Challenging all students with cognitively-demanding tasks: Samples and key insights

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The shopping problem

- A man goes into a shop and says to the owner: "give me as much money as I have with me and I will spend \$10". It is done, and the man does the same thing in a second and third store, after which he has no money left. How much did he start with?

We read the problem and thought hard.
Most people thought $30, so we made lots of $10 notes and acted out the problem with one man and three shop owners.

$30 30+30=60-10=50

50+50=100-10=90

90+90=180-10=170

$20 20 + 20 = 40 - 10 = 30

30 + 30 = 60 - 10 = 50

50 + 50 = 100 - 10 = 90

$15. 15 + 15 = 30 - 10 = 20

20 + 20 = 40 - 10 = 30

30 + 30 = 60 - 10 = 50

We saw that as we went along with each problem the numbers were getting bigger.

All the numbers are too big — we'll try 10

$10 10+10=20−10=10

10+10 = 20 − 10 = 10

10+10 = 20 − 10 = 10

We are standing still.

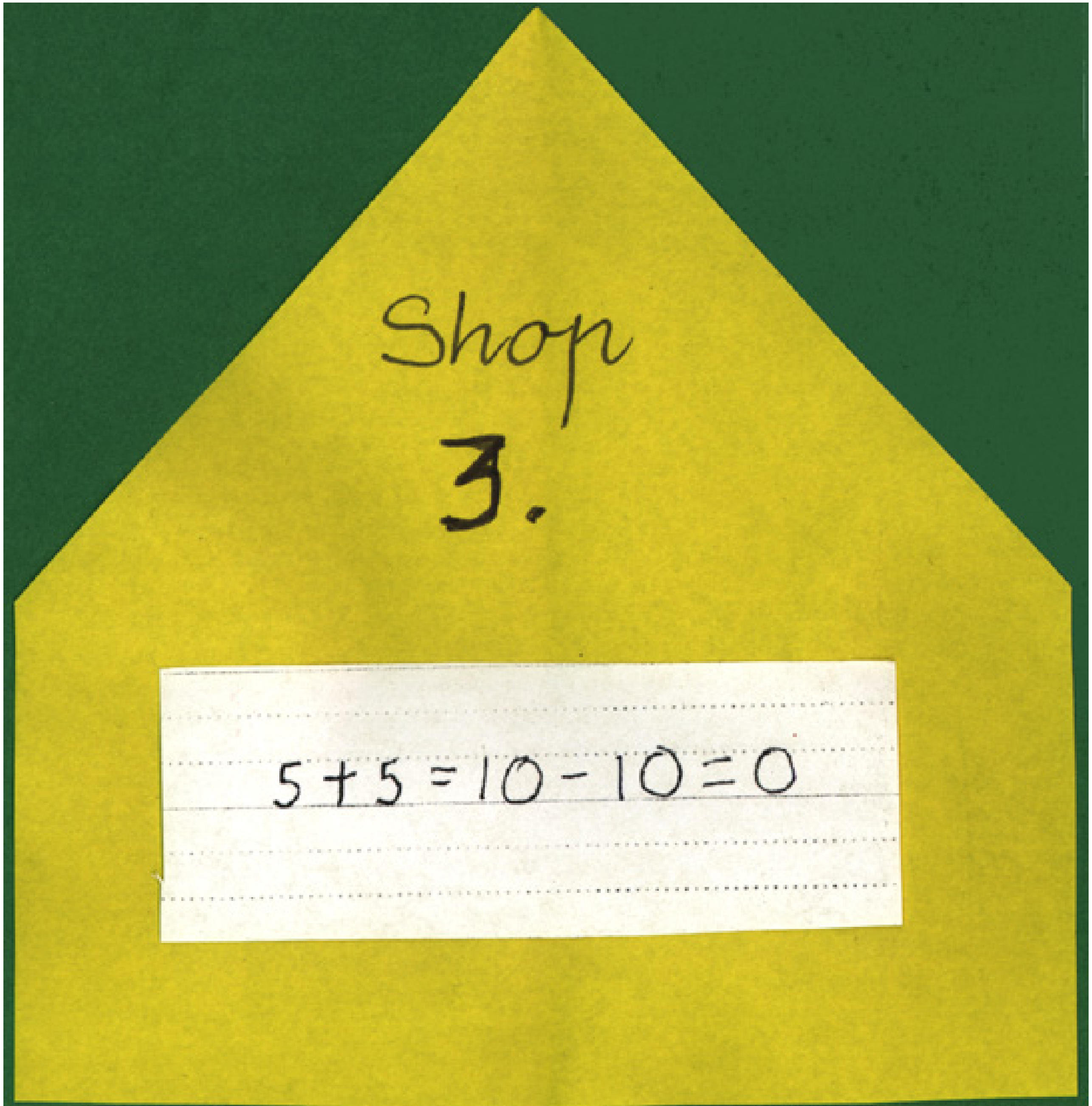
Ten is the closest so far but we need less left over.

What if we try 0?

0 + 0 = 0 − 10 We can't do this because there isn't enough. You can't spend $10 if you don't have anything.

5 Could be the answer because
5+5 = 10 − 10 = 0
This would have to be at the last shop.

We'll have to work backwards.


Shop
3.
5+5=10-10=0

Shop 2.

Our answer will need to be 5

[?] − 10 = 5

[15] − 10 = 5

We will have to use 50¢ to halve $15

$7.50 + $7.50 = 15

15 − 10 = 5.

Shop

1.

We need \$7·50 for our answer.

\$7·50 + \$10 = \$17–50.

\$17·50 – \$10 = \$7–50.

\$17·50 is close to \$18.

½ of 18 = 9.

We have 50 cents too much

We'll have to take something from each

\$9 00 to equal 50 cents.

We'll need to take 25 cents from each.

\$9 00 - 25 cents = \$8 ·75

We found out that the man started with \$8·75.

It took a lot of thinking and working out.

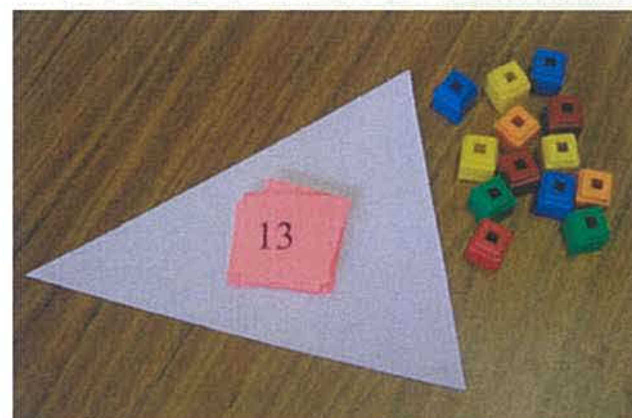
Encouraging Perseverance in Elementary Mathematics: A Tale of Two Problems

According to professor Alan Schoenfeld, many students hold the following beliefs about mathematics:

- Mathematics problems have one and only one correct answer.
- There is only one correct way to solve any mathematics problem—usually the rule that the teacher has most recently demonstrated to the class.
- Ordinary students cannot be expected to understand mathematics; they simply memorize it and apply what they have learned mechanically.
- Mathematics is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics

Figure 1

The materials that students used



Sometimes when students struggle with a mathematics task they choose not to persist. What kinds of things do you believe a teacher could do in the planning stage of a lesson and during the lesson that would help those students to persist? ***Please record as many as you can.***

1. In the planning stage, teachers could ....

•	
•	
•	
•	
•	
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•	

2. During the lesson, teachers could ....

•	
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•	

Productive Struggle

- Struggle is important for students if real learning is to take place. As Hiebert and Grouws (2007) noted, "we use the word *struggle* to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do *not* use *struggle* to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems" (p. 387).

