But My Kids Don't Think That Way!

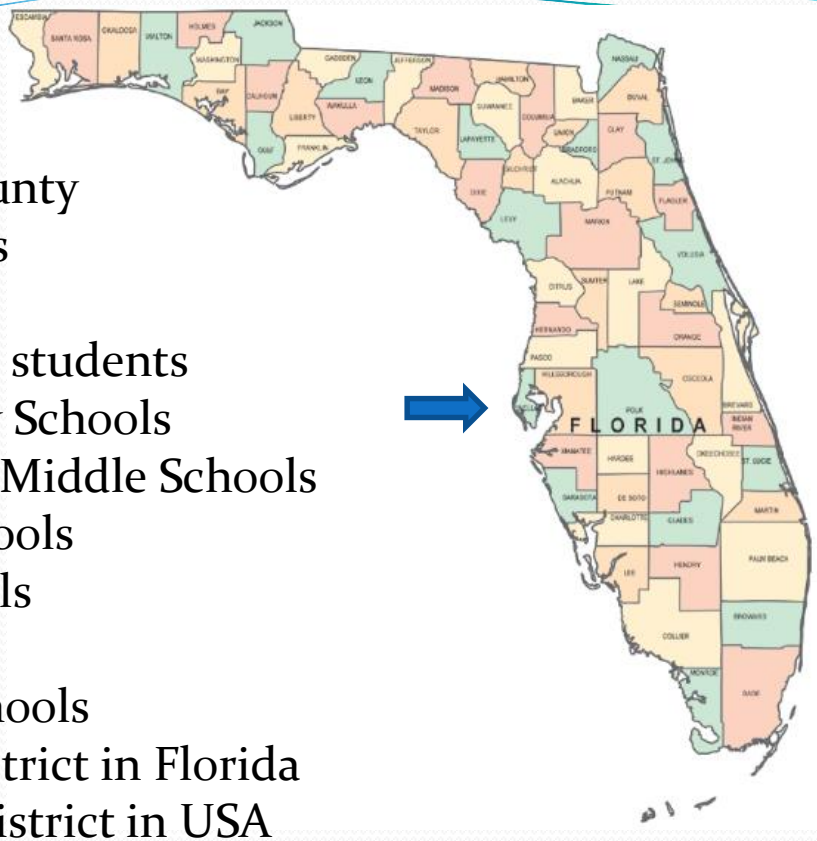
My ESE Students Need the Fewest Number of Steps to Solve a Problem Correctly.

Kevin Larkin
larkinke@pcsb.org

Adrienne DeLong
delonga@pcsb.org

Staff Developers
Elementary Mathematics
Pinellas County School, Florida

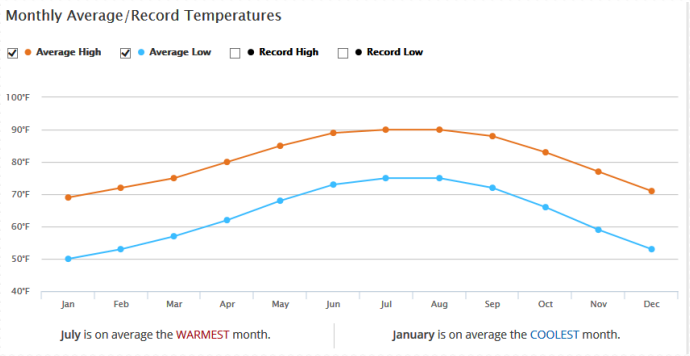
Pinellas County Schools



About 103,000 students
74 Elementary Schools
2 Elementary/Middle Schools
21 Middle Schools
18 High Schools
5 ESE Schools
23 Charter Schools
7th Largest District in Florida
26th Largest District in USA

Pinellas County

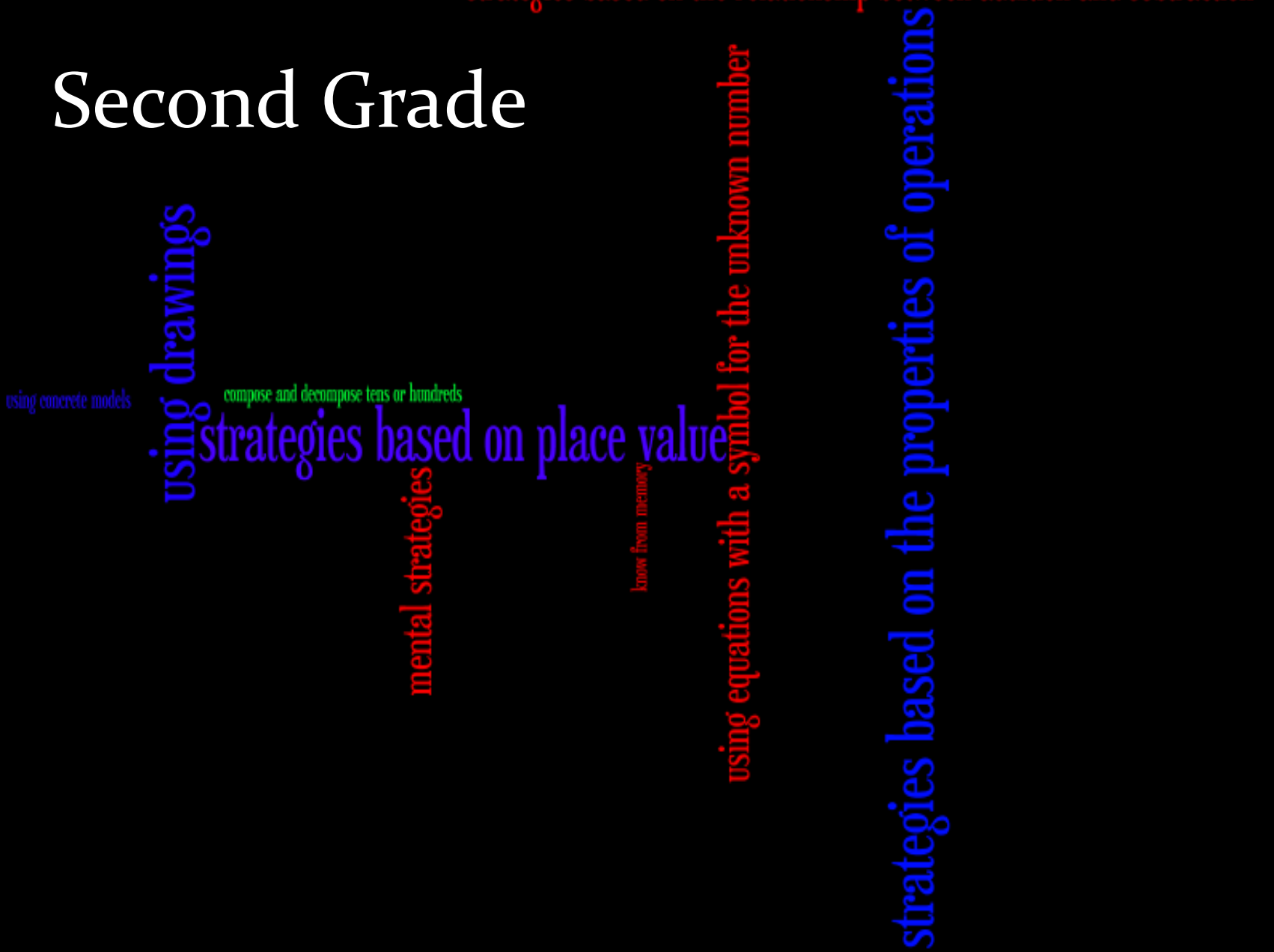
Over 900,000 people
588 miles of coastline
35 miles of sandy beaches
11 barrier islands




STRATEGIES BASED ON PROPERTIES OF OPERATIONS
USING OBJECTS
USING DRAWINGS
MAKING TEN
DECOMPOSING A NUMBER LEADING TO A TEN
USE PROPERTIES OF OPERATIONS AS STRATEGIES
USING THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION
EQUATIONS WITH A SYMBOL FOR THE UNKNOWN NUMBER
UNKNOWN=ADDEND
MENTALLY
COUNTING ON
STRATEGIES BASED ON PLACE VALUE
CREATING EQUIVALENT BUT EASIER OR KNOWN SUMS

First Grade

Second Grade



But My Kids Don't Think That Way!

Goals:

- Learn why we use various alternative/invented addition and subtraction strategies.
- Use tools to help students progress from a concrete to an abstract representation of a strategy.
- Elaborate on student thinking to develop and connect computation strategies
- SMPs
- PtA Teaching Practices

What is Fluency?

Principles to Action (pages 42-45)

- Fluency depends on and extends from conceptual understanding.

- To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

What is Fluency?

Principles to Action: Continued

- Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems.

What is Fluency?

Principles to Action: continued

- In meaningful learning of basic number combinations, students progress through well-documented phases towards fluency.
  - Using objects
  - Visual representations
  - Verbal counting
  - Reasoning strategies using number relationships and properties

What is Fluency?

Number Talks by Sherry Parish (p 38)

- Fluency is much more than fact recall.
- Fluency is knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.

But My Kids Don't Think That Way! Questions

1. Which standards promote students' thinking in this way?
2. How did we develop the connections between the various tools, the student invented strategies, and the abstract computational recording of the strategies?
3. Which Mathematical Practices could be highlighted within lessons that promote student thinking?
4. Looking at the Effective Teaching Practice about fluency from Principles to Action, which teacher and student moves would you expect to see during a lesson that emphasize invented strategies and algorithms?
5. What other Effective Teaching Practices would you expect to surface in a lesson incorporating alternative strategies, such as those seen today?
6. Looking back at the computational strategies mentioned throughout the common core standards, how were they incorporated today

Developing Fluency within the Common Core Addition Standards

Tools

- Base Ten Blocks
- Ten Frames
- Hundreds Chart
- Open Number Line

Alternative Strategies

- Adding Up
- Over-shooting
- Partial Sums

Creating Anchor Charts that capture student thinking to support learning.

So . . .

What are the differences between math tools and math strategies?

Anchor Charts

Do you create your anchor charts before the lesson to help make your teaching easier?

or

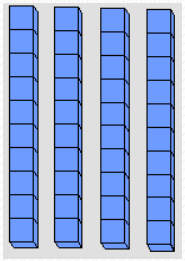
Do you create your anchor charts based on student work during the lesson to support student learning?

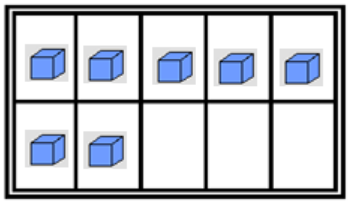
Addition Strategies

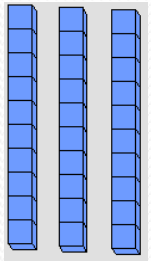
Use the base ten blocks (along with double ten frames) to add the problem below. Do not use the standard algorithm.

