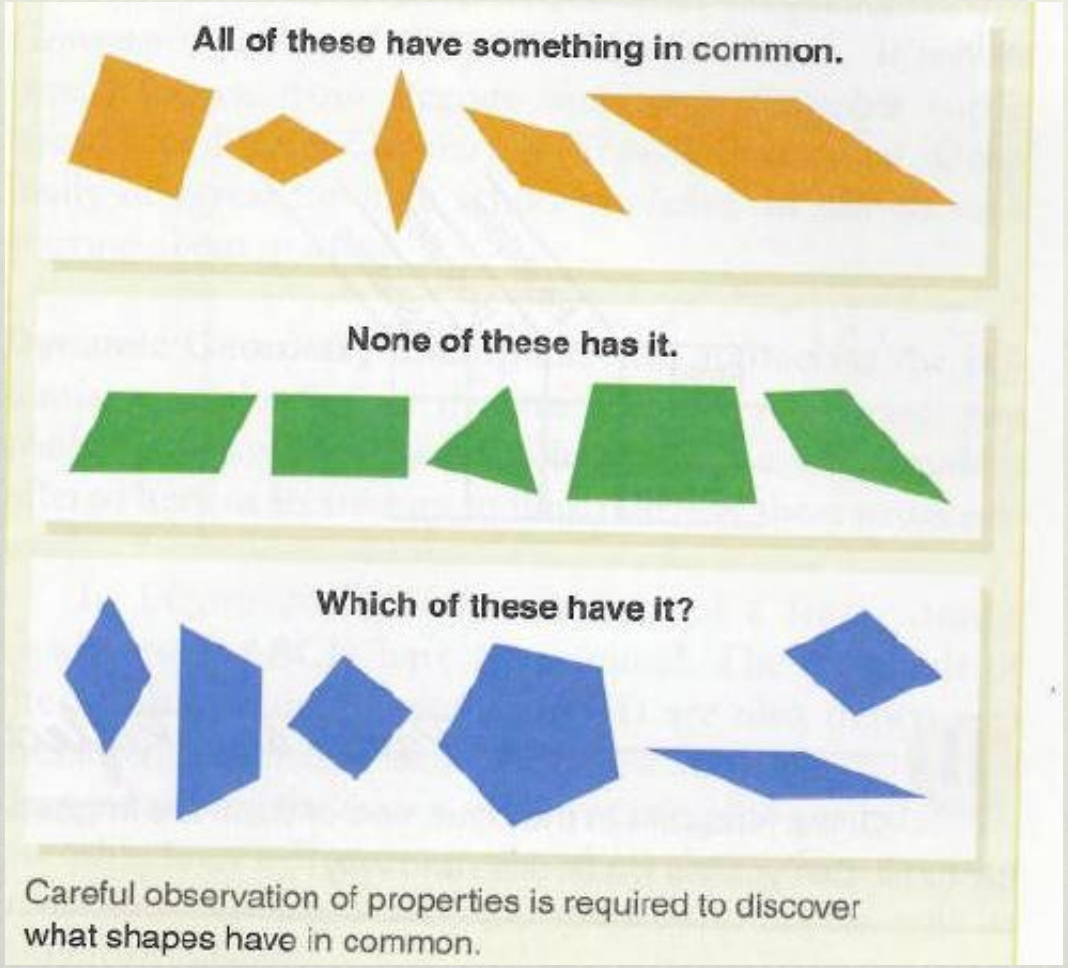
- Do different activities that challenge students in different manners
- Think about your student's knowledge as a puzzle to design instruction towards

Clements & Samara (2009)