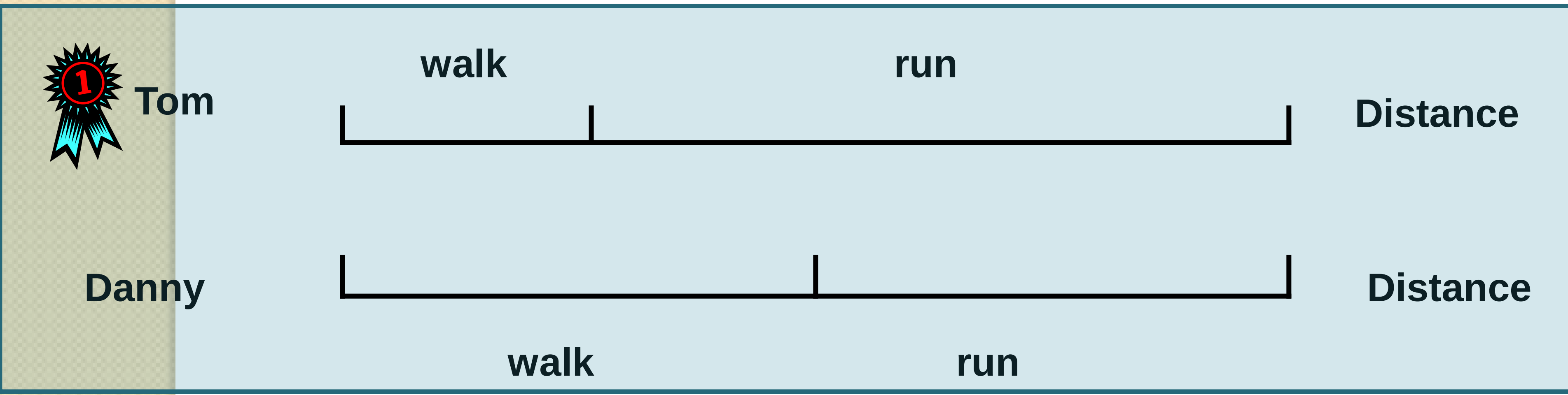
- Pogrow (1988) warned that by protecting the self-image of under-achieving students through giving them only "simple, dull material" (p. 84), teachers actually prevent them from developing self-confidence. He maintained that it is only through success on complex tasks that are valued by the students and teachers that such students can achieve confidence in their abilities. There will be an inevitable period of struggling while the students begin to grapple with problems but Pogrow asserted that this "controlled floundering" is essential for students to begin to think at higher levels.

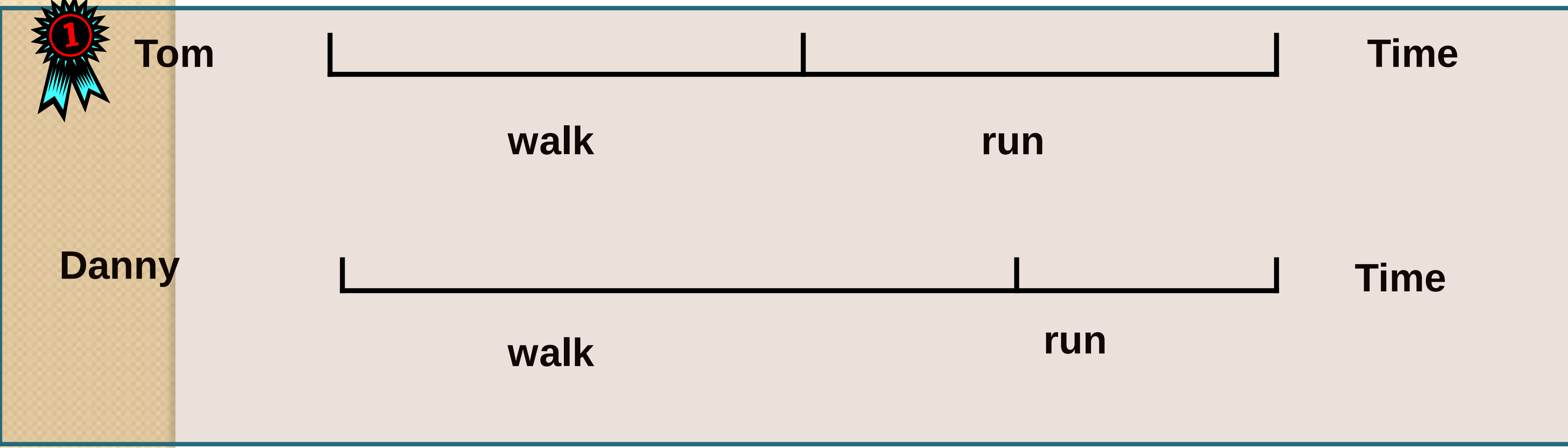
“The zone of confusion”

- **“I’m in the zone”**

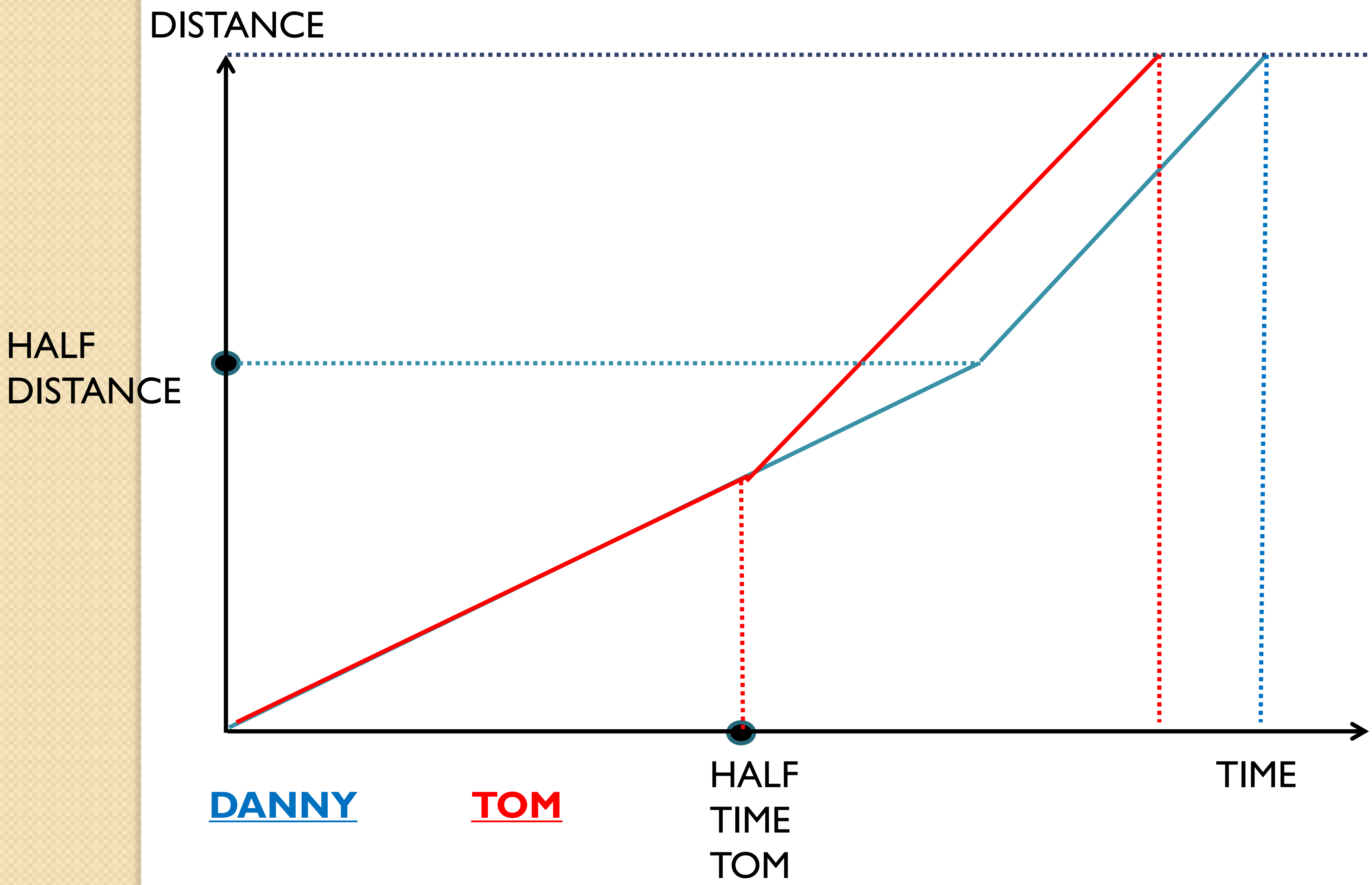
Danny and Tom

- Danny and Tom travelled from Acton to Beamsville on foot.
- Danny walked half the distance and ran half the distance.
- Tom walked half the time and ran half the time.
- They started at the same time, and walked at the same speed as each other and ran at the same speed as each other.
- Who arrived first, or was it a tie?


walk
run
Tom
Distance
Danny
Distance
walk
run


Tom
Time
walk
run
Danny
Time
walk
run

Danny and Tom



The Fields Medal: the greatest honour a mathematician can receive.