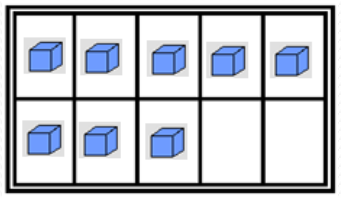
47

+ 38









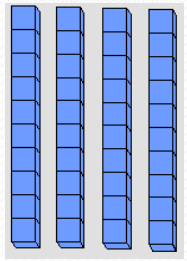
Let's look at the various ways you may have solved this problem.

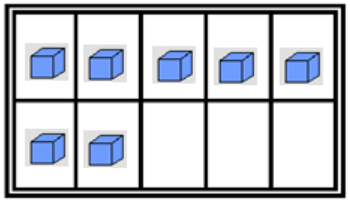
Adding Up:
Add the Tens First, then Add the Ones

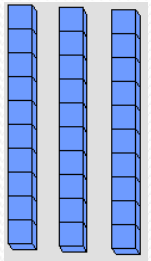
- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

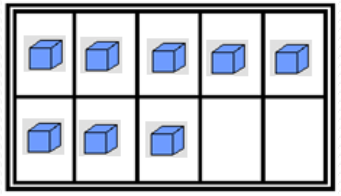
Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$







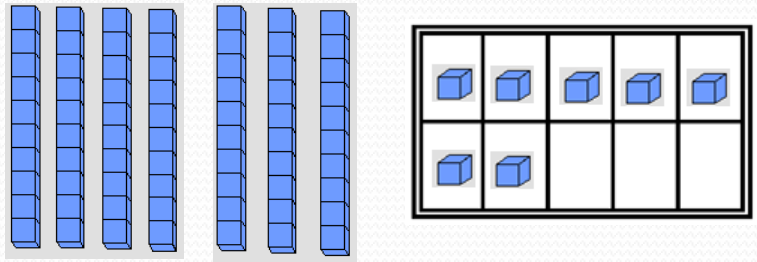


You added 3 tens or 30 to 47.
Now you have 77.

$$\begin{array}{r} 47 \\ +\ 30 \\ \hline 77 \end{array}$$

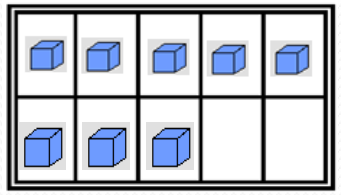
Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 77 \\ 80 \\ 85 \end{array}$$



You added 3 ones to 77.
Now you have 80.

You add 5 ones to 80.
Now you have 85.

$$\begin{array}{r} 47 \\ +\ 30 \\ \hline 77 \end{array} \qquad \begin{array}{r} 77 \\ +\ 3 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ +\ 5 \\ \hline 85 \end{array}$$

Addition Strategies:
Adding Up Using a Hundreds Chart

47

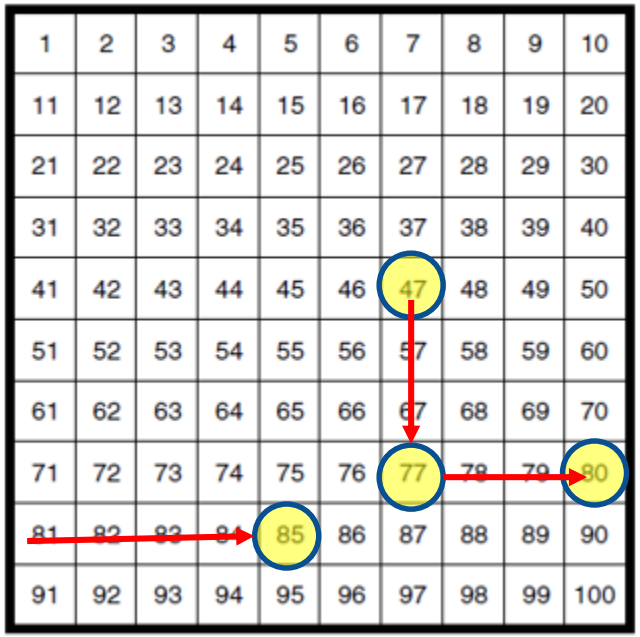
+ 38

How can we capture this thinking on the hundreds chart?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 77 \\ 80 \\ \hline 85 \end{array}$$

Student thinking

$$\begin{array}{r} 47 \\ +\ 30 \\ \hline 77 \end{array} \qquad \begin{array}{r} 77 \\ +\ 3 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ +\ 5 \\ \hline 85 \end{array}$$

Addition Strategies: Adding Up
Using an Open Number Line

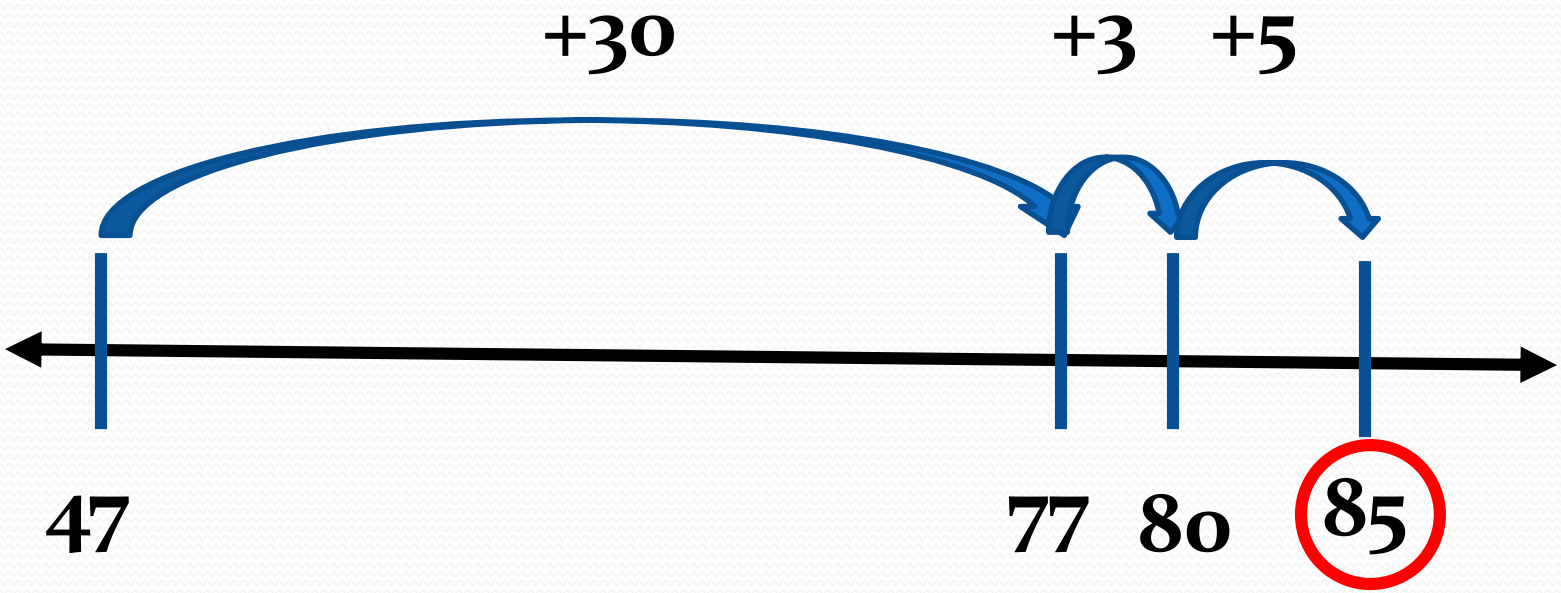
How can we capture this thinking on the open number line?

47

+ 38

Adding Up:
Add the Tens First, then Add the Ones

47
+ 38



Symbolic representation

47
+ 38
77
80
85

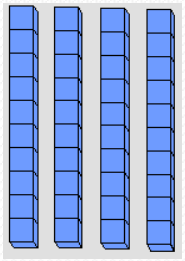
Adding Up . . . A Variation:

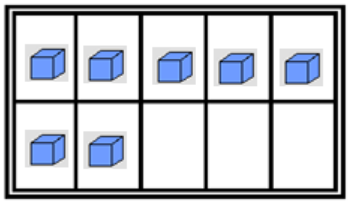
Add Some Ones to Make the Next Ten,
Add the Tens, then Add the Rest of the Ones

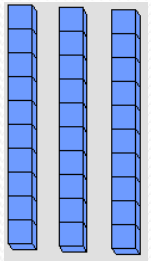
- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

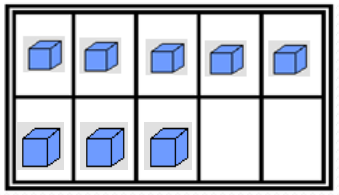
Adding Up, A Variation: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$







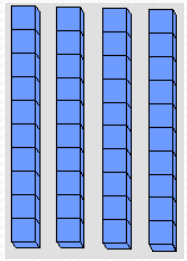


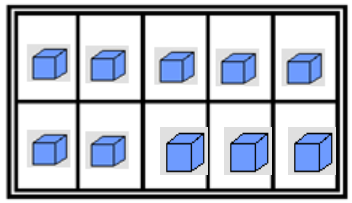
You added 3 ones to 47.
Now you have 50.

$$\begin{array}{r} 47 \\ +\ \ 3 \\ \hline 50 \end{array}$$

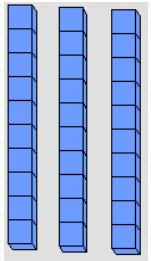
Adding Up, A Variation: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

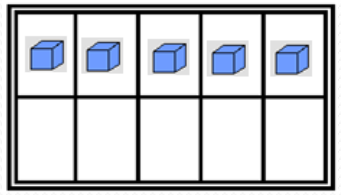
$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$





You added
3 tens to 50.
Now you have
80.





You add 5 ones to 80.
Now you have 85.