Mystery Definition Activity

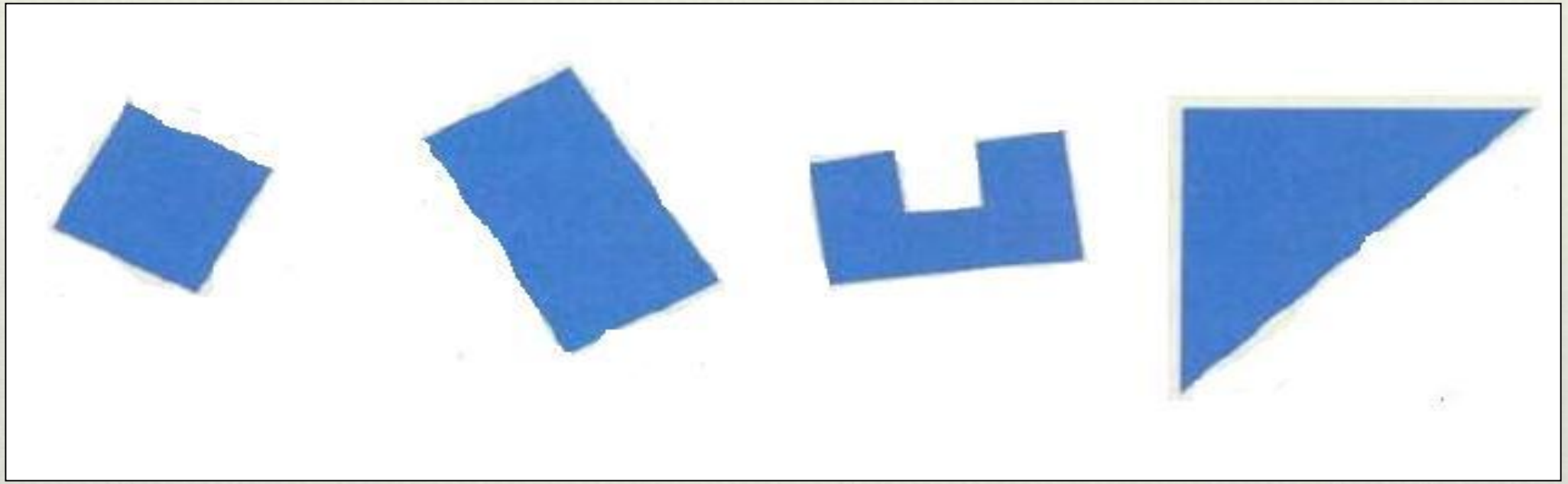


Mystery Definition Activity



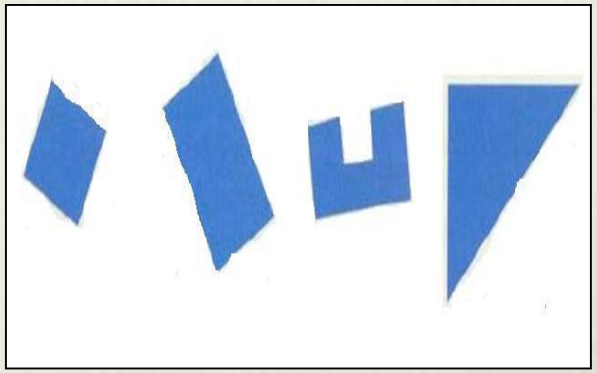
- Teacher Notes:
- Account for variations in shapes meaning use different orientations, extreme examples, and special cases.
- The shapes in the mixture set should include challenging non-examples, meaning examples that are only off by one characteristic.

Secret Sort Activity



- Create a small collection of four or five shapes that fit a secret rule. Leave others that meet the rule in pile and ask students to try to find additional pieces that belong to the set and to guess the secret rule.

