

Effective Tier 2 Intervention for Multi-Digit Multiplication and Division

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Create Your Problems Carefully

If students are challenged by basic multiplication facts, create problems that require them to draw upon only the easier facts.

Choose your digits carefully. Start with 1, 2, 4, and 5. Then introduce 0. (Over time, add one more digit at a time until students can work with them all: 3, 6, 7, 8, and 9.)

$$\begin{array}{r} 21 \\ \times 14 \\ \hline \end{array} \quad \begin{array}{r} 24 \\ \times 12 \\ \hline \end{array} \quad \begin{array}{r} 42 \\ \times 14 \\ \hline \end{array} \quad \begin{array}{r} 51 \\ \times 42 \\ \hline \end{array} \quad \begin{array}{r} 41 \\ \times 24 \\ \hline \end{array} \quad \begin{array}{r} 52 \\ \times 22 \\ \hline \end{array} \quad \begin{array}{r} 51 \\ \times 44 \\ \hline \end{array} \quad \begin{array}{r} 24 \\ \times 15 \\ \hline \end{array} \quad \begin{array}{r} 24 \\ \times 51 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ \times 22 \\ \hline \end{array} \quad \begin{array}{r} 41 \\ \times 20 \\ \hline \end{array} \quad \begin{array}{r} 50 \\ \times 42 \\ \hline \end{array}$$

Create more problems using the following 2-digit numbers. Create some that involve no regrouping and some that do.

11, 12, 14, 15, 10 21, 22, 24, 25, 20 41, 42, 44, 45, 40 51, 52, 54, 55, 50

Front-Load Basic Computation

When students are working with more challenging facts, front load the basic computation that will be required. When it is appropriate to do so, have students identify the facts they'll need to draw upon to solve a given problem.

Computation to Front-Load	Multiplication Problems
7×2 3×2 $8 + 6$ 7×4 3×4 $9 + 6$	24 42 24 42 $\times 37$ $\times 37$ $\times 73$ $\times 73$
8×3 5×3 $8 + 8$ 8×6 5×6	63 36 63 36 $\times 58$ $\times 58$ $\times 85$ $\times 85$
Now you do it ... start with this problem. (Or make your own!) Which computation could you front-load? What other problems could students solve using those facts?	
	46 $\times 38$

Practice Multiplication with Multiples of 10

- 2×3
- 2×30
- 2×300
- 2×3000
- 20×3
- 200×3
- 20×30
- 20×300

Hot Spot 1: Basic Computation

- **Wait** Wait until students are more fluent with addition with regrouping and/or basic multiplication facts.
- **Scaffold** Let students refer to a completed table of multiplication facts.
- **Craft Carefully** Craft problems for students carefully. Start with only the digits 1, 2, 4, and 5, for example.
- **Front-Load** Before completing multi-digit problems, front-load the basic computation students will need to be successful with the problems.

Hot Spot 2: Difficulty Multiplying Multiples of 10

- **Practice** Practice multiplying with multiples of 10.
- **Connect** Connect that practice to basic facts and multiplication with powers of 10. Use **problem strings** to draw out those connections using **visual models** and **symbolic notation**. Emphasize the **associative property**: express combinations as the product of a basic fact and a power of 10.
$$30 \times 20 = (3 \times 10) \times (2 \times 10) = (3 \times 2) \times 10 \times 10 = (3 \times 2) \times 100$$

Hot Spot 3: Losing Track of Partial Products

- **Array** Model combinations with an array. **Label the dimensions** clearly.
- **Context** Use **context as a tool** to help students make sense of the problem and account for all partial products.

Hot Spot 4: Understanding What Is Reasonable

- **Anticipate & Estimate** Anticipate errors and use the process of estimating before calculating to address them before they arise in students' work.
- **Build on Strengths** Use tables or other organizers to help students **build from known combinations** to make reasonable estimates and accurate calculations.

The Scaffold Algorithm for Long Division

What is $390 \div 15$?

Use a Ratio Table

We use a ratio table to keep track of what we know about multiplying by 15. We multiply each number in the left column to get the product in the right column. We can add to the table as we use the scaffold algorithm, depending on which products are helpful. Notice that we can make use of doubling and halving to build from row to row in this table.

	$\times 15$	
1	15	$1 \times 15 = 15$
2	30	$2 \times 15 = 30$
4	60	$4 \times 15 = 60$
10	150	$10 \times 15 = 150$
20	300	$20 \times 15 = 300$

... With the Scaffold Algorithm

We use the scaffold algorithm for long division to carry out the computation. We use the products from the ratio table to help.

