Welcome
NCTM Annual Meeting & Exposition

Response to Intervention: Supporting Students Who Struggle in Mathematics

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Topics for Today

- Brief overview of **RtI Model**, one version of a multi-tiered system of support (**MTSS**)
- What helps students with disabilities build **cognitive structures** and **connections** in mathematics?
- Research based **Interventions** to try (not buy)
- **Diagnostic interviews** - a way to gather feedback on **students' mathematical thinking**
- **Strategies** for teaching math that **DON'T EXPIRE!!**

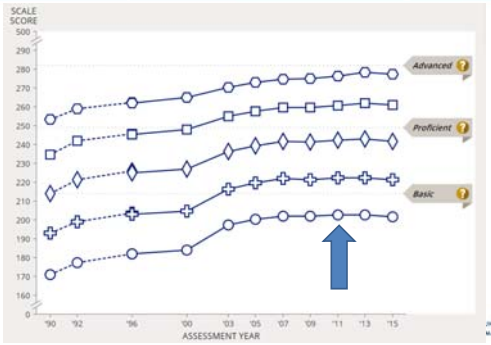
Foundational Questions

Content – what comes before the Common Core State Standards for Mathematics at your grade level?

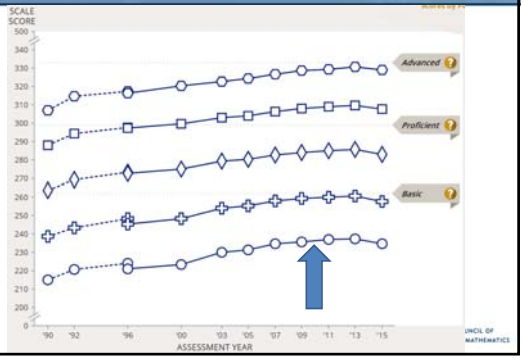
What are the foundational ideas in mathematics that students can build on? (not dead ends)

How do you teach these foundational concepts to students who struggle?

Has RtI Implementation Closed the Math Gap? Let's look at 4th grade.



Are we doing any better at 8th grade? NO



Why aren't Tier 2 Interventions Helping?

- Recent studies reveal that teachers providing Tier 2 mathematics interventions to elementary and middle grade students largely used **worksheets** (Foegen & Dougherty, 2010; Swanson, Solis, Ciullo & McKenna, 2012)
- In my travels to classrooms and schools many use a **one-size-fits-all generic computer program** (a worksheet on a computer).

Worksheets + computer programs ≠ understanding for students who struggle

Slide 6

3 you could probably say: "In my travels many others are using a one-size fits all drill and kill computer program.

Francis (Skip) Fennell, 1/20/2013

RtI in Reading
Practice Falls Short of Promise

- Studied 20,000 students in 13 states and found "first grade students who received RtI performed worse than a similar peer group that did not receive the interventions"
- Not adequately evaluating students in an effort to match intervention to the child
- RtI interventions were rigid and standardized for all students

Education Week – November 2015

What might a student's brain look like?

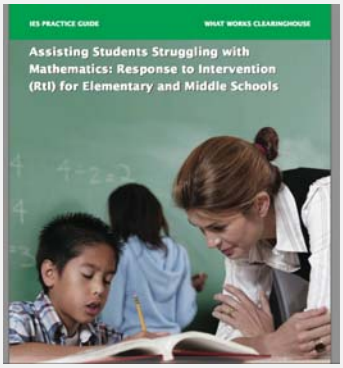
What if one student had a good understanding of a mathematical concept and the other student had just memorized it (or lacked the ability to memorize – like a student with disabilities)?

Task Analysis? Maybe Not?

- When you or the materials you use break skills down into small pieces, it requires students to put the pieces together to form the whole.
- Are students who struggle good at that?

