

# NCTM Annual Meeting, 2016



***“As teachers, we want to minimize the ‘wait-to-fail’ approach and instead improve math learning through high-quality instruction and interventions.”***

**Gresham & Little**

**Effective Tier 2 Intervention for  
Multi-Digit Multiplication  
&  
Division**

**Allyn Fisher, Director of Curriculum Development**

**Martha Ruttle, Senior Editor**

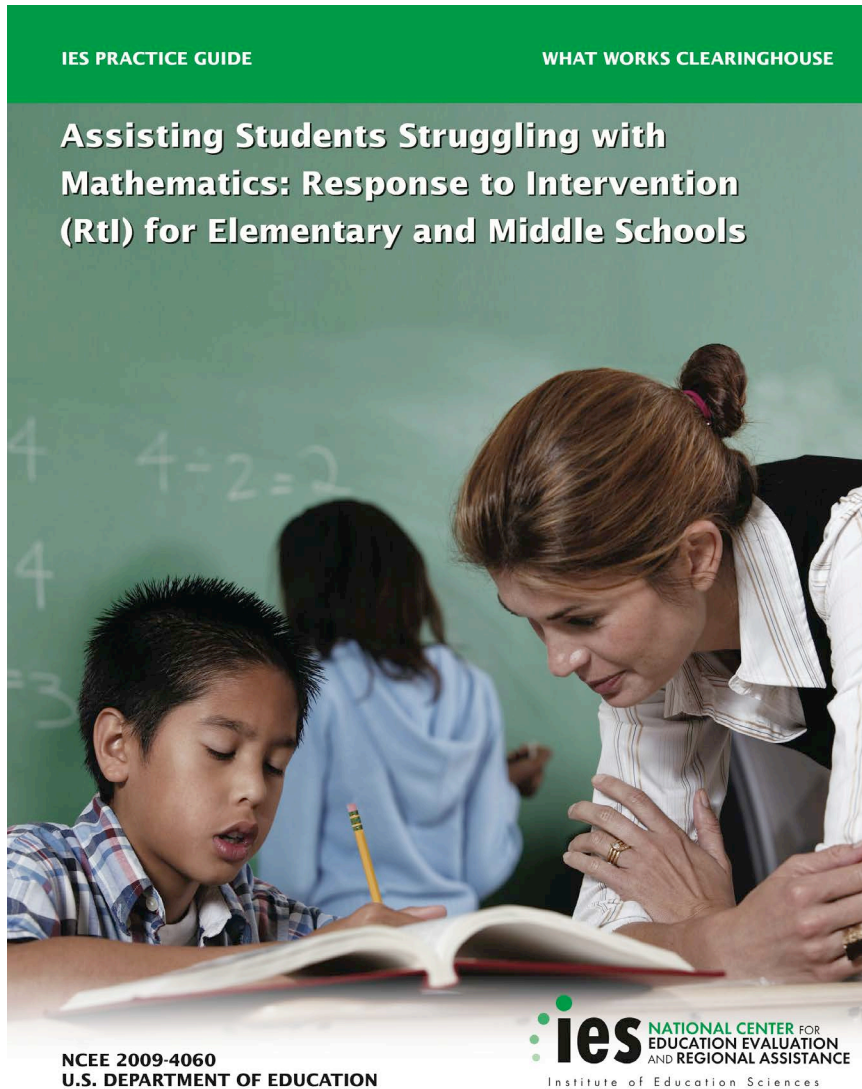
**The Math Learning Center • Salem, Oregon**

**[www.mathlearningcenter.org](http://www.mathlearningcenter.org)**

# Agenda

- ▶ **IES Recommendations for Intervention**
- ▶ **Thinking about Multi-Digit Multiplication**
- ▶ **Identifying & Addressing the Hot Spots**
- ▶ **Multiplying to Divide**
- ▶ **The Array/Area Model for Division**
- ▶ **The Scaffold Algorithm: A Success Story**
- ▶ **Questions & Comments**