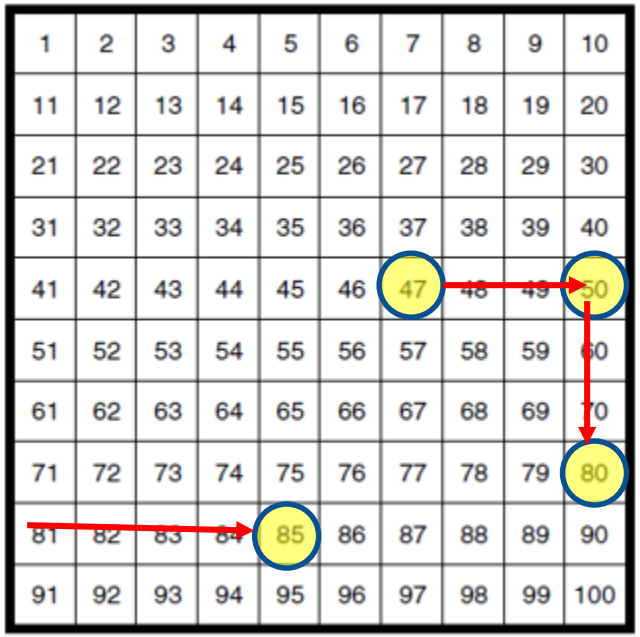
Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 80 \\ 85 \end{array}$$

$$\begin{array}{r} 47 \\ +\ 3 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ +\ 30 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ +\ 5 \\ \hline 85 \end{array}$$

Adding Up, A Variation: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



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51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 80 \\ \hline 85 \end{array}$$

$$\begin{array}{r} 47 \\ +\ 3 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ +\ 30 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ +\ 5 \\ \hline 85 \end{array}$$

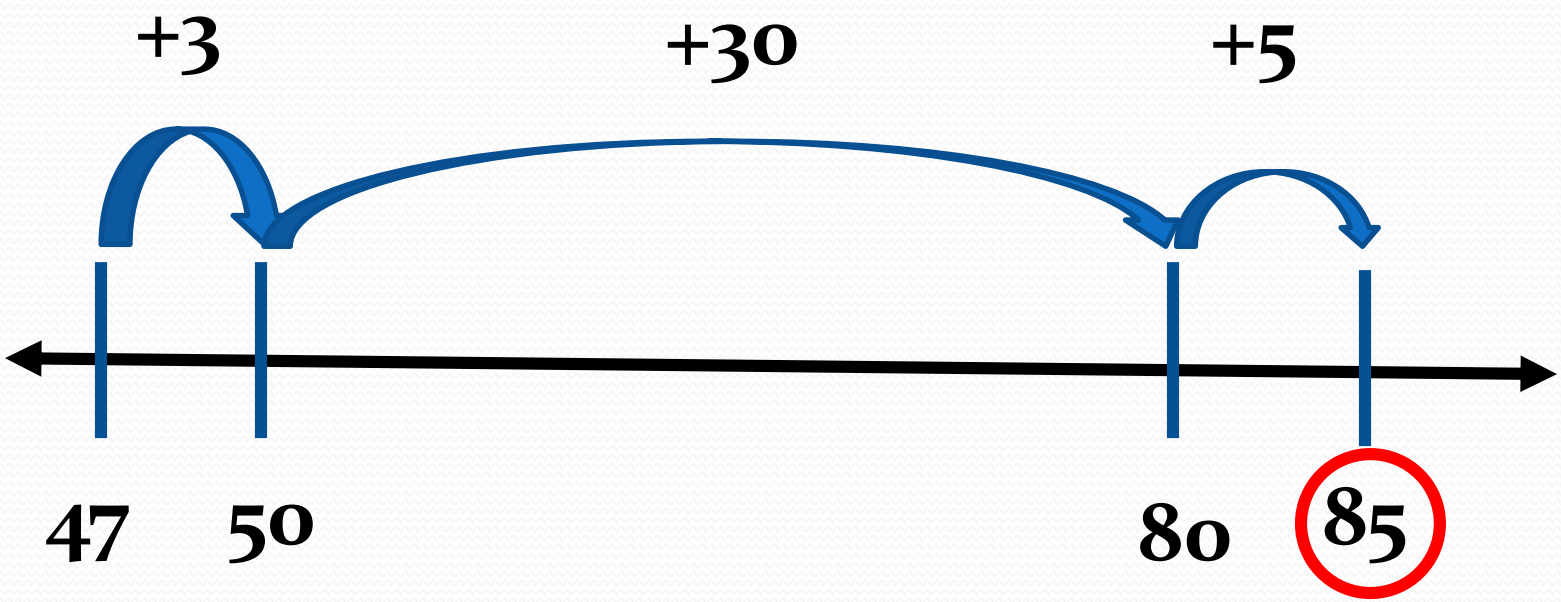
Adding Up, A Variation: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

How can we capture this thinking on the hundreds chart?

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$

Adding Up, A Variation: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 80 \\ 85 \end{array}$$

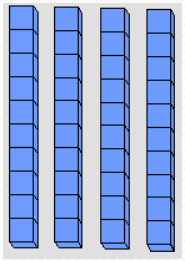
Adding Up . . . Another Variation –

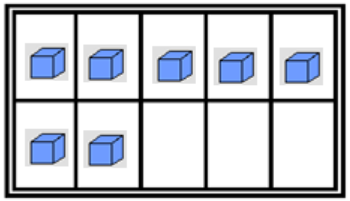
Add the Ones First and then Add the Tens

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

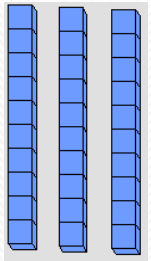
Adding Up: Another Variation - Add the Ones First, then Add the Tens

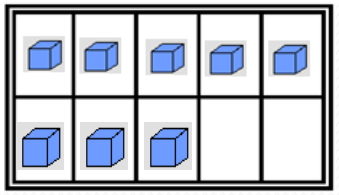
$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$





You added 3 ones to 47. Now you have 50.





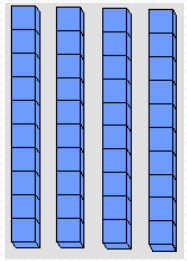
Then you added 5 ones to 50. Now you have 55.

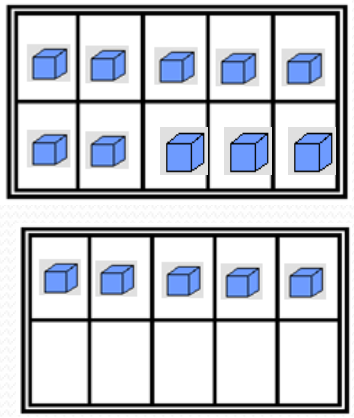
$$\begin{array}{r} 47 \\ +\ 3 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ +\ 5 \\ \hline 55 \end{array}$$

Adding Up:
Add the Ones First, then Add Tens

Symbolic representation

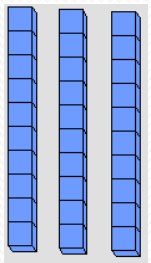
$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$





$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 55 \\ 85 \end{array}$$

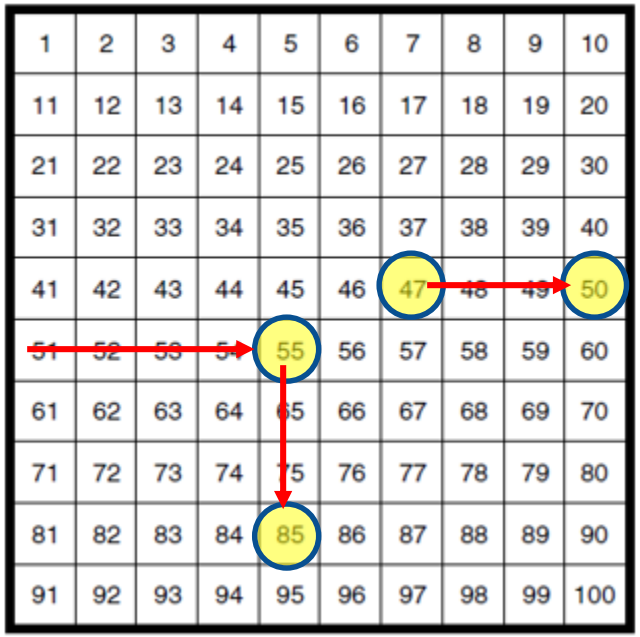
You added 3 tens to 55. Now you have 85.



$$\begin{array}{r} 47 \\ +\ 3 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ +\ 5 \\ \hline 55 \end{array} \qquad \begin{array}{r} 55 \\ +\ 30 \\ \hline 85 \end{array}$$

Adding Up: Another Variation – Add the Ones First, then Add the Tens

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



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31	32	33	34	35	36	37	38	39	40
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51	52	53	54	55	56	57	58	59	60
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 55 \\ 85 \end{array}$$

$$\begin{array}{r} 47 \\ +\ 3 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ +\ 5 \\ \hline 55 \end{array} \qquad \begin{array}{r} 55 \\ +\ 30 \\ \hline 85 \end{array}$$

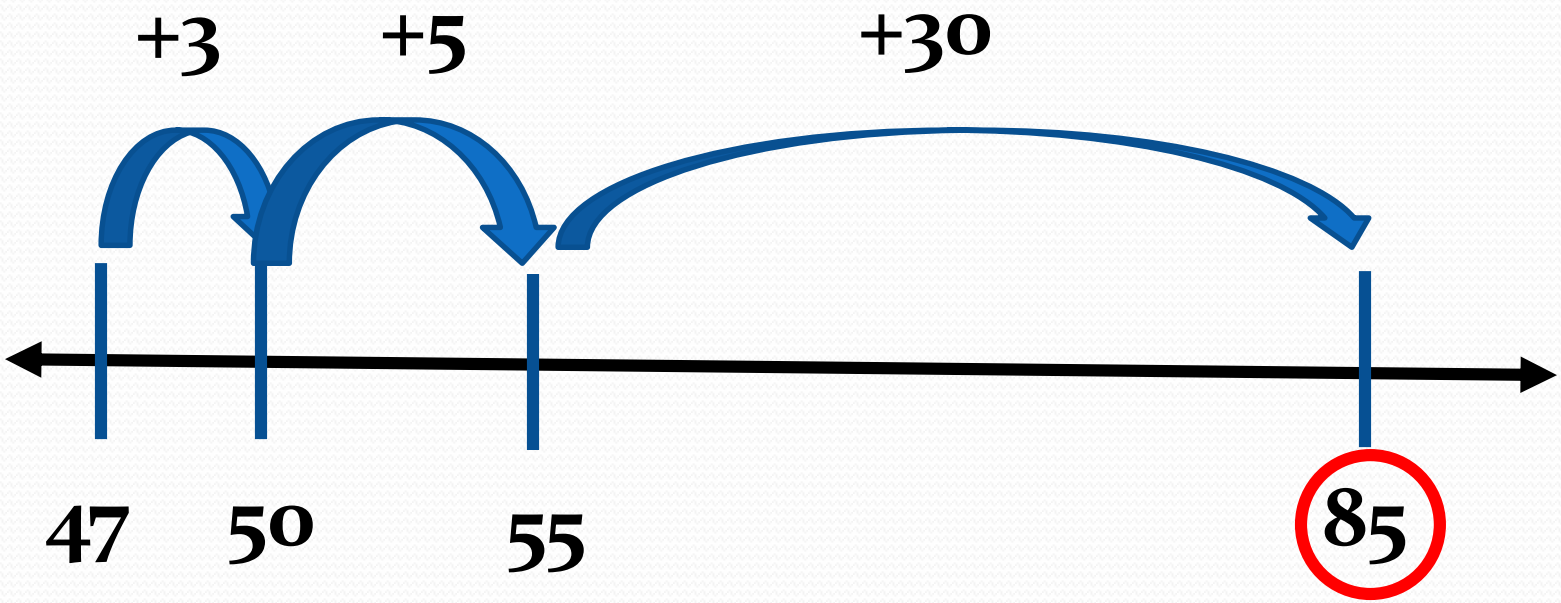
Adding Up, A Variation:
Add the Ones, and Then Add the Tens

How can we capture this thinking on the open number line?