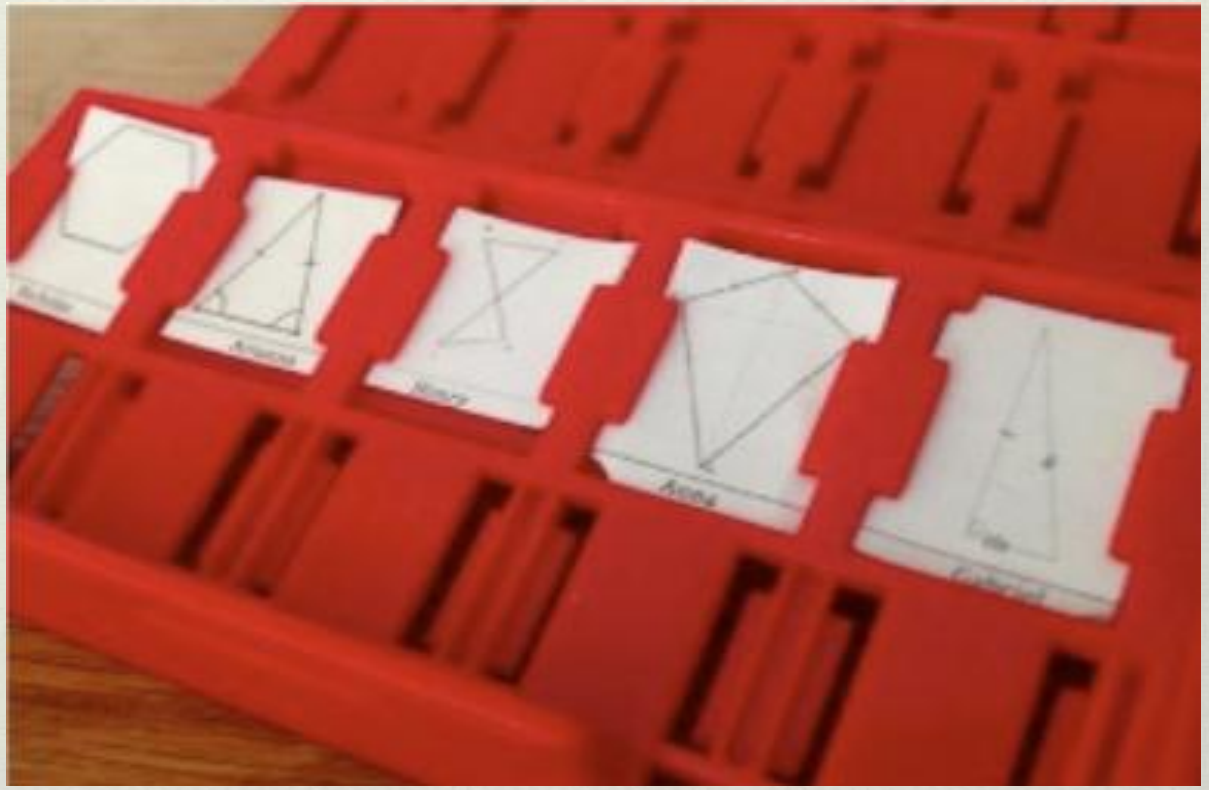
Secret Sort Activity



- Teacher Notes:
- This can be done as a whole class activity. You could do this in pairs where students alternate making their own collections and quiz each other.
- This is also a great way to introduce a new property
- This can be done to differentiate instructions because you can specifically give students secret sorts for properties that they are struggling with.
- Properties to introduce include but are not limited to – concave/convex, right angles/perpendicular, parallel lines, straight/curved (polygons/not polygons), number of sides, equal sides, equal angles.

So how many of you have played the board game *Guess Who?*

Guess What?



*Game produced by Milton Bradley/Hasbro and available wherever games are sold

Rules for Guess What

- Objective is to guess your opponent's mystery shape before your opponent guesses your mystery shape.
- Players take turns asking yes or no questions about character attributes like "Does your shape have at least 1 right angle?"
- Shapes that no longer fit the description of the opponents' mystery shape are eliminated by flipping card holders over.
- The first player to correctly guess the other players' mystery shape wins!

Variations - Use word bank for ESL students

- Parallel
- Right Angle
- Perpendicular
- Vertex
- Sides
- Angles
- Open Figure
- Closed Figure

- Quadrilateral
- Triangle
- Rectangle
- Square
- Parallelogram
- Rhombus
- Trapezoid
- Pentagon
- Hexagon
- Octagon
- Isosceles
- Equilateral