# IES Recommendations



- ✓ **IES (Institute of Education Sciences)**
- ✓ **Published, 2009**
- ✓ **8 recommendations backed by research-based evidence**

[http://ies.ed.gov/ncee/wWc/pdf/practice\\_guides/rti\\_math\\_pg\\_042109.pdf](http://ies.ed.gov/ncee/wWc/pdf/practice_guides/rti_math_pg_042109.pdf)

# Recommendation #1

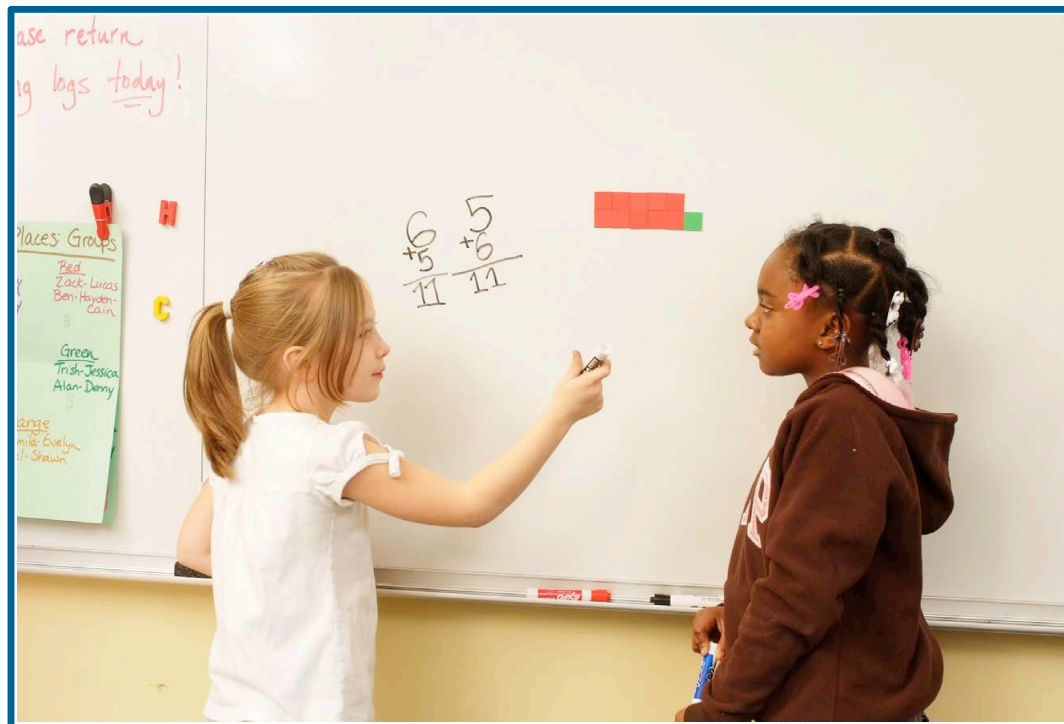
**Screen all students to identify those at risk for potential difficulties. Provide interventions to students identified as at risk.**





# Recommendation #2

**Focus on in-depth treatment of whole numbers in K–5 and on rational numbers in grades 4–8.**



# Recommendation #3

## Provide instruction that is explicit and systematic:

- ✓ Models of proficient problem solving
- ✓ Verbalization of thought processes
- ✓ Guided practice
- ✓ Corrective feedback
- ✓ Cumulative review



# Recommendation #4

**Include instruction on solving story problems that is based on common underlying structures.**

*Dustin and Tomas are playing a video game. After a few minutes, Dustin had 20 points, and Tomas had 4 times as many points as that. How many points did Tomas have?*