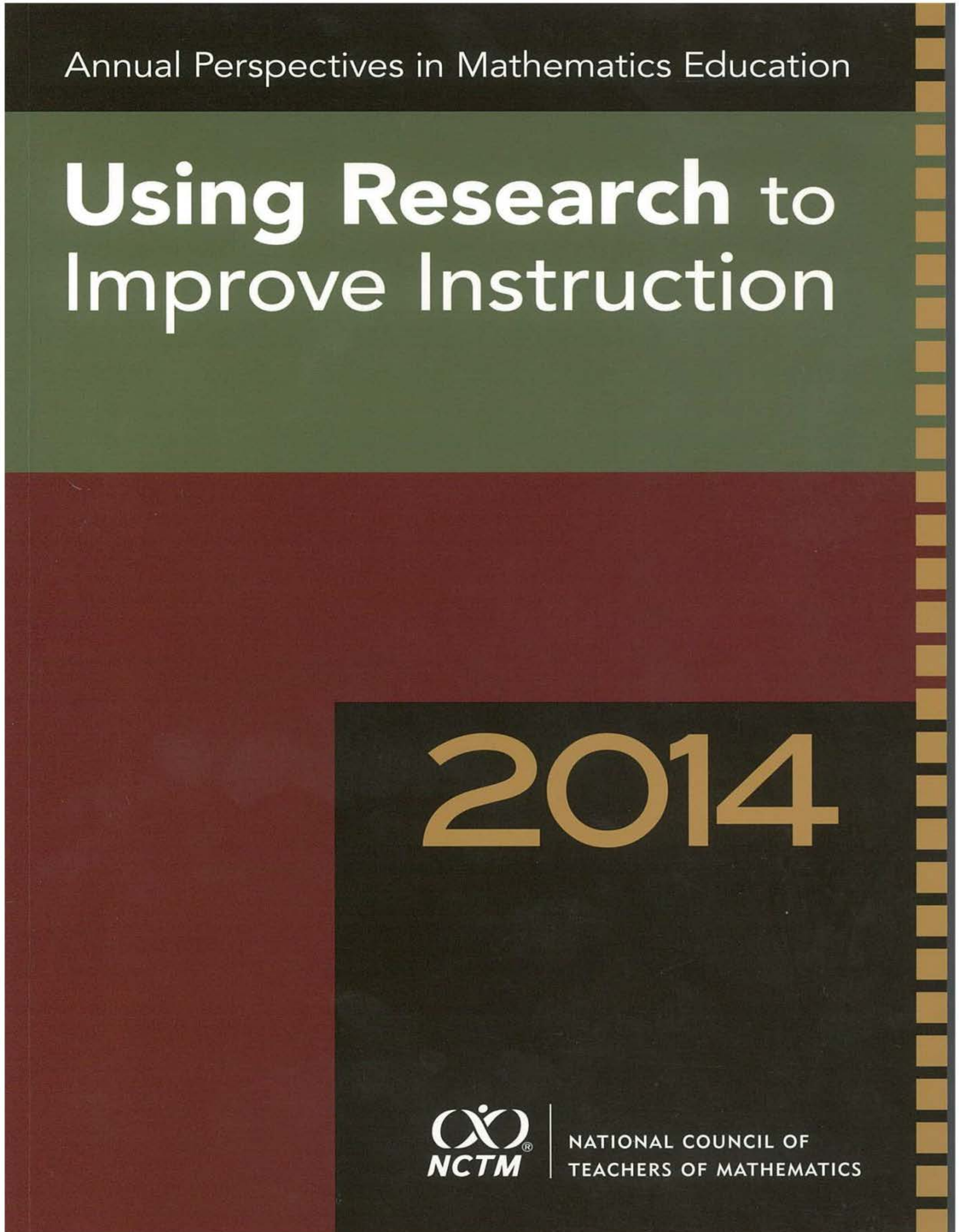
Awarded to two to four people in the world every four years.

Maryam Mirzakhani: the first woman to win the Fields Medal (2014)

What do you find most rewarding or productive?

Of course, the most rewarding part is the "Aha" moment, the excitement of discovery and enjoyment of understanding something new - the feeling of being on top of a hill and having a clear view.

But most of the time, doing mathematics for me is like being on a long hike with no trail and no end in sight.
I find discussing mathematics with colleagues of different backgrounds one of the most productive ways of making progress.


Annual Perspectives in Mathematics Education
Using Research to Improve Instruction
2014
NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Creating a Classroom Culture That Encourages Students to Persist on Cognitively Demanding Tasks

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Anne Roche, *Australian Catholic University (Melbourne Campus)*
Peter Sullivan, *Monash University (Clayton Campus), Australia*
Jill Cheeseman, *Monash University (Peninsula Campus), Australia*

For many years, the importance of problem solving in mathematics education has been well recognized. As Thompson and colleagues (2009) noted, "Good problems challenge students to develop and apply strategies, serve as a means to introduce new concepts, and offer a context for using skills. Problem solving is not a specific topic to be taught but perme-

Division with remainders

Activity sheet ✎ Division with Remainders

1 Work out the answers to each of these problems.
You can use a calculator, but you must explain your reasoning.
(Hint: all the answers are different in some way)

a. We need to book buses to take all the students in the school to a concert. There are 1144 students and each bus can take 32 students. How many buses do we need to order?

Answer:

Our reasoning:

b. We are making up packets of chocolates. Each packet must have exactly 32 chocolates. If I have 1144 chocolates, how many complete packets can we make?

Answer:

Our reasoning:

c. Our basketball club won a prize of $1144. The 32 members decided to share the prize exactly between them. How much money will each of them get?

Answer:

Our reasoning:

d. The train from Melbourne to Sydney travels at an average speed of 32 km/hr. How long would it take to travel the 1144 km to Sydney if the train does not stop?

Answer:

Our reasoning:

e. Our year level of 32 students together won a prize of 1144 pizzas. If we share the prize equally, how much pizza do we each get?

Answer:

Our reasoning:

f. A dairy farm produced 1144 litres of milk, and has 32 containers in which to store the milk. If the containers are filled exactly, how much milk should go into each container?

Answer:

Our reasoning:

g. There are 1144 people who need to cross a crocodile infested river. The ferry can carry 32 people each trip. If everyone is in a hurry to cross the river, how many people will be left for the last trip?

Answer:

Our reasoning:

2 In what ways are these problems similar to each other?

3 In what ways are these problems different from each other?

4 Make up another problem to add to this set.

“I THINK”

Division with remainders- answers

- A) 36 buses
- B) 35 packets
- C) $35.75
- D) 35 hours and 45 minutes
- E) 35 $\frac{3}{4}$ pizzas
- F) 35.75 litres
- G) 24 people

Year 5 students

1. Work out the answers to each of these problems. You can use a calculator, but you must explain your reasoning. (Hint: all the answers are different in some way)

a. We need to book buses to take all the students in the school to a concert. There are 1144 students and each bus can take 32 students. How many buses do we need to order?