Variations

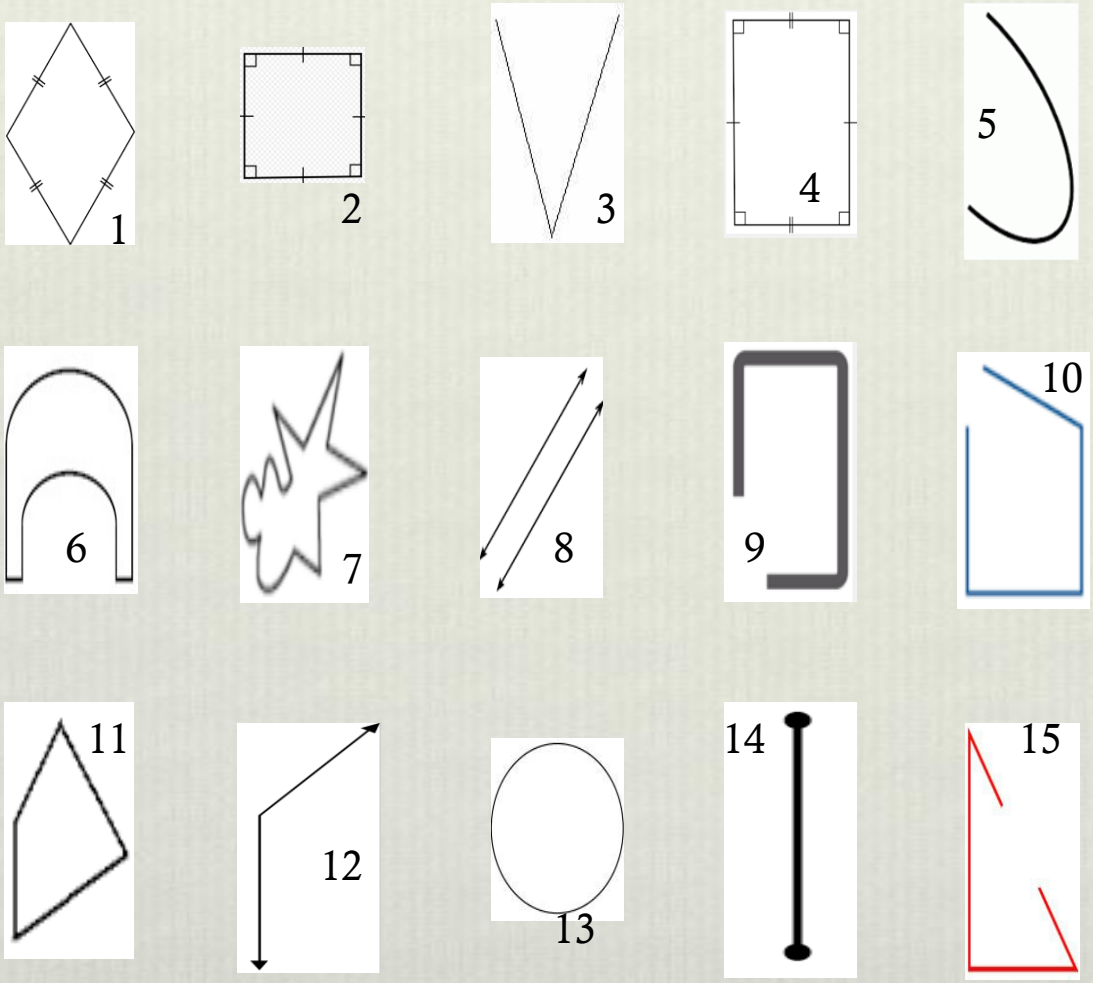
- Allow students to only use each vocab word once.
- Use during Response to Intervention (RTI) time or with students that need additional support.