Graphic Organizer 3 Compare Problems

4  
times as much/many

Reference Set 20

✓ ✓ ✓ ✓

Total: 80

Equation:  
 $4 \times 20 = 80$

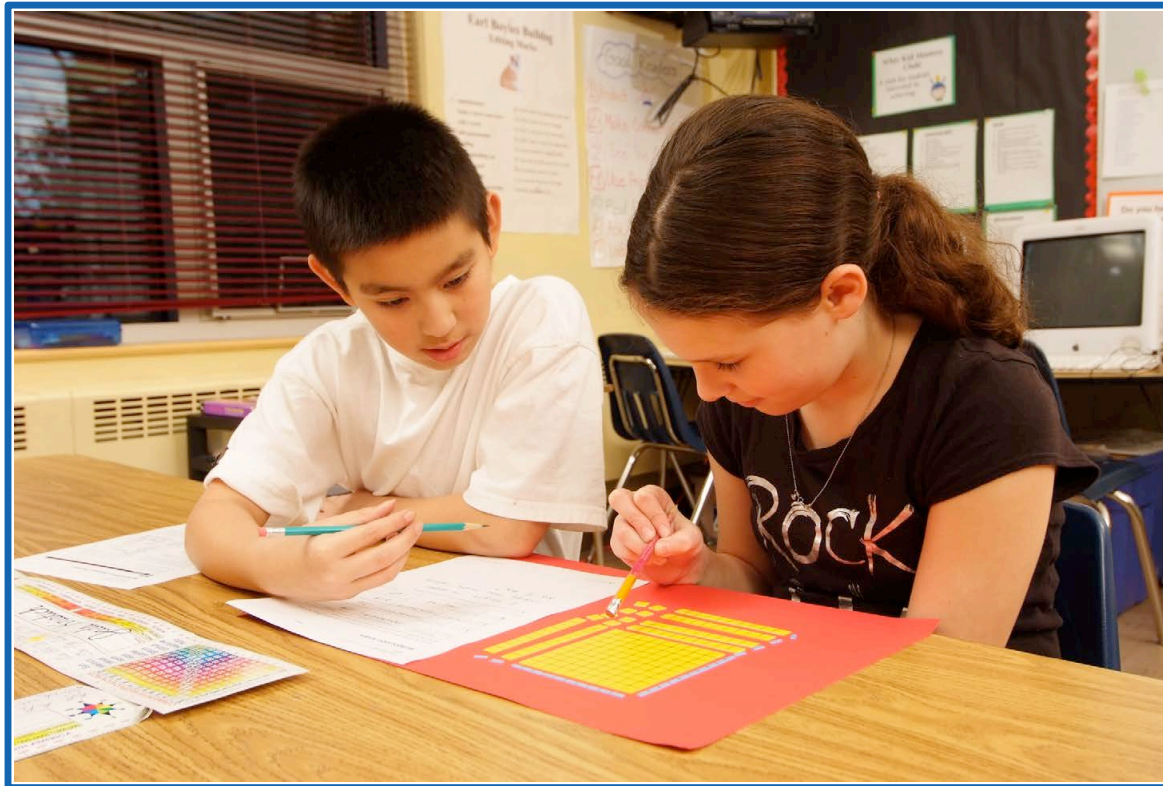
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The graphic organizer is titled "Graphic Organizer 3 Compare Problems". It contains several elements: a blue number "4" with the text "times as much/many" below it; a grey square labeled "Reference Set" with the number "20" written next to it; a ten-frame with the first four cells containing blue checkmarks; the text "Total:" followed by the number "80" enclosed in a red box; and a box labeled "Equation:" containing the equation  $4 \times 20 = 80$ , where the "80" is circled in red. At the bottom, there is a copyright notice "© The Math Learning Center", the text "WATER SOLUBLE MARKERS ONLY", and the code "QCI2373".