Answer: 36 r 8

Our reasoning: We time

32 x 32 = 1024 - too low

32)1144

35 x 32 = 1120 too low

36 x 32 = 1152 - too high

there are 36 buses

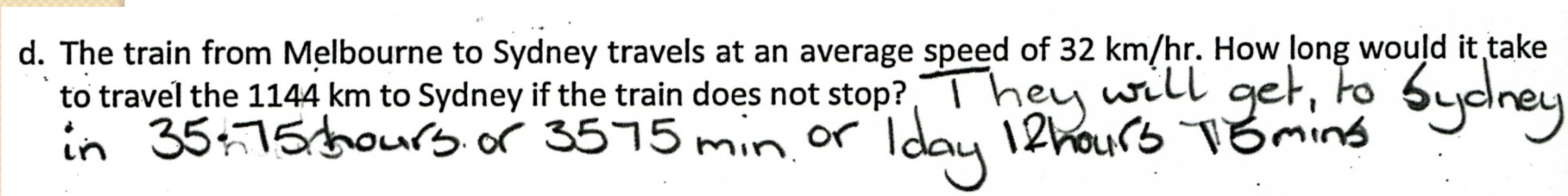
b. We are making up packets of chocolates. Each packet must have exactly 32 chocolates. If I have 1144 chocolates, how many complete packets can we make?

Answer: 35 packets

Our reasoning: we divied 1144 to 32 and we got 35.75 we didn't use the 3 quarters because it said that each packet must have 32 chocolates.

c. Our basketball club won a prize of $1144. The 32 members decided to share the prize exactly between them. How much money will each of them get?

35 dollars and. 2.34375c



d. The train from Melbourne to Sydney travels at an average speed of 32 km/hr. How long would it take to travel the 1144 km to Sydney if the train does not stop? They will get to Sydney in 35.75 hours. or 3575 min. or 1day 12hours 75mins

Three different answers that are not equivalent!!

'THINK' Framework

Get students to use mnemonic to help them to improve their problem solving.

- ***Talk*** about the problem
- ***How*** can it be solved?
- ***Identify*** a strategy to solve the problem.
- ***Notice*** how the strategy helped you solve the problem.
- ***Keep*** thinking about the problem. Does it make sense? Is there another way to solve it?

Thomas, K. R. (2006). Students THINK: A framework for improving problem solving. *Teaching Children Mathematics, 13*(2), 86-95.

Framework "I THINK"

- ***I*** **–** independent thinking
- ***Talk*** about the problem
- ***How*** can it be solved?
- ***Identify*** a strategy to solve the problem.
- ***Notice*** how the strategy helped you solve the problem.
- ***Keep*** thinking about the problem. Does the solution make sense? Is there another way to solve it?

Thomas, K. R. (2006). Students THINK: A framework for improving problem solving. *Teaching Children Mathematics, 13*(2), 86-95.

DOING ADDITION AND SUBTRACTION WITH A BROKEN CALCULATOR

In each of these calculations, pretend that you are using a calculator that has the "4" button broken. In each case, show the buttons you would press to work out the answer.

Write down two different methods for each one.

341 + 276

361 - 274

Enabling prompt(s) for students experiencing difficulty

- How could you do these on a calculator if the '4' button is broken?

- 4 + 7 14 - 2

Extending prompt(s) (for those who finish quickly)

- Explain how you could work these out on a calculator with both the ‘4’ and the ‘5’ button broken (use the calculator to check that you are correct).

- 274 + 345 742 - 345

CALCULATOR WITH A BROKEN BUTTON (PART 1)

NAME:____

In each of these calculations, assume that you are using a calculator that has the "4" button broken. In each case, give two different methods.

341 + 276

Method 1	Method 2

“It’s better to solve one problem five different ways than to solve five different problems.”

George Polya, 1965

CALCULATOR WITH A BROKEN BUTTON (PART 2)

In each of these calculations, assume that you are using a calculator that has the "4" button broken. In each case, give two different methods.

421 + 256

Method 1	Method 2

441 + 246

Method 1	Method 2

441 – 257

Method 1	Method 2

441 – 254

Method 1	Method 2

Music Cards

Which card is the better value?

Give two different mathematical justifications for your answer.

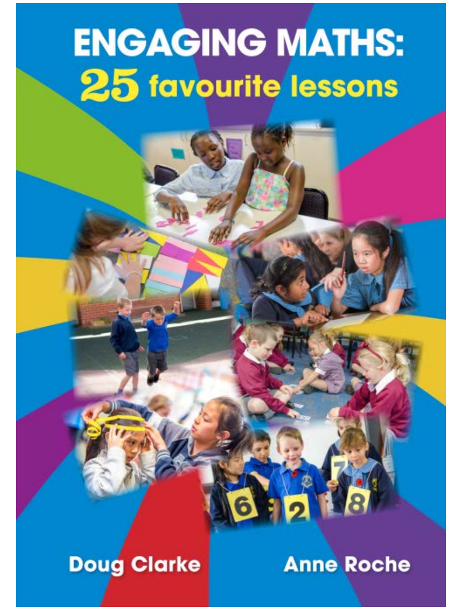
Activity sheet ✎ Music Cards

Which music card is the better value? Please explain your thinking. If possible, solve this problem in two different ways.

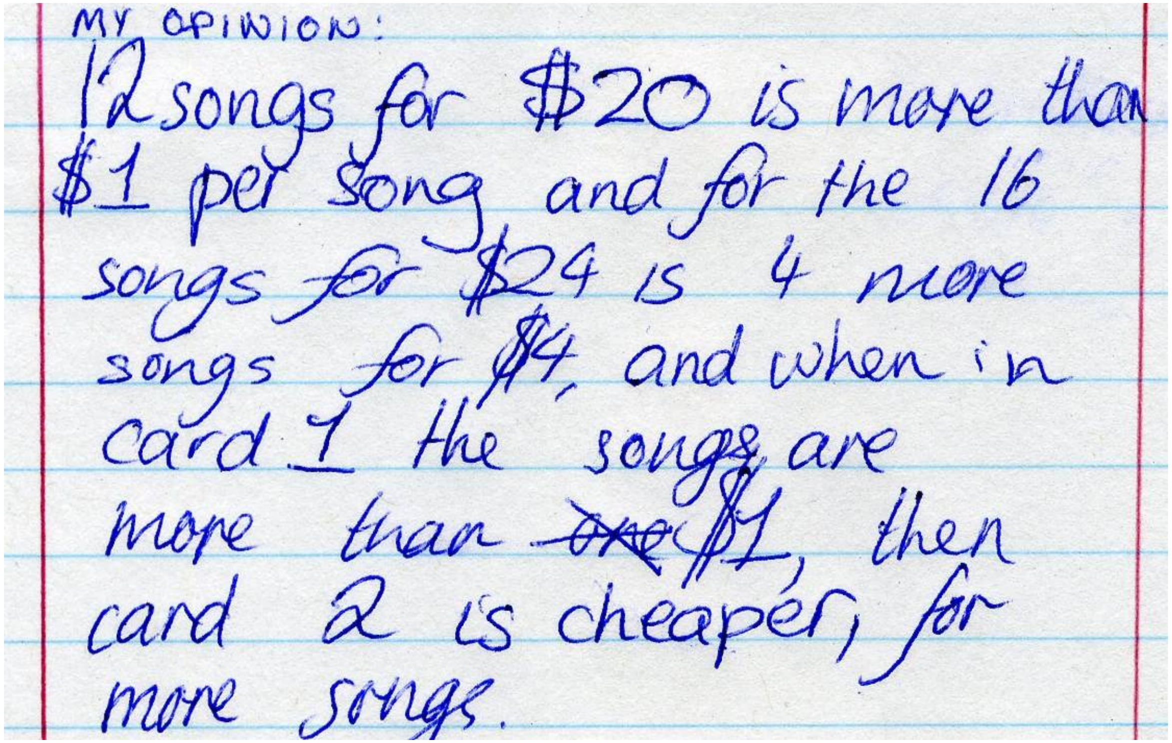
Answer: MySongs MyTunes Same value *(Please circle one)*

First method:

Second method:



Four for four improves the average price



MY OPINION:
12 songs for $20 is more than $1 per song and for the 16 songs for $24 is 4 more songs for $4, and when in card 1 the songs are more than ~~one~~ $1, then card 2 is cheaper, for more songs.