Dougherty, 2012 *Access for All – Using Response to Intervention Techniques*, NCTM Algebraic Thinking Institute

Search for **IES** Practice Guides



Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC:* National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Recommendation
1. Universal screening (Tier 1)
2. Focus instruction on whole number for grades k-5 and rational number for grades 4-8
3. Systematic instruction
4. Solving word problems
5. Visual representations
6. Building fluency with basic arithmetic facts
7. Progress monitoring
8. Use of motivational strategies

Intervention Recommendations from Research

- **C**oncrete--**S**emi-Concrete--**A**bstract (CSA) approach
- Explicit instruction
- Underlying mathematical structures
- Examples (and counterexamples)
- Feedback – Not teacher to student but students' feedback to teacher on what they know and don't know

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.
Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* New York: Routledge.

So, What did you learn in school?

- With the person sitting next to or around you, decide if the rules shown at the right are always true.
- If the rule is not always true, find a counterexample.

- **Addition and multiplication make numbers bigger.**
- **When you multiply by 10, just put a 0 on the end of the number.**
- **The longer the number, the larger the number.**

Addition and multiplication make "bigger"

32 + 67 = 99

15 x 10 = 150

– 3 + (–14) = –17

4 x 0 = 0

15 + 0 = 15

$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$

When you multiply by 10, just put a 0 on the end of the number.

15 x 10 = 150

4.5 x 10 = 45.0

4.5 x 10 ≠ 4.50

The longer the number, the larger the number.

1,278,931 > 1,469

1.3 > 1.0118743

1.02 < 1.2

Impact of Rules

- Students use rules as they have interpreted them.
- They often do not think about the rule beyond its application.
- When even the best students find that a rule doesn't work, it is unnerving and scary.

Goal – Try to AVOID DEAD ENDS

"13 Rules that Expire" (Karp, Bush & Dougherty August 2014 in *Teaching Children Mathematics*) TCM article of the YEAR!

What do we know?

- Telling isn't teaching.
- Told isn't taught.
- Explicit instruction isn't telling.

So, Karp

What's an example of your so called - explicit teaching based on structure, examples, concrete, semi-concrete, abstract understanding that is not "telling" and will reach my many students who are struggling bringing them to higher levels of mathematical understanding through the creation of blue lines?

Let's Start with Word Problems

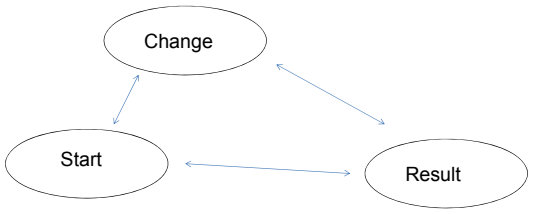
At all grades students who struggle see each problem as a **separate endeavor**

They **focus on steps** to follow rather than the behavior of the operations

They tend to use **trial and error** – (disconnected thinking – not relational thinking)

They need to focus on **actions**, **representations** and **general properties of the operations**

Explicit Schema for Additive Structures



Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.
Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

CCSSM Appendix – Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

Put Together/ Take Apart[2]	Total Unknown	Addend Unknown	Both Addends Unknown[3]
	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

Creating Mental Residue

- Establishing foundational understanding
- Modeling the physical action is the important part and doesn't go away
- Acting and "doing" the process supports students' thinking about the operation

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahwah, NJ: Erlbaum.

Other options?

Would your students be able to discern which of the following three options would be the correct answer?

- The shepherd is 30 years old
- The shepherd is 125 years old; and
- It is not possible to tell the shepherd's age from the information given in the problem.

Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2.* NCTM.

Danger - Key Words ahead

Mark has 3 packages of pencils. There are 6 pencils in each package. How many pencils does he have in all?

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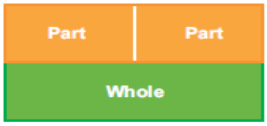
The Myth of Keywords

- Keywords do not—
  - Develop of sense making or support making meaning
  - Build structures for more advanced learning
  - Appear in many problems
- Students consistently use key words inappropriately
- Multi-step problems are impossible to solve with key words (and two step problems start in 2nd grade)

Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson

Part-Part- Whole Problem Structure



Two parts that are combined into one whole

Lynnette has 4 fiction and 3 nonfiction books. How many books does she have?