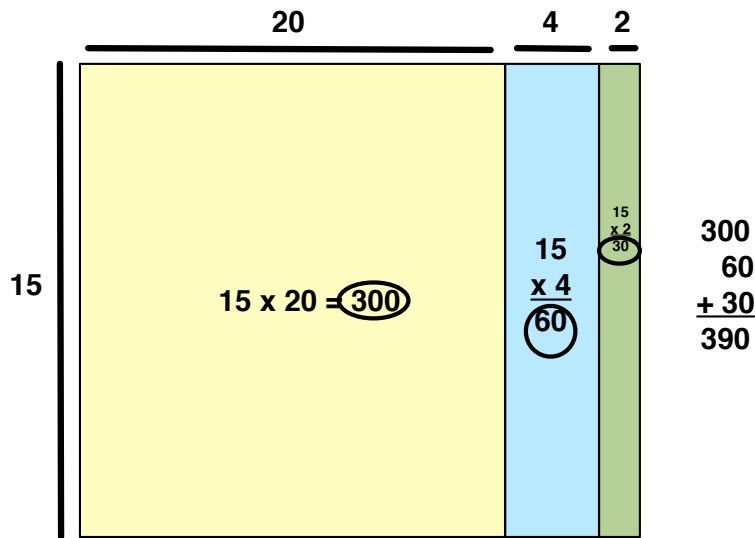
$$\begin{array}{r} 2 \\ 4 \\ 20 \\ \hline 15 \overline{)390} \\ - \underline{300} \\ 90 \\ - \underline{60} \\ 30 \\ - \underline{30} \\ 0 \end{array}$$

$$\begin{aligned} 15 \times 26 &= 390, \text{ so} \\ 390 \div 15 &= 26 \end{aligned}$$

Note that the products from the ratio table are color coded so that it's easy to see where they were used in the scaffold algorithm.

Connecting the Array to the Scaffold Algorithm

Again, we've used color-coding to help you connect the parts of the array to the ratio table and scaffold algorithm. You can use the array to demonstrate visually how we're building up to the quotient to solve the division problem. (In other words, what is the missing dimension of an array with one dimension of 15 and a total area of 390?)



The total area is 390, so $15 \times 26 = 390$.

	$\times 15$	
1	15	$1 \times 15 = 15$
2	30	$2 \times 15 = 30$
4	60	$4 \times 15 = 60$
10	150	$10 \times 15 = 150$
20	300	$20 \times 15 = 300$

$$\begin{array}{r}
 2 \\
 4 \\
 20 \\
 \hline
 15 \overline{)390} \\
 \underline{- 300} \\
 90 \\
 \underline{- 60} \\
 30 \\
 \underline{- 30} \\
 0
 \end{array}$$

$15 \times 26 = 390$, so
 $390 \div 15 = 26$

Before moving on to the next page, try using the scaffold algorithm with these combinations. First, use a ratio table to record what you know about the relevant multiplication combinations. Then use the scaffold algorithm. (Solutions and methods are shown on the following page.)

$$414 \div 23$$

$$884 \div 52$$

$$414 \div 23$$

$\times 23$			$\begin{array}{r} 3 \\ 5 \\ 10 \end{array} \Bigg) 18$
1	23	$1 \times 23 = 23$	$23 \overline{)414}$
2	46	$2 \times 23 = 46$	$\begin{array}{r} - 230 \\ \hline 184 \end{array}$
10	230	$10 \times 23 = 230$	$\begin{array}{r} - 115 \\ \hline 69 \end{array}$
5	115	$5 \times 23 = 115$	$\begin{array}{r} - 69 \\ \hline 0 \end{array}$
3	69	$3 \times 23 = 69$	$23 \times 18 = 414 \text{ so}$ $414 \div 23 = 18$

Note that you can use whatever multiples of 23 are useful and comfortable for you.

$$884 \div 52$$

$\times 52$			$\begin{array}{r} 2 \\ 5 \\ 10 \end{array} \Bigg) 17$
1	52	$1 \times 52 = 52$	$52 \overline{)884}$
2	104	$2 \times 52 = 104$	$\begin{array}{r} - 520 \\ \hline 364 \end{array}$
10	520	$10 \times 52 = 520$	$\begin{array}{r} - 260 \\ \hline 104 \end{array}$
5	260	$5 \times 52 = 260$	$\begin{array}{r} - 104 \\ \hline 0 \end{array}$
			$52 \times 17 = 884 \text{ so}$ $884 \div 52 = 17$

Note that you can use whatever multiples of 52 are useful and comfortable for you.

Effective Tier 2 Intervention for Multi-Digit Multiplication and Division References

Chval, K., Jones, D., Lannin, J. (2013) *Putting Essential Understanding of Multiplication and Division into Practice in Grades 3–5*. Reston, VA: NCTM.

Dolk, M. and Fosnot, C. (2001) *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann.

Fisher, A. (2015) *Bridges Intervention, Volume 6: Multiplication & Division of Multi-Digit Numbers*. Salem, OR: The Math Learning Center

Fuson, K. (2003) Toward Computational Fluency in Multidigit Multiplication and Division. *Teaching Children Mathematics* 9 (6): 300–305.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J.R., and Witzel, B. (2009) *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. Washington, D.C.: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Gresham, G. and Little, M. (2012) “RtI in Math Class.” *Teaching Children Mathematics*, 19 (1): 20–29.

Van de Walle, J.A., Karp, K.S. and Bay-Williams, J.M. (2010) *Elementary and Middle School Mathematics: Teaching Developmentally* (7th Edition). Boston: Allyn & Bacon.