Unitising (per song)

$20 ÷ 12 songs = $1.66

$24 ÷ 16 songs = $1.50

Divide the ~~number~~ money by the songs and the cheapest one will be your answer.

Unitising (per 48 songs)

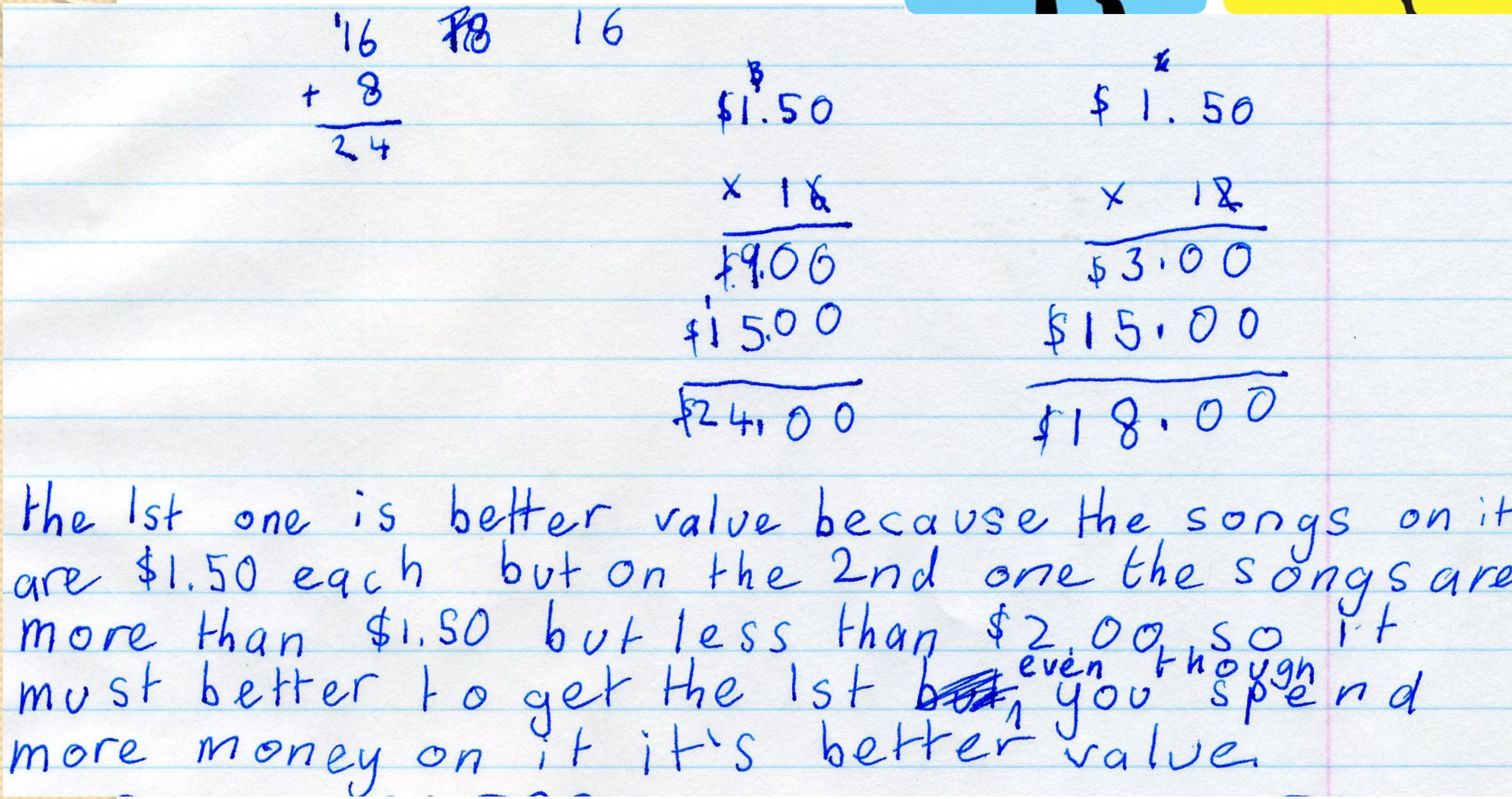
well 16 x 3 = 48

so $24.00 x 3 = $72.00

then 12 goes into 48 4 times

so $20.00 x 4 = $80.00

Compares to Pod Tunes



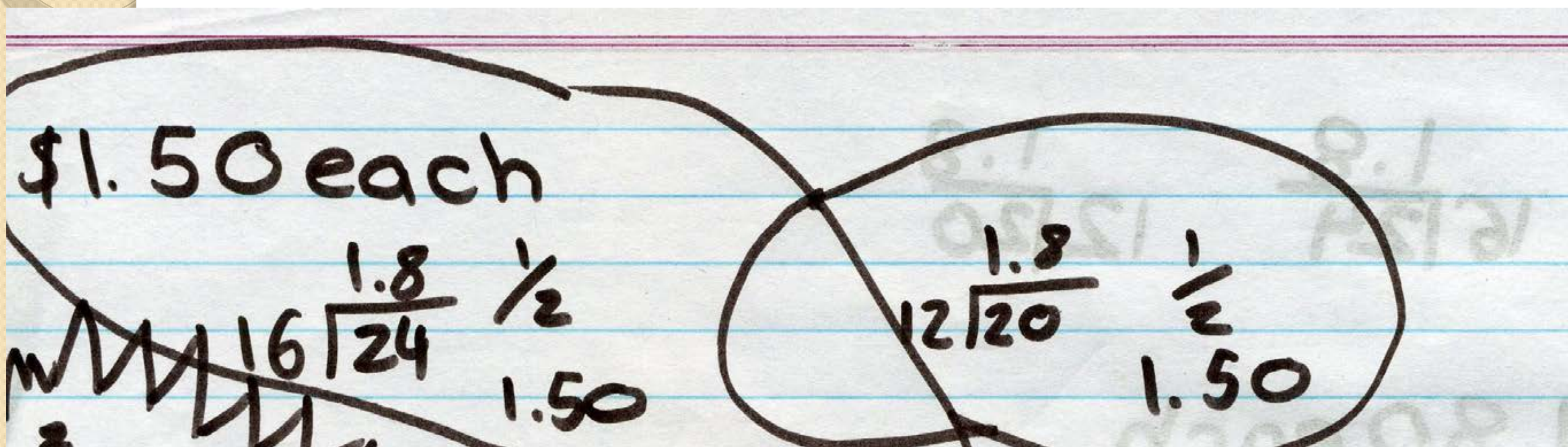
16 + 8 = 24

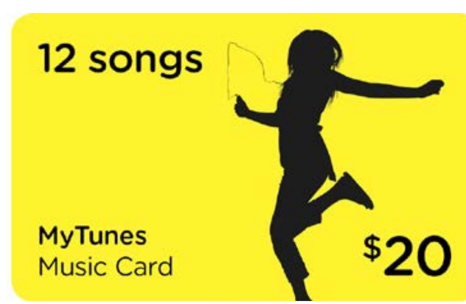
$1.50 × 16: $9.00 + $15.00 = $24.00

$1.50 × 12: $3.00 + $15.00 = $18.00

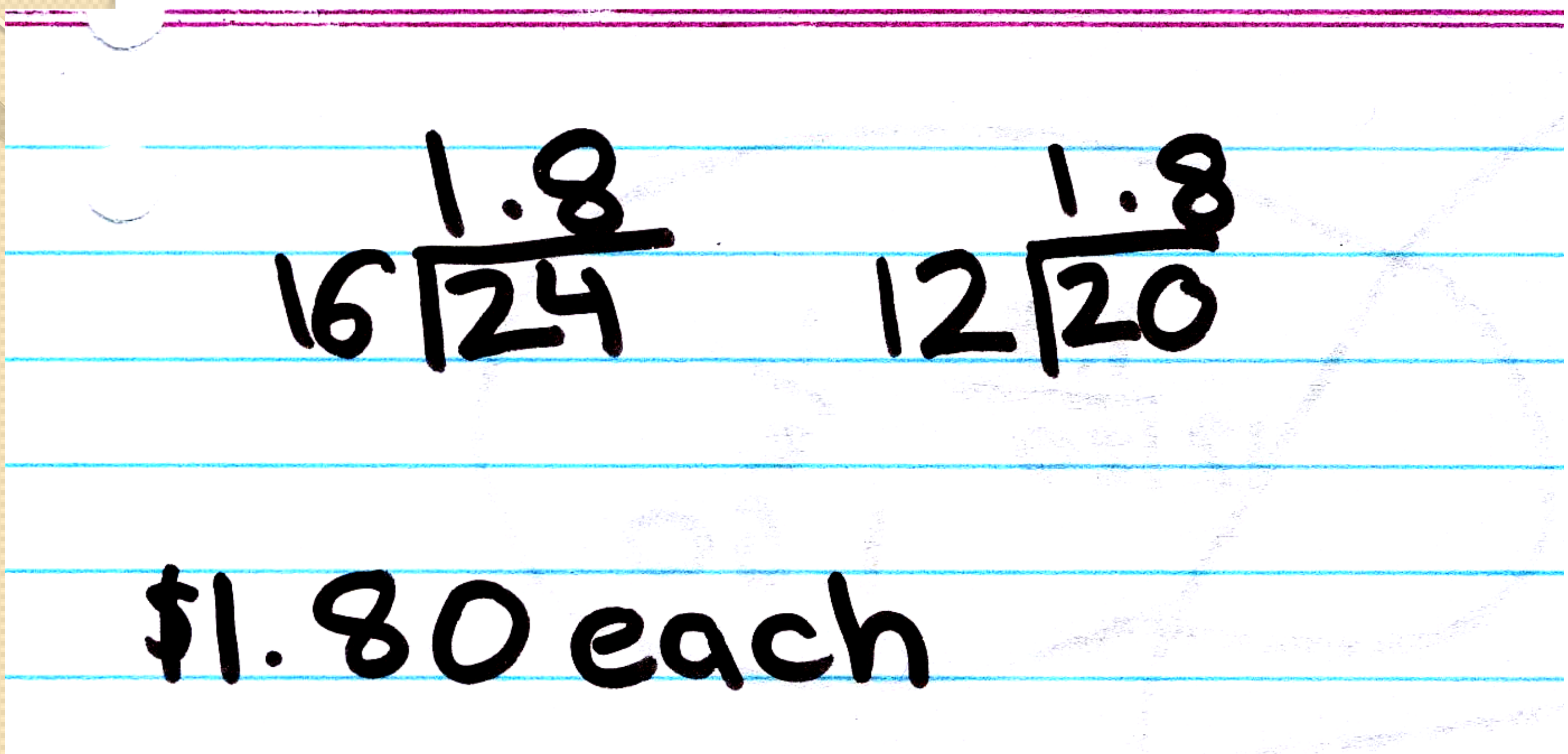
the 1st one is better value because the songs on it are $1.50 each but on the 2nd one the songs are more than $1.50 but less than $2.00 so it must better to get the 1st ~~but~~ even though you spend more money on it it's better value.

Incorrect use of remainders





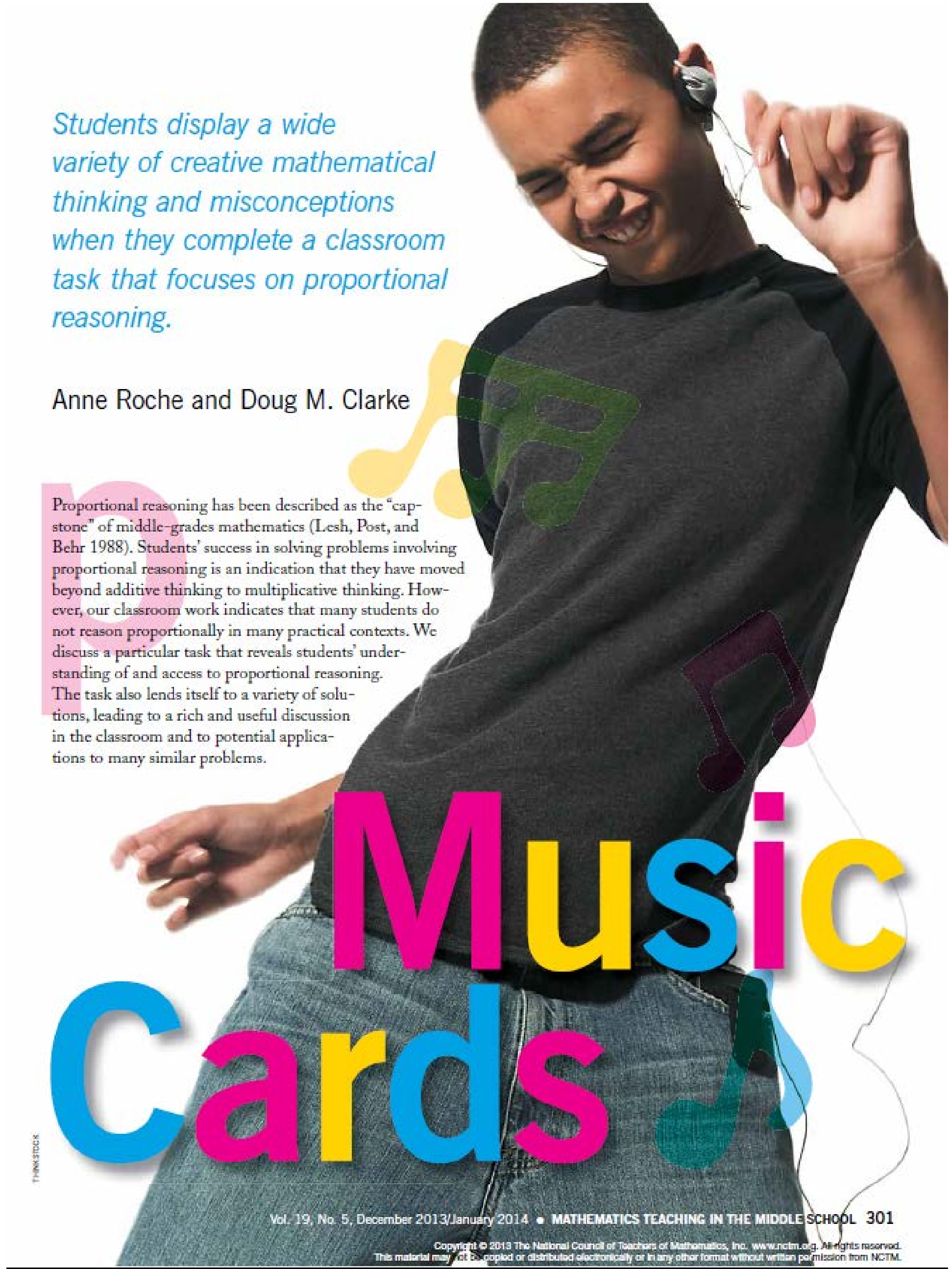
Incorrect use of remainders



Additive Thinking

The Same!

because the difference between the MySongs music card & MyTunes music card is 4.
And the difference between the money is 4.
The difference between the songs & the money is 8 on both. (12&20 and 16&24)



Music Cards

Students display a wide variety of creative mathematical thinking and misconceptions when they complete a classroom task that focuses on proportional reasoning.

Anne Roche and Doug M. Clarke

Proportional reasoning has been described as the "capstone" of middle-grades mathematics (Lesh, Post, and Behr 1988). Students' success in solving problems involving proportional reasoning is an indication that they have moved beyond additive thinking to multiplicative thinking. However, our classroom work indicates that many students do not reason proportionally in many practical contexts. We discuss a particular task that reveals students' understanding of and access to proportional reasoning. The task also lends itself to a variety of solutions, leading to a rich and useful discussion in the classroom and to potential applications to many similar problems.