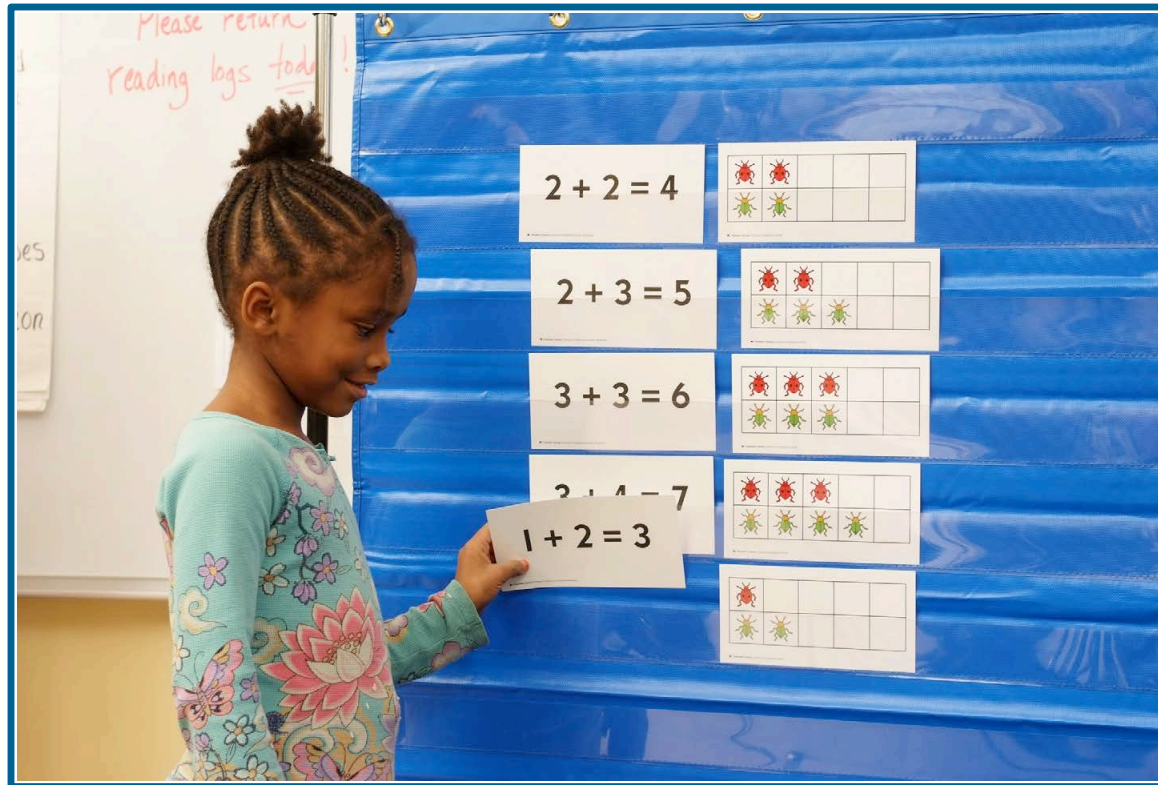
# Recommendation #5

**Include opportunities for students to work with visual representations of mathematical ideas.**



# Recommendation #6

**Devote about 10 minutes per intervention session to building fluent retrieval of basic arithmetic facts.**



# Recommendation #7

**Monitor the progress of students receiving supplemental instruction and other students who are at risk.**



# Recommendation #8

**Use motivational strategies.**





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# A Tale of Two Problems

Take a moment to solve these two multiplication problems in whatever way you like.

$$\begin{array}{r} 11 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \times 83 \\ \hline \end{array}$$

- ✓ How are they alike, how are they different?
- ✓ Which problem would be more challenging for your students? Why?
- ✓ What specific aspects would be most challenging?



# A Tale of Two Problems, cont.

- ✓ How would your struggling students approach the problems?
- ✓ What kinds of strategies might they use?
- ✓ What kinds of errors would you see?
- ✓ What patterns, if any, do you notice in the errors they make?

$$\begin{array}{r} 11 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \times 83 \\ \hline \end{array}$$

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# Hot Spots = Common Errors & Challenges

**Here's our list. How does it compare with yours?**

- ✓ **Lack of basic computation skills: addition with regrouping and multiplication facts**
- ✓ **Inability to accurately multiply with multiples of 10**
- ✓ **Losing track of partial products when working with numbers that are not multiples of 10**
- ✓ **Inability to determine whether results are reasonable**

# Hot Spot #1: (Lack of) Basic Computation Skills

- 1 Use the standard multiplication algorithm to find  $36 \times 83$ . Show all your work.