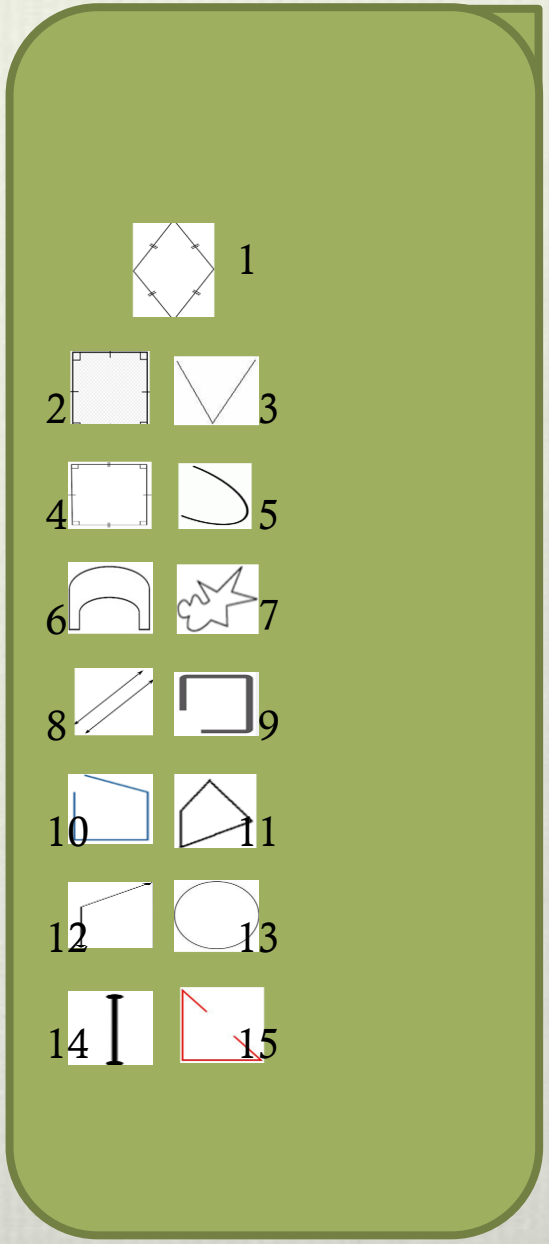
. . . Ya, but it probably took you a year to cut all these shapes out and paste them on each board! YOU'RE RIGHT! ☺

Take it to Your Classroom

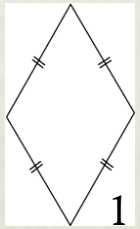
- $10 a set online
- Consider writing a small grant for your school.

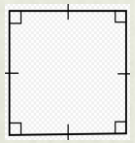
Lines, Closed vs Open Figures, & Beginning Quads