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$

Adding Up, A Variation:
Add the Ones, and Then Add the Tens

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 50 \\ 55 \\ 85 \end{array}$$

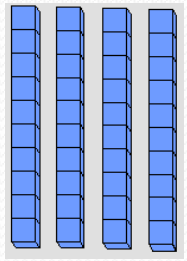
Adding Up - Overshoot and Come Back

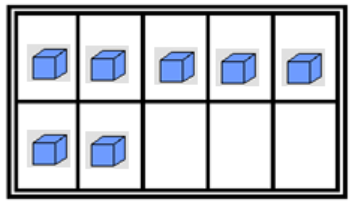
- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Adding Up: Another Variation - Overshoot and Come Back

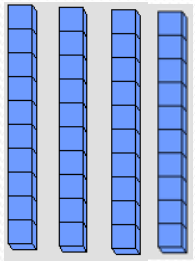
47

+ 38



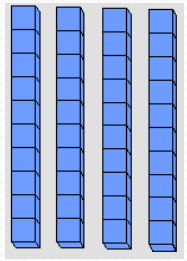


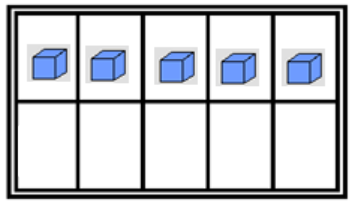
38 is close to 40.
Add 40 to 47. . .



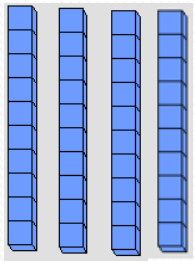
Adding Up:
Overshoot and Come Back

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$





38 is close to 40. Add 40 to 47 which gives you 87.

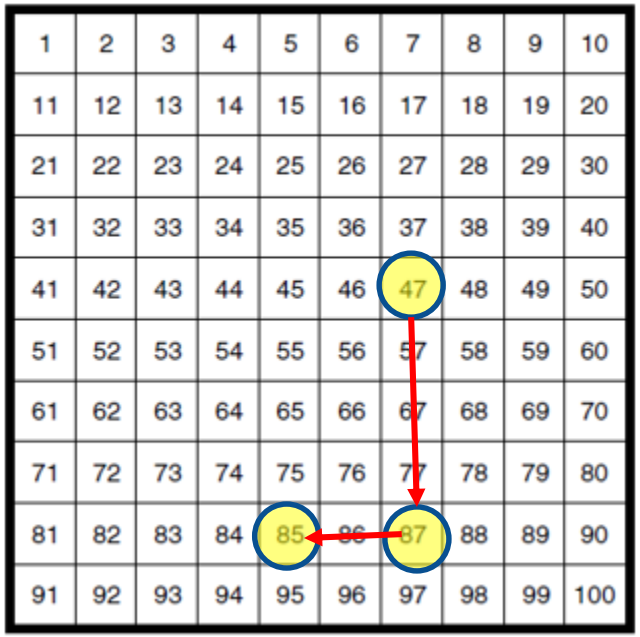


Take away the extra 2 from 87. Now you have 85

$$\begin{array}{r} 47 \\ +\ 40 \\ \hline 87 \end{array} \qquad \begin{array}{r} 87 \\ -\ 2 \\ \hline 85 \end{array}$$

Adding Up: Another Variation – Overshoot and Come Back

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 87 \\ \hline 85 \end{array}$$

$$\begin{array}{r} 47 \\ +\ 40 \\ \hline 87 \end{array} \qquad \begin{array}{r} 87 \\ -\ 2 \\ \hline 85 \end{array}$$

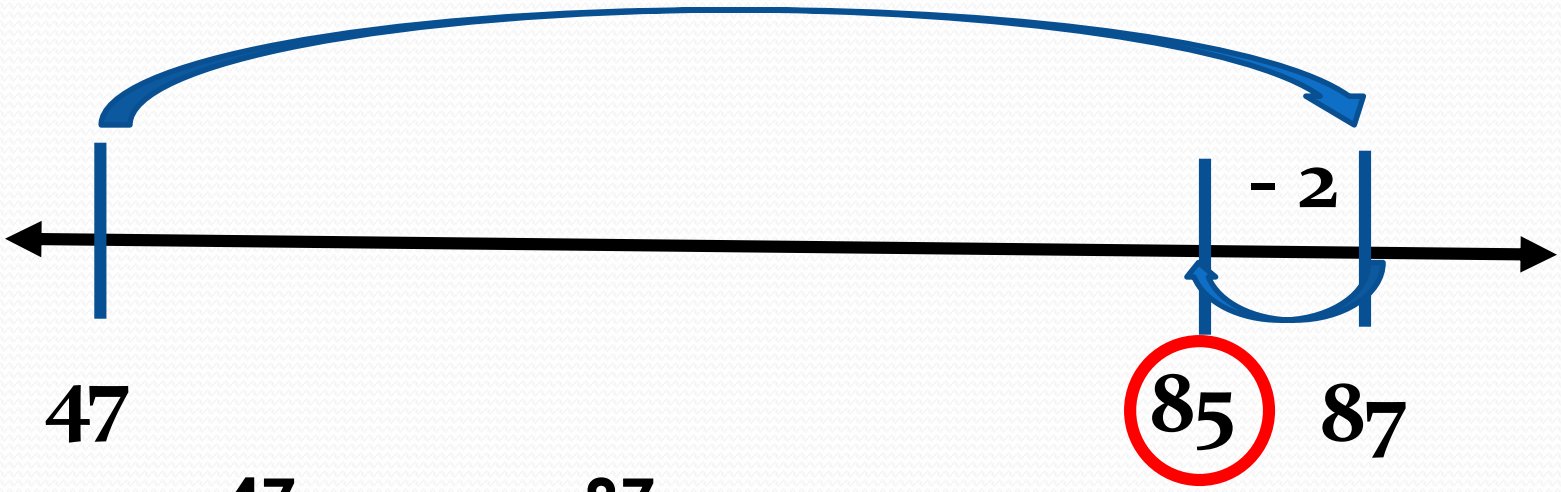
Adding Up: Another Variation – Overshoot and Come Back

47
+ 38

+40

Symbolic representation

47
+ 38
87
85



47
+ 40
87

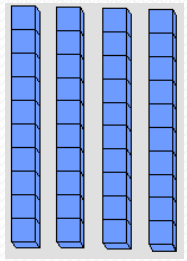
87
- 2
85

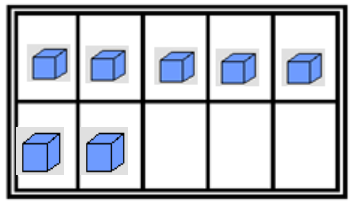
Moving Towards the Standard Algorithm: Partial Sums

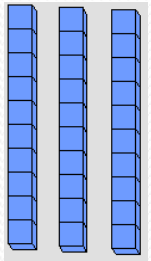
- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

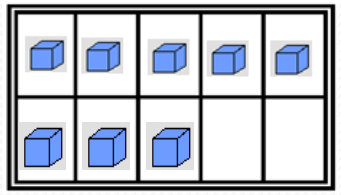
Adding by Place Value – Partial Sums

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$









What do students typically do?

They push the tens together and the ones together.

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 70 \\ +\ 15 \\ \hline 85 \end{array}$$

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 70 \\ 10 \\ +\ 5 \\ \hline 85 \end{array}$$

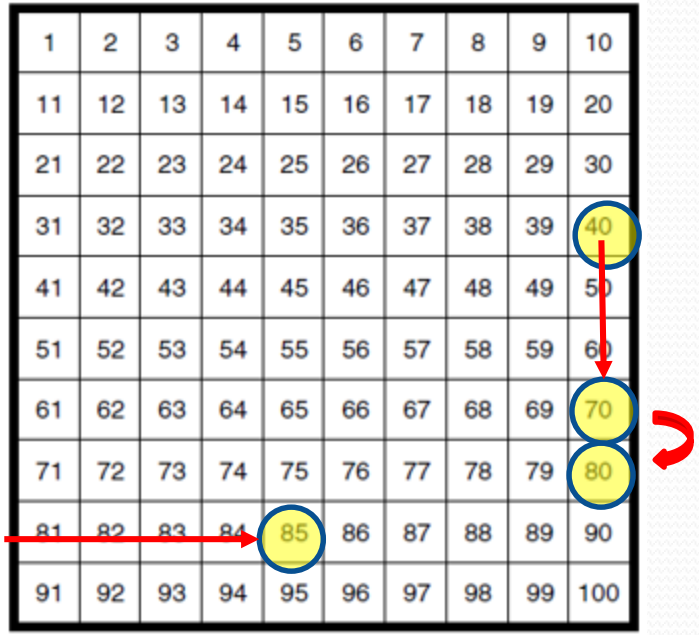
4 tens plus 3 tens equals 7 tens or 70.

7 ones plus 8 ones equals 15 ones.

Partial Sums on the Hundreds Board

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ +\ 30 \\ \hline 70 \end{array} \qquad \begin{array}{r} 70 \\ +\ 15 \\ \hline 85 \end{array}$$