THINKSTOCK

SECTION 3: CHANGES TO PLANNING

In this project, you have trialled a number of challenging mathematics tasks and encouraged students to persist when working on them. We are interested in what you believe is the most important change in your practice that contributes to students persisting, both in the planning stage and during the lesson.

1. In terms of your planning

a. Please describe **one** aspect of **your planning for these lessons,** *that is different from the way you planned previously,* and which you believe has helped some students to persist.

b. Please explain how you know it has helped

2. In terms of your teaching

a. Please describe one aspect of your **teaching behaviour during the lessons**, *that is different from the way you taught previously*, and which you believe has helped some students to persist.

Primary – In the planning stage

(“students” removed)

- 35 teachers

Primary – During the lesson

("students" removed) 35 teachers

Comments made about "Time"

- Sit in the zone for a longer period of time
- Time to think
- More time for enquiry learning
- Less teacher talk time
- Allowing time for students to solve the problems without interfering
- Don't over teach during working time
- Giving them time to discuss with other children
- Students share more of their thinking more of the time
- Making and trying to allocate time to the summary phase
- Give more time to the share/summary [phase]
- Allow students thinking time
- Thinking time
- Thinking time
- Question time
- Discussion time

In summary, you do differently:

- holding back from telling;
- allowing some time for students to think by themselves and work on the task;
- allowing students to share ideas and work collaboratively;
- discussing the behaviours of persistence (being in the zone of confusion, risk taking, not giving up, facing the challenge);

In summary, you do differently:

- providing challenging problem solving tasks that engage students;
- planning for and using enabling and extending prompts; and
- working through the tasks in advance of the lesson.

Encouraging persistence as students work on challenging mathematics problems

It seems desirable that:

- Tasks are chosen which have the potential to engage students in worthwhile, challenging and interesting mathematics;
- the ways of working are explained to the students, including the type of thinking in which they are expected to engage and what they might later report to the class;
- some attempt is made to connect the task with the students' experience;

EPMC
Encouraging Persistence
Maintaining Challenge

- the teacher communicates enthusiasm about the task, including encouraging the students to persist with it;
- students have a choice of ways that they can approach the task, and perhaps on the level of difficulty of the task itself;
- processes and expectations for recording are made clear;
- there is time allowed for lesson review so that students see the strategies of other students and any summaries from the teacher as learning opportunities.

EPMC
Encouraging Persistence
Maintaining Challenge

- the teacher moves around the class, predominantly observing students at work, selecting students who might report and giving them a sense of their role, intervening only when necessary to seek clarification of potential misconceptions, to support students who cannot proceed, and to challenge those who have completed the task.

EPMC
Encouraging Persistence
Maintaining Challenge

Teacher Questioning

A question about questioning

- What are effective questions that you use in general to probe children's thinking and engage them in dialogue?

Useful Questions

- How did you work that out?
- Could you do that a different way? … a quicker way?
- What is the same and what is different about …?
- What am I going to ask you next?
- Where have you seen something like this before?
- What's your "brain picture"? What do you see in your mind when I say …


31
28
29
30
NON SMOKING FLOOR
25
26
27
22
23
24
NON SMOKING FLOOR
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B
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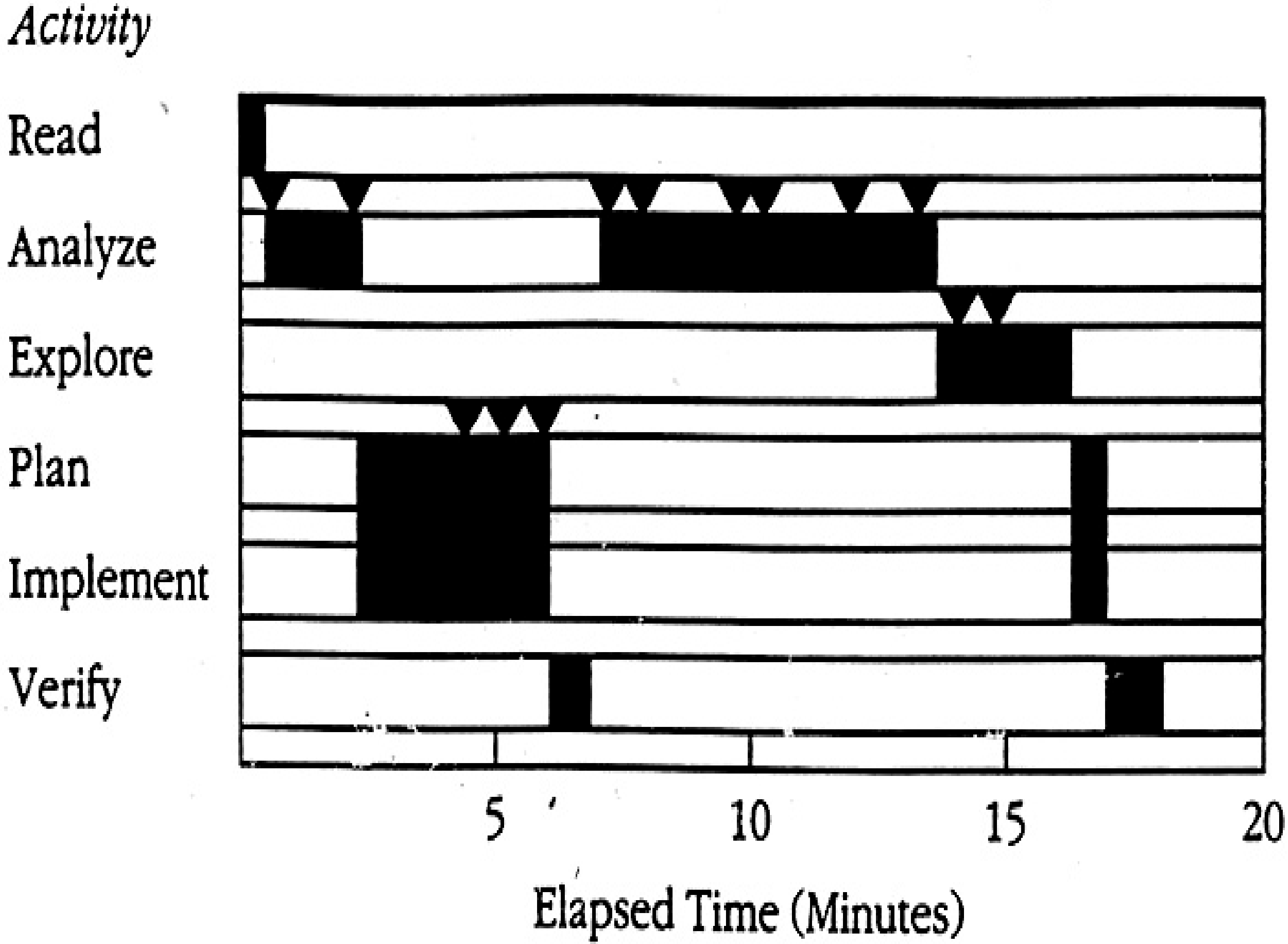
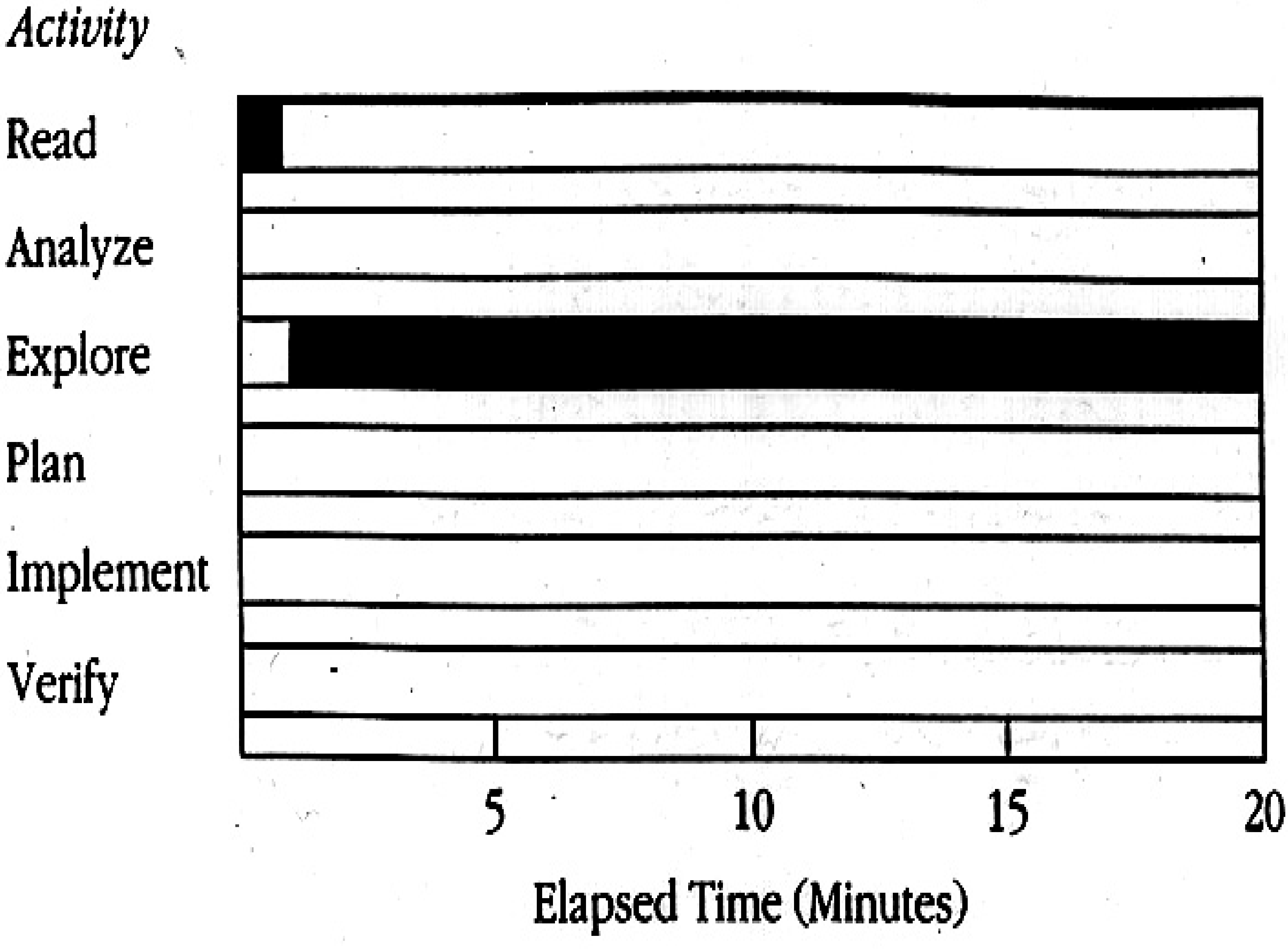
CAR PARK
RECEPTION
FUNCTION ROOMS