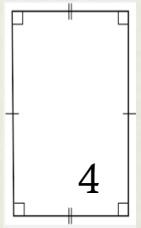


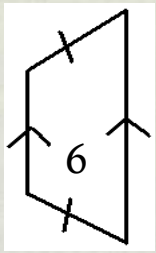


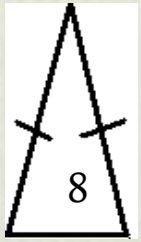
Quads & Triangles

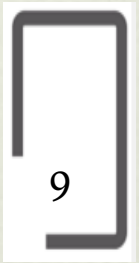


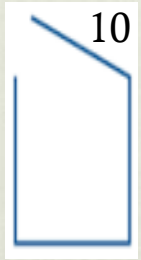


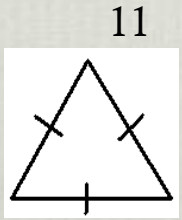


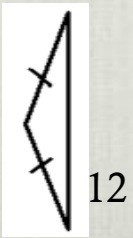


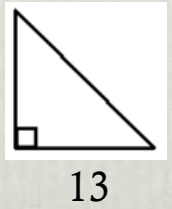


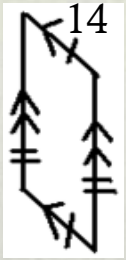


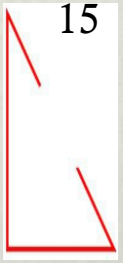


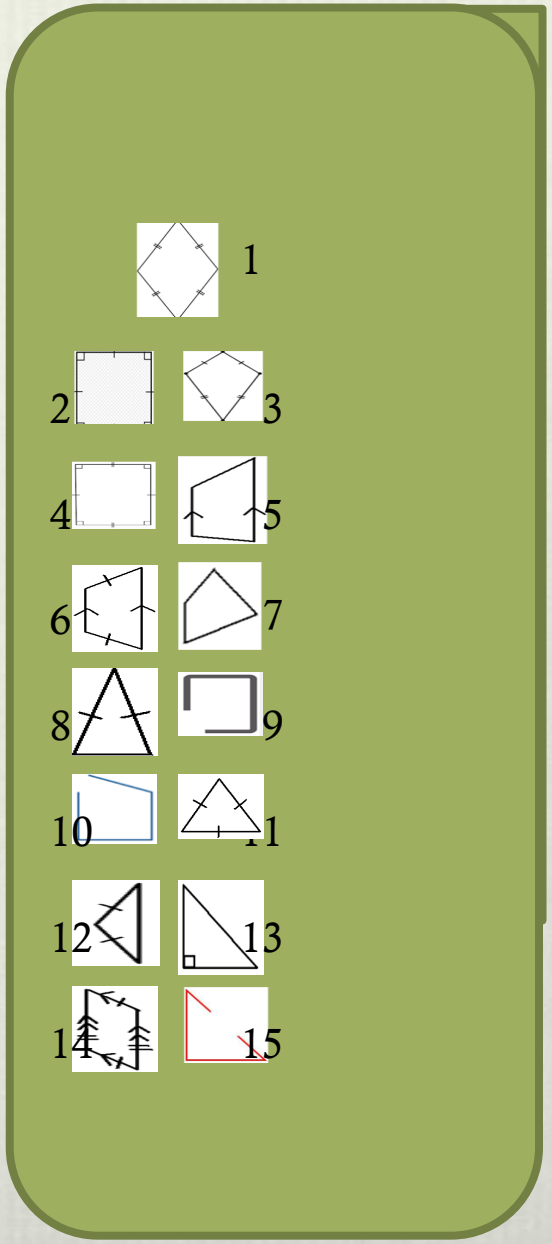




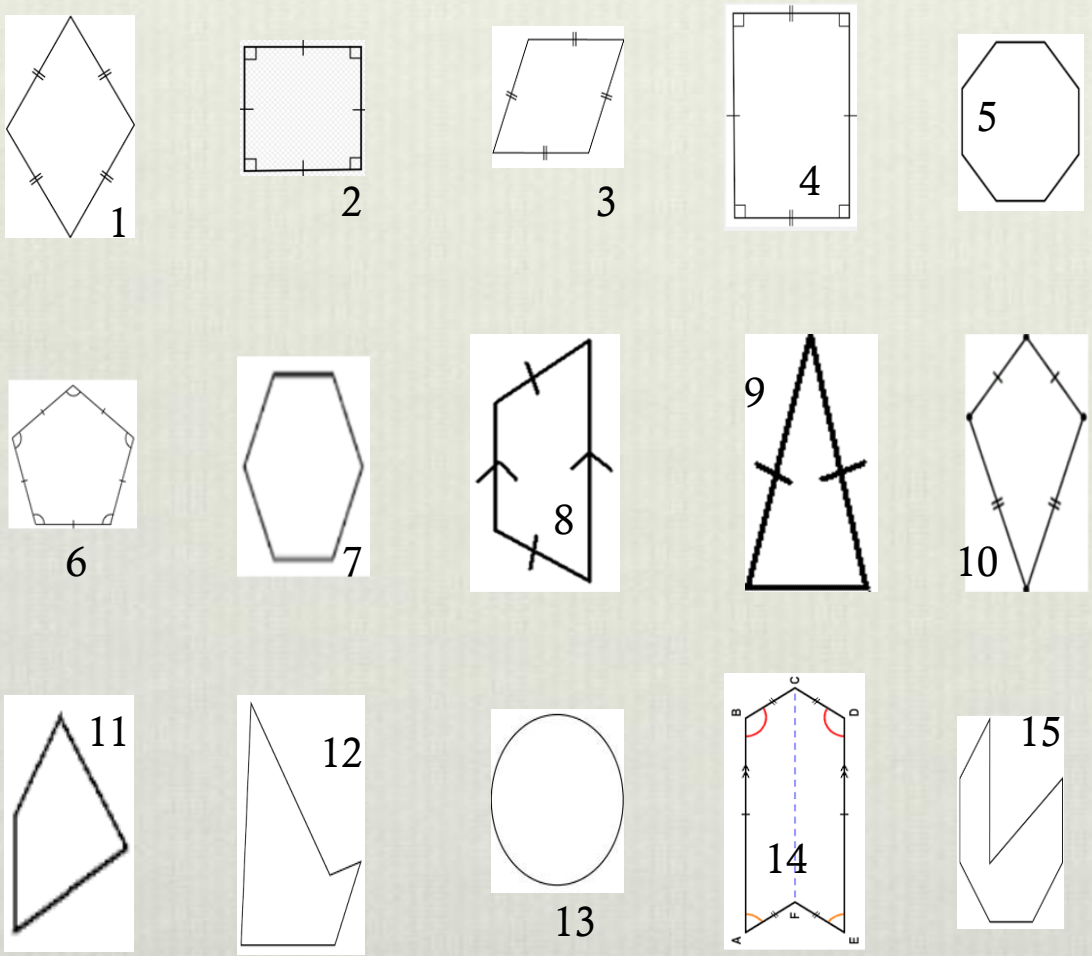


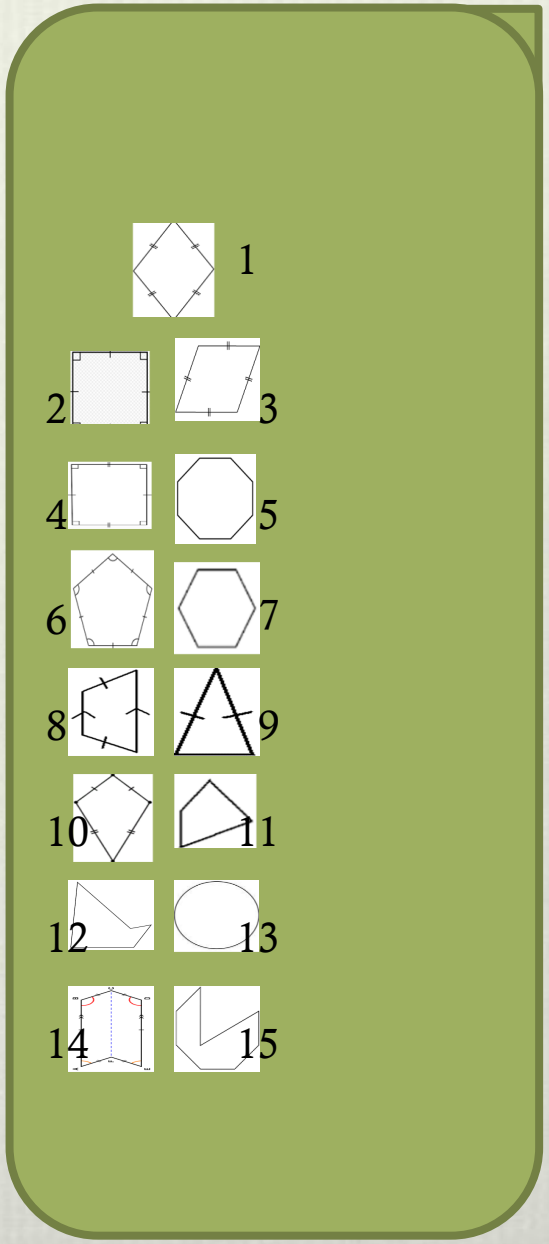






Closed Figures focused on Sides & Vertices





Other Resources for further Reading

- **NCTM Publication:** Currently in preparation and approved for publication by NCTM is C. Walcott (Ed.), "Using NAEP in the Classroom". Reston, VA: National Council of Teachers of Mathematics.

- **Daiga, M.**, Dilworth, L., Vesperman, C., & Creager, M. (2014, April). *Guess What?:* Adapting *Guess Who* to Challenge Students' Geometric Knowledge. *Wisconsin Teacher of Mathematics 65*(2), pp. 8-12.

- Van de Walle, J. A., Lovin, L. A. H., Karp, K. H., & Williams, J. M. B. (2013). *Teaching Student-Centered Mathematics: Pearson New International Edition: Developmentally Appropriate Instruction for Grades Pre K-2* (Vol. 1). Pearson Higher Ed.

- Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. Teaching children mathematics, 5(6), 310.

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