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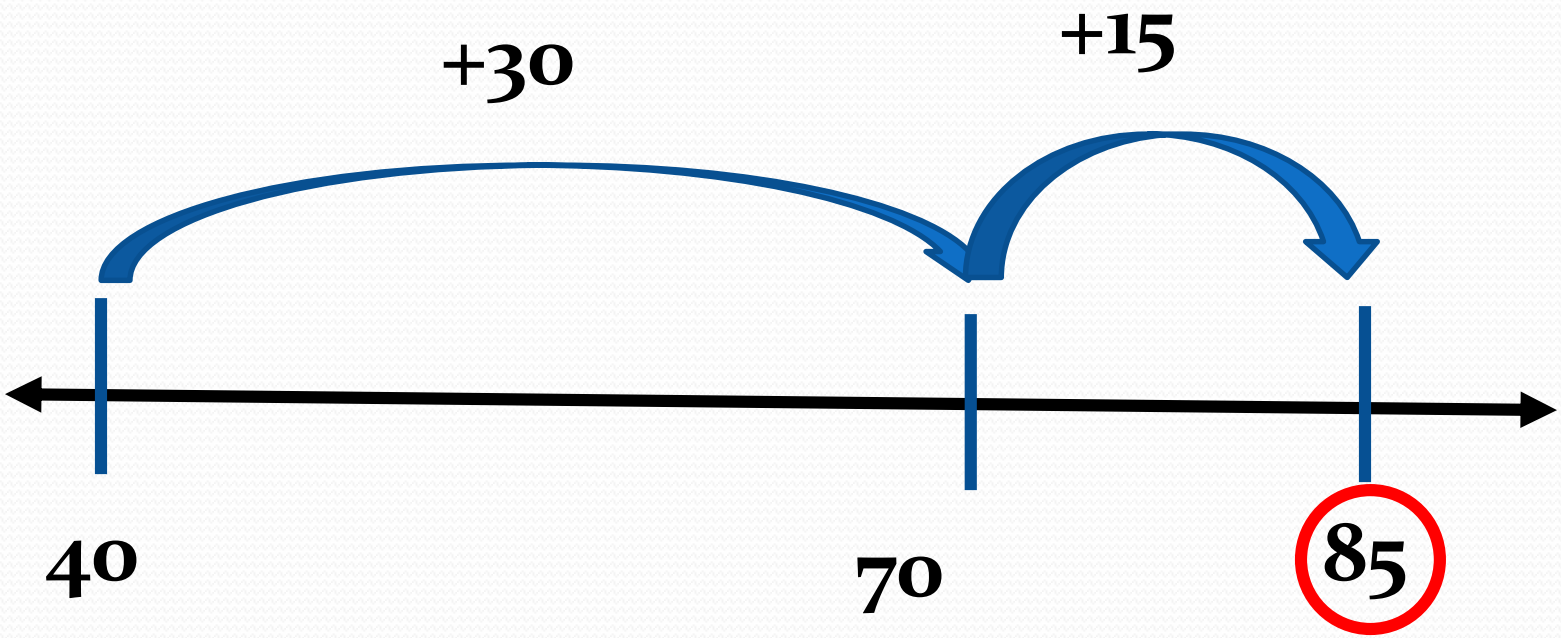
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 70 \\ +\ 15 \\ \hline 85 \end{array} \qquad \begin{array}{r} 47 \\ +\ 38 \\ \hline 70 \\ 10 \\ +\ 5 \\ \hline 85 \end{array}$$

Partial Sums on the Open Number Line

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 47 \\ +\ 38 \\ \hline 70 \\ +\ 15 \\ \hline 85 \end{array}$$

Developing Fluency within the Common Core Subtraction Standards

Tools

- Base Ten Blocks
- Ten Frames
- Hundreds Chart
- 99 Chart
- Open Number Line

Alternative Strategies

- Adding Up
- Take Away
- Over-shooting

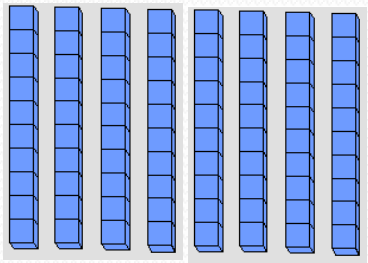
Creating Anchor Charts that capture student thinking to support learning.

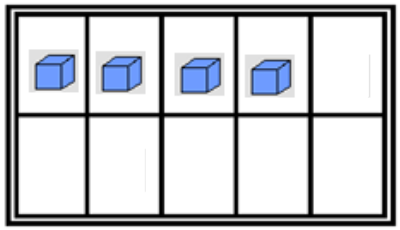
Subtraction Strategies

Use the base ten blocks (along with double ten frames) to subtract the problem below. Do not use the standard algorithm.

84

- 37





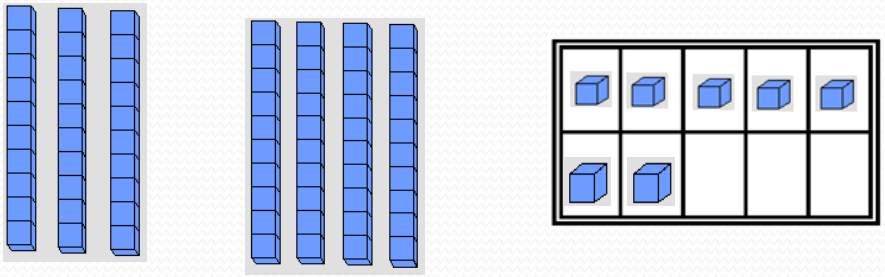
Let's look at the various ways you may have solved this problem.

Subtraction: Adding Up

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

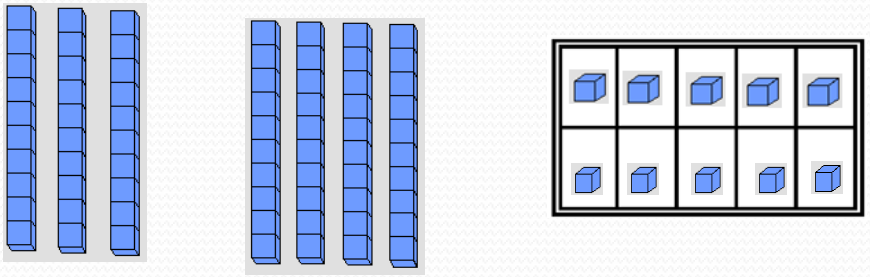


Add up to reach 84.

$$\begin{array}{r} 37 \\ +\ 40 \\ \hline 77 \end{array}$$

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

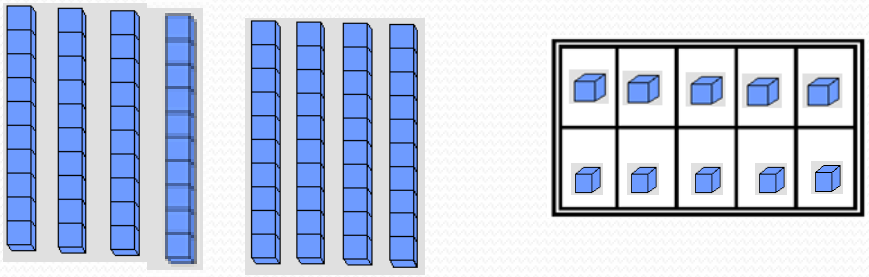
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 37 \\ +\ 40 \\ \hline 77 \end{array} \qquad \begin{array}{r} 77 \\ +\ 3 \\ \hline 80 \end{array}$$

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

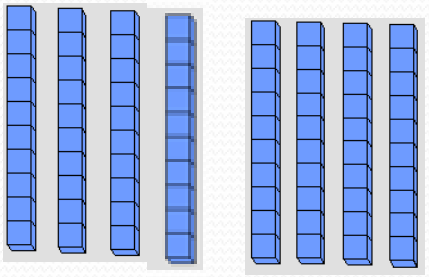


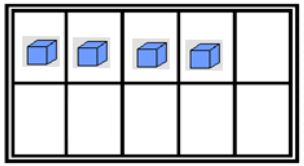
$$\begin{array}{r} 37 \\ +\ 40 \\ \hline 77 \end{array} \qquad \begin{array}{r} 77 \\ +\ 3 \\ \hline 80 \end{array}$$

Trade the 10 ones for one ten

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$





Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline 40 \\ 3 \\ +\ \ 4 \\ \hline 47 \end{array}$$

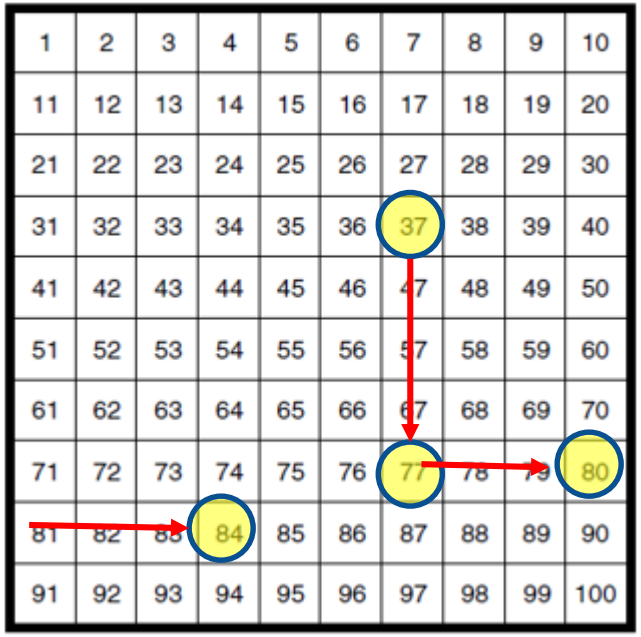
$$\begin{array}{rrrr} 37 & 77 & 80 & \\ +\ 40 & +\ 3 & +\ 4 & = 47 \\ \hline 77 & 80 & 84 & \end{array}$$

or

$$\begin{array}{rrrr} +40 & +3 & +4 & = 47 \\ \hline 37 \ldots & 77 \ldots & 80 \ldots & 84 \end{array}$$

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline 40 \\ 3 \\ +\ 4 \\ \hline 47 \end{array}$$

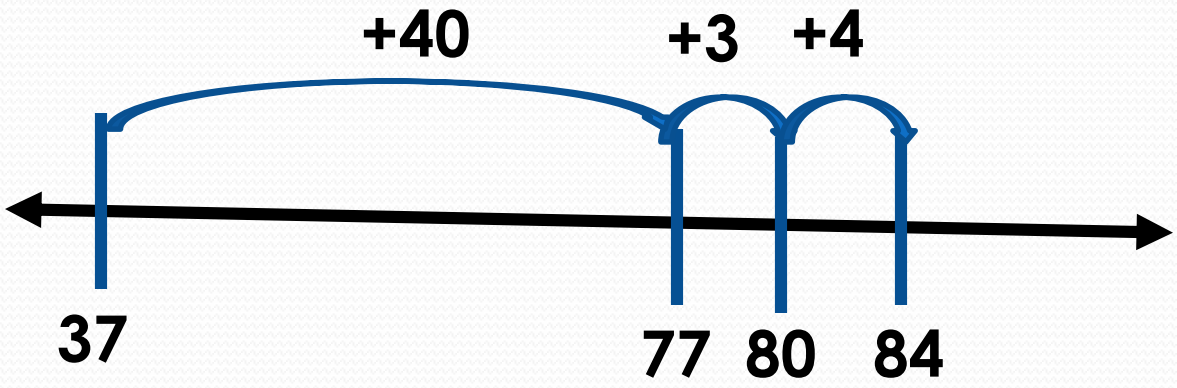
$$\begin{array}{rrrl} 37 & 77 & 80 & \\ +\ 40 & +\ 3 & +\ 4 & = 47 \\ \hline 77 & 80 & 84 & \end{array}$$

+40 +3 +4 = 47

37 . . . 77 . . . 80 . . . 84

Subtraction - Adding Up:
Add the Tens First, then Add the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline 40 \\ 3 \\ +\ \ 4 \\ \hline 47 \end{array}$$

$$\begin{array}{rrrr} 37 & 77 & 80 & \\ +\ 40 & +\ \ 3 & +\ \ 4 & = 47 \\ \hline 77 & 80 & 84 & \end{array}$$

or

$$\begin{array}{rrrr} +40 & +3 & +4 & = 47 \\ 37 \ldots & 77 \ldots & 80 \ldots & 84 \end{array}$$