Lynnette and her friend Victoria put 7 books into a backpack. Lynnette put in 4 books. How many books did Victoria put in the backpack?

Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson

Which number sentences would students say are True? False?

7 = 7

2 + 5 = 4 + 3

5 + 1 = 7

7 = 2 + 5

- Which equation formats would confuse your students?

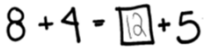
Diagnostic Interviews for Progress Monitoring

Give a task - collect students' mental strategies

No instruction – just ask questions

Capture student feedback on their thinking

Let students use tools for demonstrating reasoning



Use information gathered to improve instruction

Equal Sign - Two Levels of Understanding

Operational - students mistakenly see the equal sign as signaling something they must "do" with the numbers such as "give me the answer."

Relational - students use the relationships between the two quantities to balance the sides of the equation.

- Do students use relational thinking to generalize rather than actually computing the individual amounts?
- Do students see the equal sign as relating to the symbols for "greater than," "less than," and "not equal to?"

Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson

Well-Child Checkup

- All students need to have their temperature taken on this important concept
- Must identify students who harbor this misconception about the equal sign

What is the long term danger?

- If middle grades students think the equal sign means "the answer is coming up next," what happens when they move to algebraic equations such as $3x = 2x + 3$?

Common Core State Standards

Grade 1:
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Let's go back to Tier 1 Core Instruction Headline – *New York Times*

July 27, 2014

(New Math) – (New Teaching) = Failure

Why do Americans Stink at Math - Green

What were the Main Points?

- Common Core State Standards – Fosters intuitive thinking through **real-world examples** – that **is** the best way to teach mathematics
- Real Problem – Teachers are being asked to teach in ways they've **not experienced** as students – or have **not been effectively taught**
- America invented the best ways of teaching math as espoused by NCTM and supporting research – yet **not enough teachers are using these methods**

NCTM's *Principles to Actions* - Mathematics Teaching Practices for Effective Teaching and Learning

- **Establish mathematics goals to focus learning**.
- **Implement tasks that promote reasoning and problem solving**.
- **Use and connect mathematical representations**.
- **Facilitate meaningful mathematical discourse**.
- **Pose purposeful questions**.
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics**.
- **Elicit and use evidence of student thinking.**

Another Option - from Magdalene Lampert

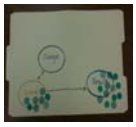
- **You** – Start the problem (on your own)
- **Y'All** – Share ideas with peers
- **We** – Share ideas and strategies with the whole class – including misunderstandings (let's not wait for the test to find out what we don't know)

What is the Whole School Agreement?

- Decide on the language and models everyone will use – be precise and consistent
- Prepare all students, from the beginning to walk out of the building
- Think about the level of teaching – are challenging students at the highest level?
- Get kids "doing mathematics" so they can build mental residue and long lasting understanding

Karp, Bush & Dougherty (in press) *Establishing The Whole School Agreement.* NCTM.

Agree on Language, Models, and Notation Across Your School

- Language ~~Borrowing/Carrying~~ → Regrouping/Trading
- Models  → 
- Notation $4 + 5 = \square$ → $17 + x = 26$

Shifts in Thinking

- Teacher talking and doing TO **students talking and doing – never say anything a kid can say**
- Using key words TO **building student understanding with reasoning and sense making**
- Learning disconnected rules and algorithms TO **engaging students in productive struggle with rich, high-quality problems**

An Article coming soon – to a TCM journal near you!!!

Establishing The Whole School Agreement

Karp, Bush & Dougherty – In press in **Teaching Children Mathematics**

Did you notice we focused on BIG IDEAS?

Interventions should emphasize BIG IDEAS THROUGH Explicit instruction:

- Structure of word problems
- Meaning of the operations
- Meaning of symbols – equal sign