308



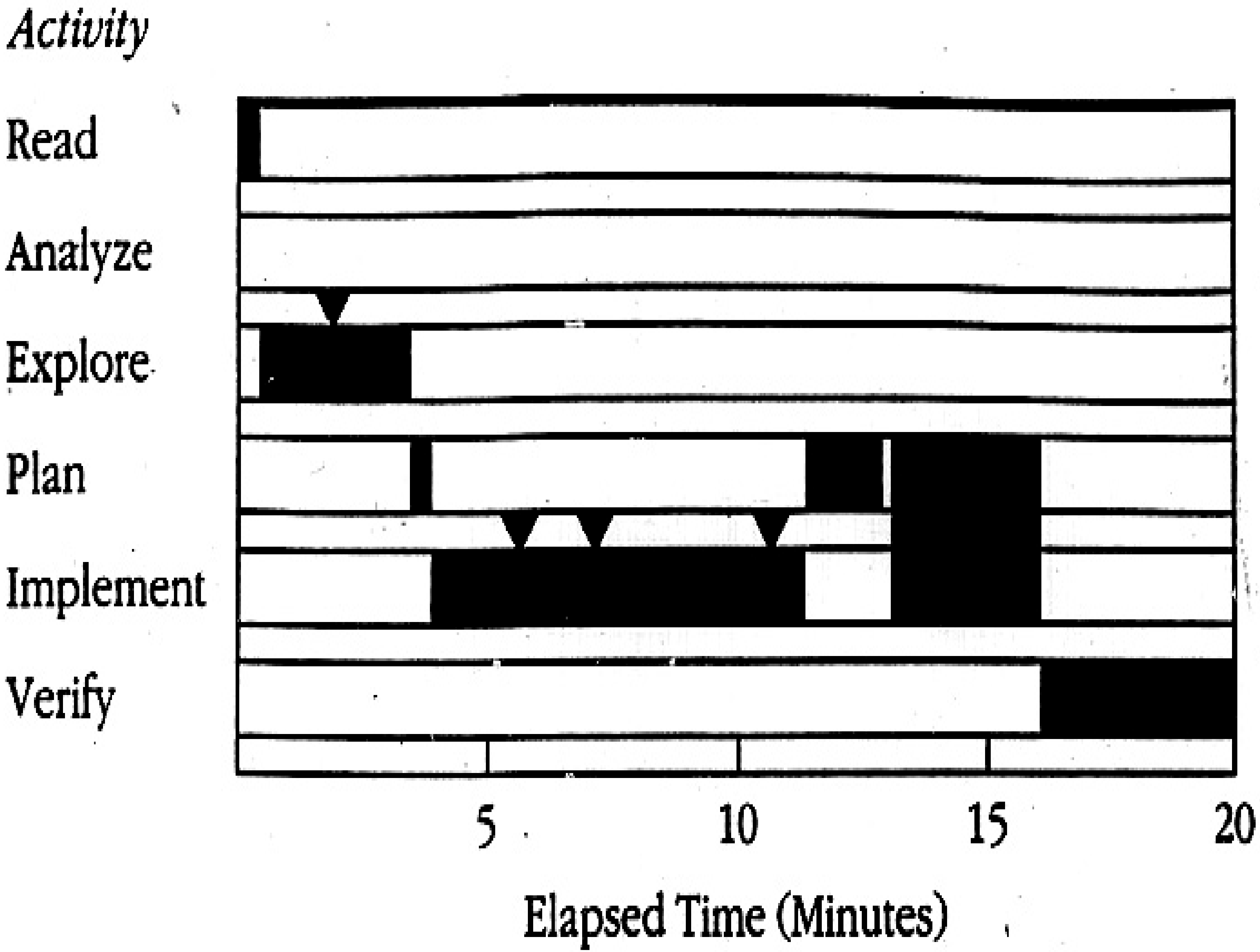
Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370)New York: Macmillan Publishing Company.



Activity
Read
Analyze
Explore
Plan
Implement
Verify
5
10
15
20
Elapsed Time (Minutes)

Questions the teacher asks students or groups as they are working on problems:

- What (exactly) are you doing? (Can you describe it precisely?)
- Why are you doing it? (How does it fit into the solution?)
- How does it help you? (What will you do with the outcome when you obtain it?)


Activity
Read
Analyze
Explore
Plan
Implement
Verify
5
10
15
20
Elapsed Time (Minutes)

Research on wait time

- In most classrooms, students are typically given less than one second to respond to a question posed by a teacher.
- Research shows that under these conditions students generally give short, recall responses or no answer at all rather than giving answers that involve higher-level thinking.

Rowe, M. B. (1974). Wait-time and rewards as instructional variables, their influence on language, logic, and fate control: Part one—Wait time. *Journal of Research in Science Teaching, 11*(2), 81-94.