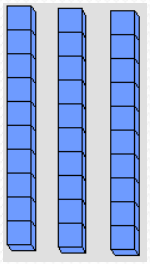
Subtraction: Adding Up in Chunks: A Variation

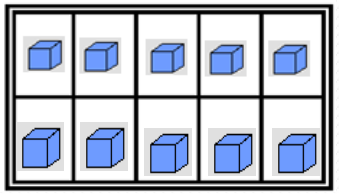
Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction - Adding Up: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

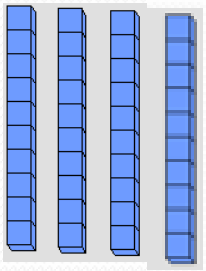


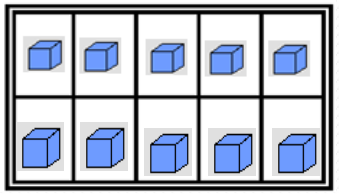


$$\begin{array}{r} 37 \\ +\ 3 \\ \hline 40 \end{array}$$

Subtraction - Adding Up: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



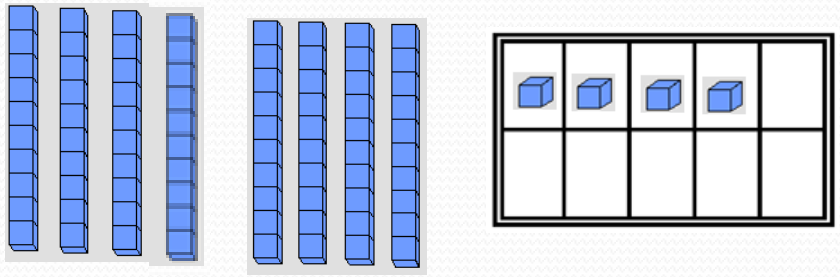


$$\begin{array}{r} 37 \\ +\ 3 \\ \hline 40 \end{array}$$

Trade the 10 ones for one ten.

Subtraction - Adding Up: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

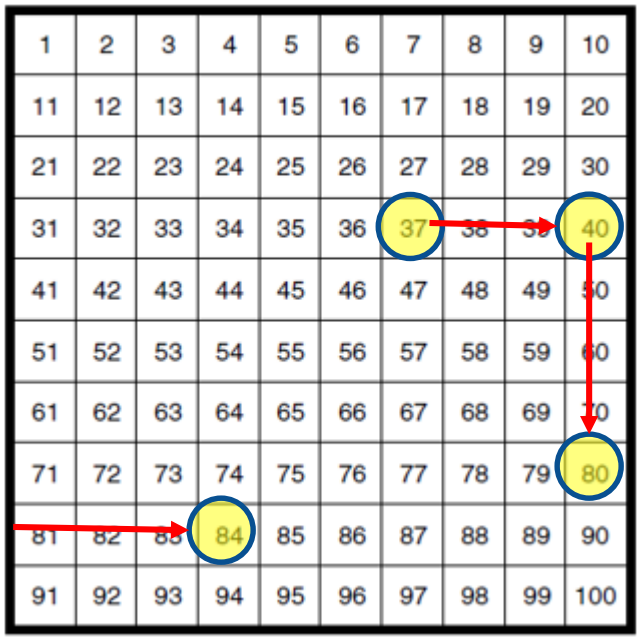
$$\begin{array}{rrrl} 37 & 40 & 80 & \\ +\ 3 & +\ 40 & +\ 4 & = 47 \\ \hline 40 & 80 & 84 & \end{array}$$

$$\begin{array}{rr} 84 & \\ -\ 37 & +\ 3 \\ & +\ 40 \\ & +\ 4 \\ \hline & 47 \end{array} \quad \text{or}$$

$$37 \ldots 40 \ldots 80 \ldots 84 \qquad +3 \quad +40 \quad +4 = 47$$

Subtraction - Adding Up: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array} \quad \begin{array}{r} +\ 3 \\ +\ 40 \\ +\ 4 \\ \hline 47 \end{array}$$

$$\begin{array}{r} 37 \\ +\ 3 \\ \hline 40 \end{array} \qquad \begin{array}{r} 40 \\ +\ 40 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ +\ 4 \\ \hline 84 \end{array} \qquad = 47$$

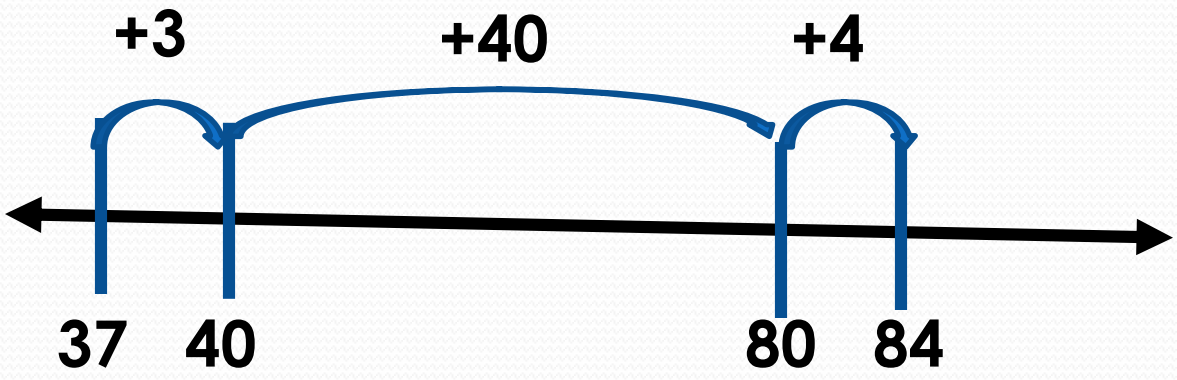
or

+3 +40 +4 = 47

37 . . . 40 . . . 80 . . . 84

Subtraction - Adding Up: Add Some Ones to Make the Next Ten, Add the Tens, then Add the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{rl} 84 & \\ -\ 37 & +\ 3 \\ & +\ 40 \\ & +\ 4 \\ \hline & 47 \end{array}$$

$$\begin{array}{rrrl} 37 & 40 & 80 & \\ +\ 3 & +\ 40 & +\ 4 & = 47 \\ \hline 40 & 80 & 84 & \end{array}$$

or

$$37 \xrightarrow{+3} 40 \xrightarrow{+40} 80 \xrightarrow{+4} 84 \quad = 47$$

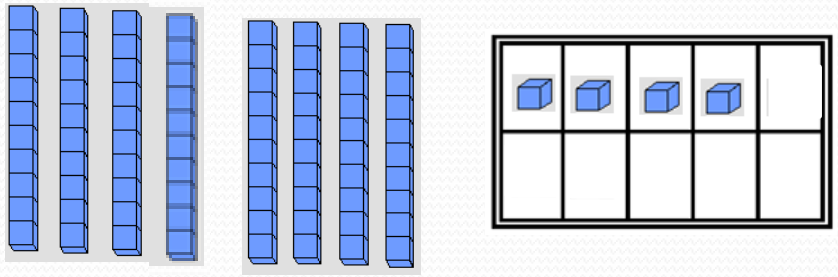
Subtraction: Adding Up: Another Variation

Overshoot and Come Back

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction – Adding Up, Another Variation: Overshoot and Come Back

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{rrr} 37 & 87 & \\ +\ 50 & -\ 3 & = 47 \\ \hline 87 & 84 & \end{array}$$

$$\begin{array}{rll} 84 & & \\ -\ 37 & +\ 50 & \\ & -\ 3 & \\ & \hline & 47 \end{array}$$

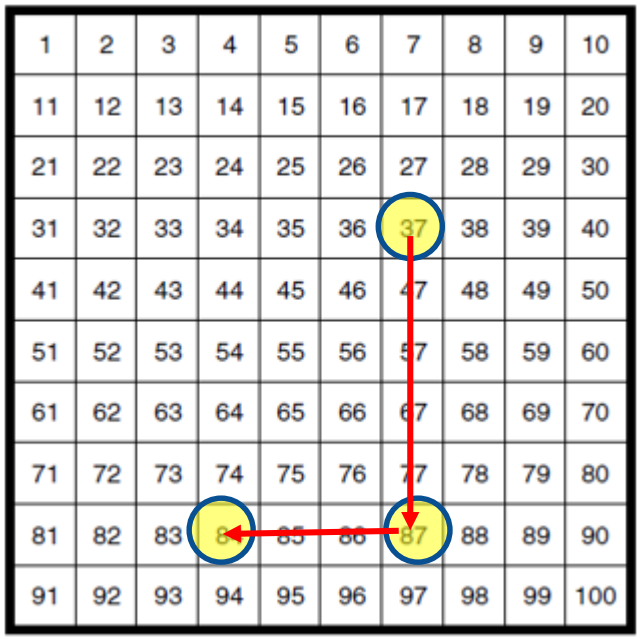
or

$$+50 \quad -\ 3 \quad = \quad 47$$

$$37 \ldots 87 \ldots 84$$

Subtraction – Adding Up, Another Variation: Overshoot and Come Back

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array} \quad \begin{array}{r} +\ 50 \\ -\ 3 \\ \hline 47 \end{array}$$

or

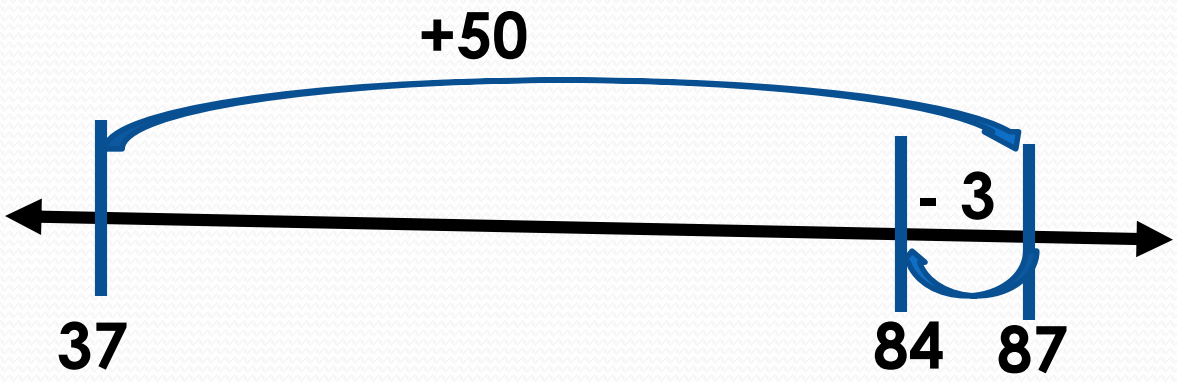
+50 -4 = 47

37 . . . 87 . . . 84

$$\begin{array}{r} 37 \\ +\ 50 \\ \hline 87 \end{array} \qquad \begin{array}{r} 87 \\ -\ 3 \\ \hline 84 \end{array} \quad = 47$$

Subtraction – Adding Up, Another Variation: Overshoot and Come Back

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{rrr} 37 & 87 & \\ +\ 50 & -\ 3 & = 47 \\ \hline 87 & 84 & \end{array}$$

Symbolic representation

$$\begin{array}{rrr} 84 & & \\ -\ 37 & +\ 50 & \\ & -\ 3 & \\ & 47 & \end{array}$$

or

$$+50 \quad -3 \quad = 47$$

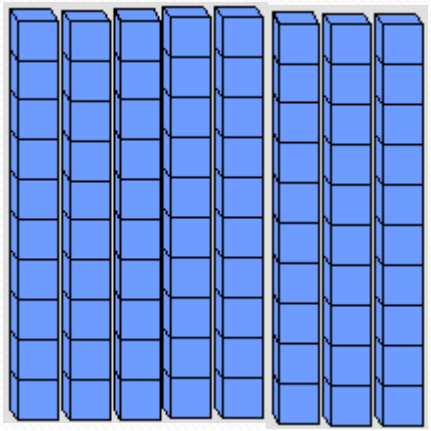
$$37 \ldots 87 \ldots 84$$

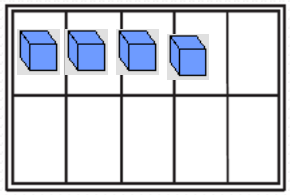
Subtraction: Take Away

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction: Take Away – Take Away Tens and Then Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

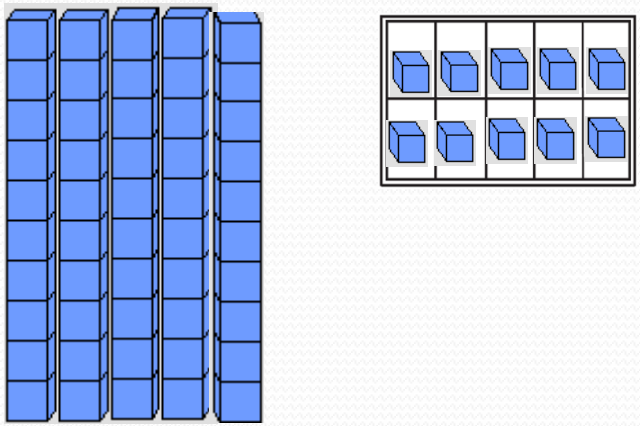




$$\begin{array}{r} 84 \\ -\ 30 \\ \hline 54 \end{array} \qquad \begin{array}{r} 54 \\ -\ \ 4 \\ \hline 50 \end{array}$$

Subtraction: Take Away – Take Away Tens and Then Ones

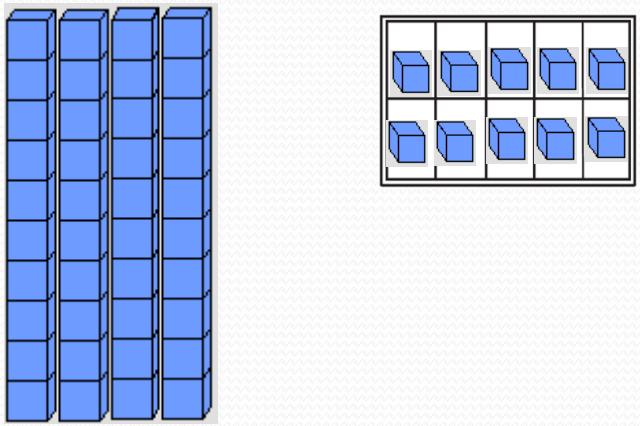
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 84 \\ -\ 30 \\ \hline 54 \end{array} \qquad \begin{array}{r} 54 \\ -\ 4 \\ \hline 50 \end{array}$$

Subtraction: Take Away – Take Away Tens and Then Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 84 \\ -\ 30 \\ \hline 54 \end{array} \qquad \begin{array}{r} 54 \\ -\ 4 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

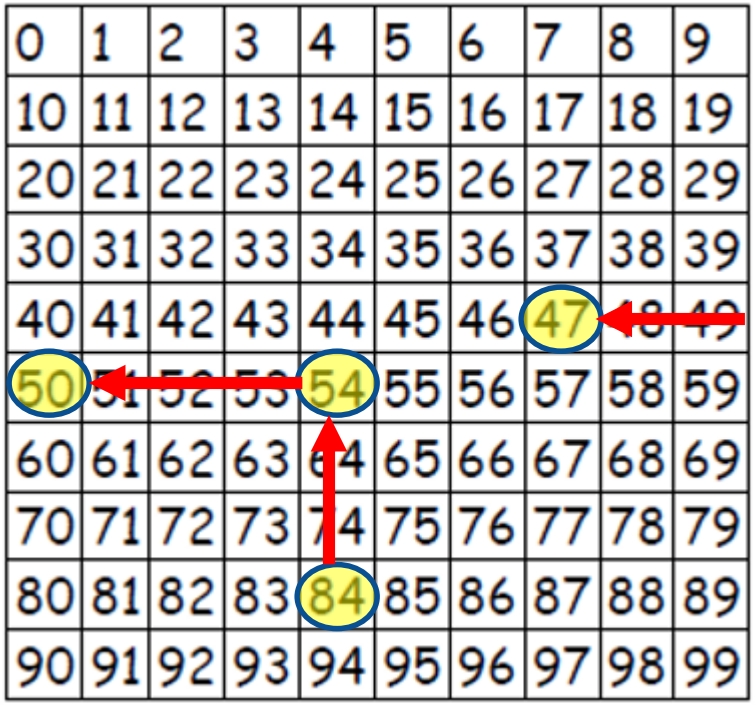
54 (-30)

50 (- 4)

47 (- 3)

Subtraction: Take Away – Take Away Tens and Then Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Symbolic representation

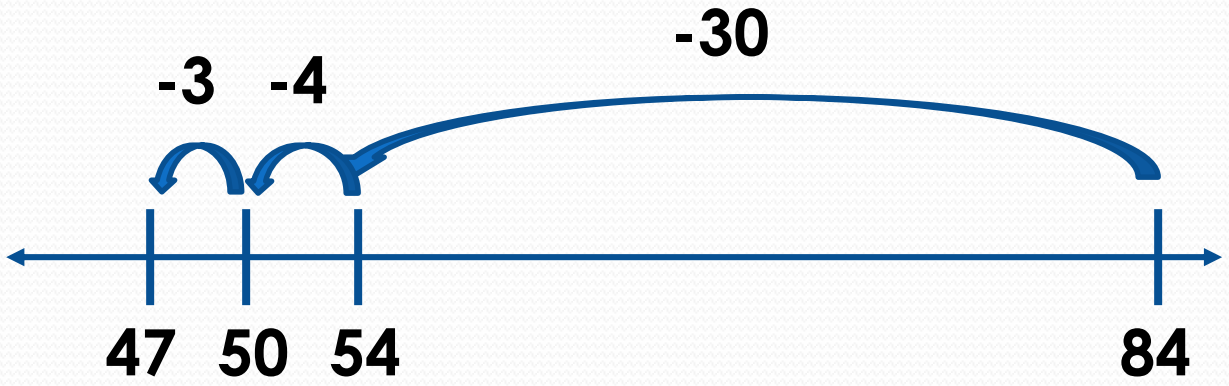
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

$$\begin{array}{rl} 54 & (-30) \\ 50 & (-\ 4) \\ 47 & (-\ 3) \end{array}$$

$$\begin{array}{r} 84 \\ -\ 30 \\ \hline 54 \end{array} \qquad \begin{array}{r} 54 \\ -\ 4 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

Subtraction: Take Away – Take Away Tens and Then Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 84 \\ -\ 30 \\ \hline 54 \end{array} \qquad \begin{array}{r} 54 \\ -\ 4 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

Symbolic representation

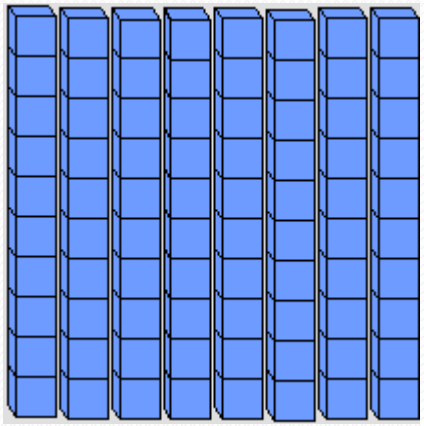
$$\begin{array}{rl} 84 & \\ -\ 37 & \\ \hline 54 & (-30) \\ 50 & (-\ 4) \\ 47 & (-\ 3) \end{array}$$

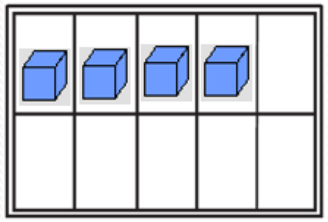
Subtraction: Take Away – A Variation

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

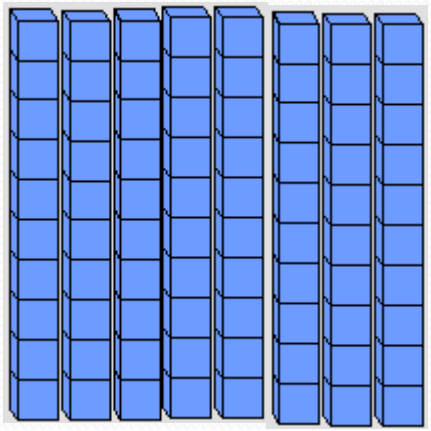
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

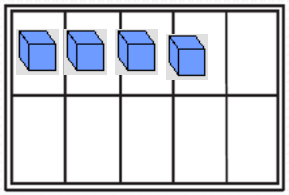




Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

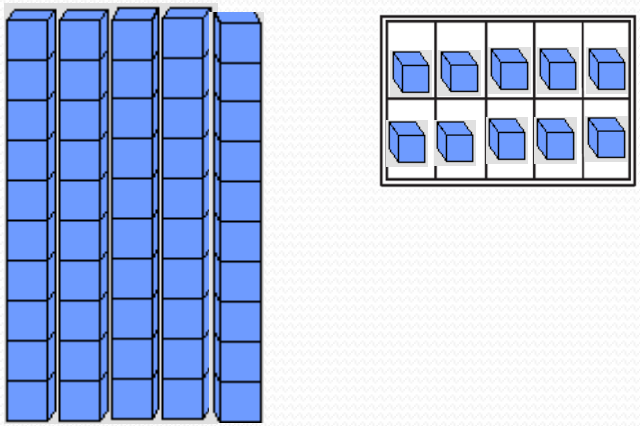




$$\begin{array}{r} 84 \\ -\ 4 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ -\ 30 \\ \hline 50 \end{array}$$

Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

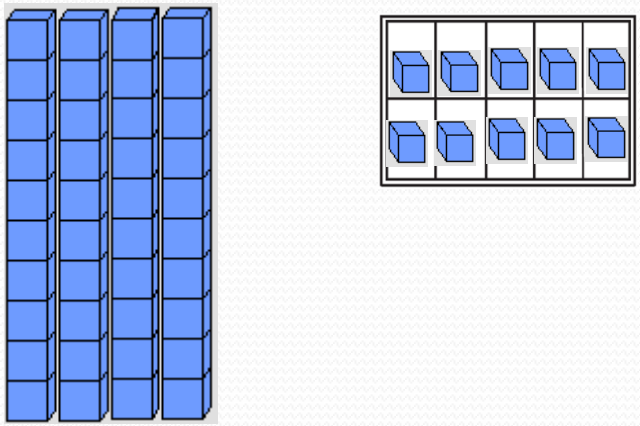
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 84 \\ -\ 4 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ -\ 30 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline \end{array}$$

Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



Symbolic representation

$$\begin{array}{r} 84 \\ -\ 4 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ -\ 30 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

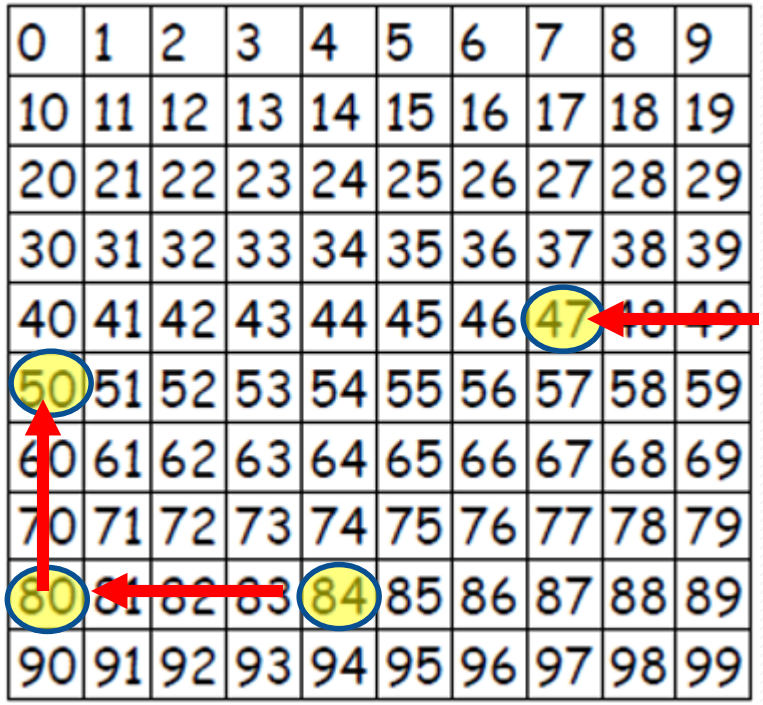
80 (- 4)

50 (-30)

47 (- 3)

Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Symbolic representation

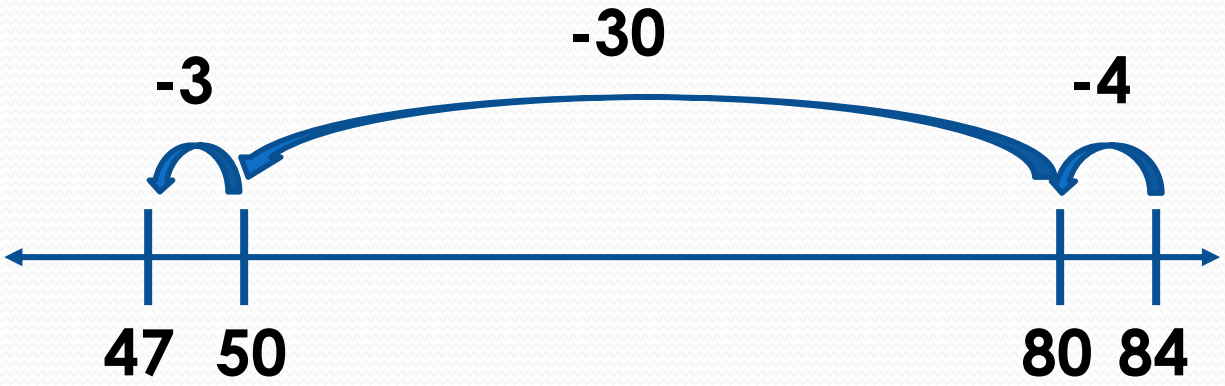
$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

80	(- 4)
50	(-30)
47	(- 3)

$$\begin{array}{r} 84 \\ -\ 4 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ -\ 30 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

Subtraction: Take Away, A Variation - Take Away Some Ones, Then the Tens, and Then the Rest of the Ones

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 84 \\ -\ 4 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ -\ 30 \\ \hline 50 \end{array} \qquad \begin{array}{r} 50 \\ -\ 3 \\ \hline 47 \end{array}$$

Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

80 (- 4)

50 (-30)

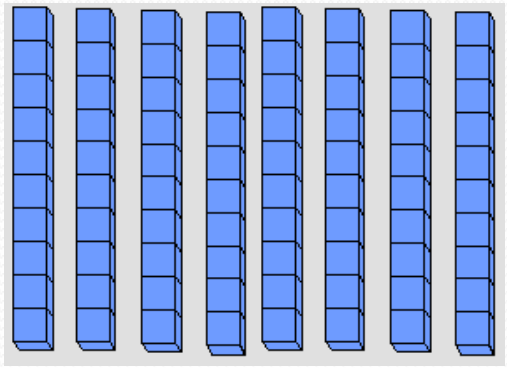
47 (- 3)

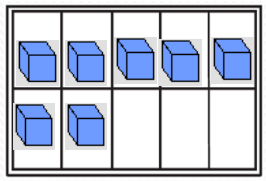
Subtraction: Take Away – Another Variation

- Base Ten Blocks
- Hundreds Chart
- Open Number Line
- Symbolic Representation

Subtraction: Take Away, Another Variation – Overshooting

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$





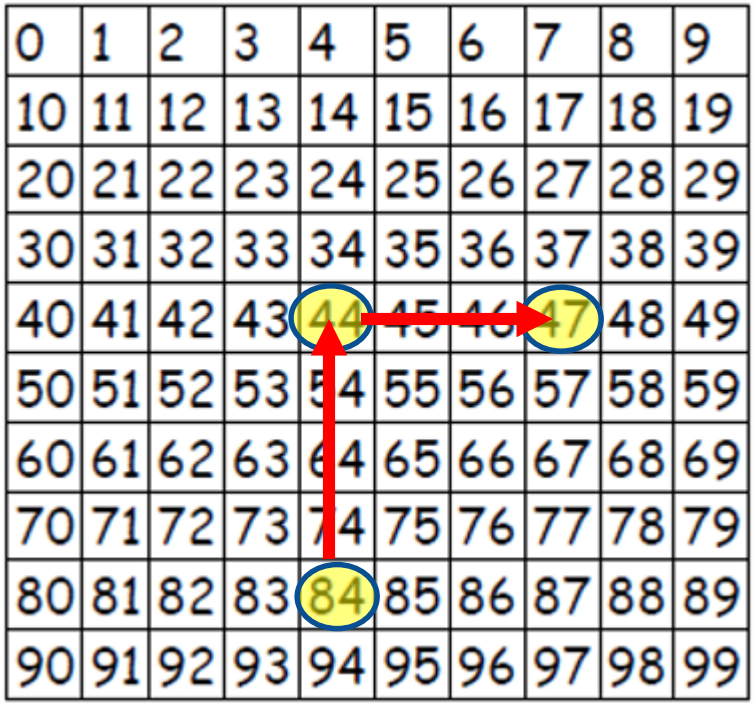
Symbolic representation

$$\begin{array}{rl} 84 & \\ -\ 37 & \\ \hline 44 & (-40) \\ 47 & (+\ 3) \end{array}$$

$$\begin{array}{r} 84 \\ -\ 40 \\ \hline 44 \end{array} \qquad \begin{array}{r} 44 \\ +\ 3 \\ \hline 47 \end{array}$$

Subtraction: Take Away, Another Variation – Overshooting

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



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50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Symbolic representation

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$

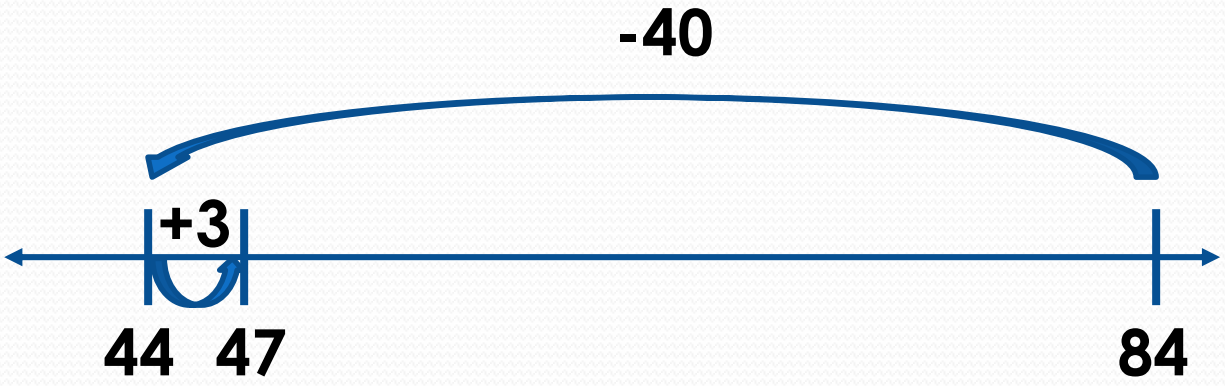
44 (-40)

47 (+ 3)

$$\begin{array}{r} 84 \\ -\ 40 \\ \hline 44 \end{array} \qquad \begin{array}{r} 44 \\ +\ 3 \\ \hline 47 \end{array}$$

Subtraction: Take Away, Another Variation – Overshooting

$$\begin{array}{r} 84 \\ -\ 37 \\ \hline \end{array}$$



$$\begin{array}{r} 84 \\ -\ 40 \\ \hline 44 \end{array} \qquad \begin{array}{r} 44 \\ +\ \ 3 \\ \hline 47 \end{array}$$

Symbolic representation

$$\begin{array}{rl} 84 & \\ -\ 37 & \\ \hline 44 & (-40) \\ 47 & (+\ 3) \end{array}$$

Questions

1. Which standards promote students' thinking in this way?

2. How did we develop the connections between the various tools, the student invented strategies, and the abstract computational recording of the strategies?

3. Which Mathematical Practices could be highlighted within lessons that promote student thinking?

Questions

4. Looking at the Effective Teaching Practice about fluency from Principles to Action, which teacher and student moves would you expect to see during a lesson that emphasize invented strategies and algorithms?

5. What other Effective Teaching Practices would you expect to surface in a lesson incorporating alternative strategies, such as those seen today?

6. Looking back at the computational strategies mentioned throughout the common core standards, how were they incorporated today?

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