Handwritten work for  $36 \times 83$  using the standard algorithm:

$$\begin{array}{r} 36 \\ \times 83 \\ \hline 108 \\ 2880 \\ \hline 2988 \end{array}$$

- 3 Fill in all of the blanks and boxe products, and the standard mult  
Don't forget to label each region i

Handwritten work for  $49 \times 46$  using the distributive property:

$$\begin{array}{r} 49 \\ \times 46 \\ \hline 6 \times 9 = 54 \rightarrow 414 \\ 6 \times 40 = 360 \\ 4 \times 9 = 360 \\ 4 \times 40 = 1600 \\ \hline 2254 \end{array}$$

$$6 \times 40 = 360$$

$$40 \times 9 = 360$$

# Hot Spot #1: Possible Solutions

- ✓ **Delay multi-digit multiplication work until students have gained proficiency with basic multiplication facts**
- ✓ **Scaffold: Provide multiplication tables & encourage students to use them**
- ✓ **Choose your digits carefully (see handout)**
- ✓ **Front Load (see handout)**



# Hot Spot #2: Difficulty Multiplying Multiples of 10

- 3 Fill in all of the blanks and boxes to solve  $25 \times 56$  with the area model, four partial products, and the standard multiplication algorithm.

Don't forget to label each region in the area model sketch with its correct area.

**a**

**b**

		56
		$\times 25$
A	$5 \times 6$	$= 30$
B	$5 \times 50$	$= 2,500$
C	$20 \times 6$	$= 1,200$
D	$50 \times 20$	$= 1,000$
	Total	$= 5,000$

**c**

$$\begin{array}{r} 0 \\ 3 \\ 56 \\ \times 25 \\ \hline 280 \\ + 1,120 \\ \hline 2,900 \end{array}$$

$$5 \times 50 = 2,500$$


$$20 \times 6 = 1,200$$





# Hot Spot #2: Possible Solutions


- ✓ **Connect Multiplication with Multiples of Ten to Basic Facts & Powers of Ten**
- ✓ **Use Visual Models**

$$2 \times 3$$


$$20 \times 3$$


$$200 \times 3$$


$$2 \times 30$$



$$2 \times 300$$


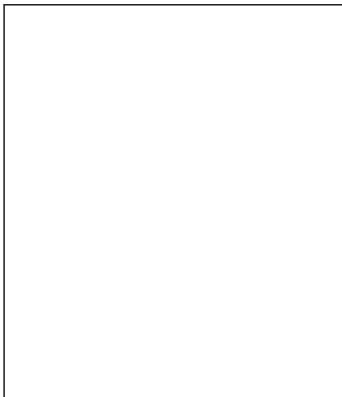
$$2 \times 3,000$$


# Hot Spot #2: Possible Solutions, cont.


- ✓ **Connect Multiplication with Multiples of Ten to Basic Facts & Powers of Ten**
- ✓ **Use Visual Models**

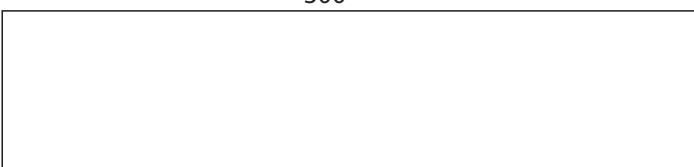
$2 \times 3$   


$20 \times 3$   


$200 \times 30$   


$2 \times 30$   


$20 \times 30$   


$20 \times 300$   


# Hot Spot #2: Possible Solutions, cont.

## ✓ Employ the Associative Property

$$2 \times 3 = 6$$

$$2 \times 30 = 2 \times (3 \times 10) = (2 \times 3) \times 10 = 6 \times 10 = 60$$

$$2 \times 300 = 2 \times (3 \times 100) = (2 \times 3) \times 100 = 6 \times 100 = 600$$

## ✓ Extend the Patterns

$$20 \times 30 = (2 \times 10) \times (3 \times 10) = (2 \times 3) \times 10 \times 10 = ?$$

$$20 \times 300 = (2 \times 10) \times (3 \times 100) = ?$$

$$200 \times 30 = (2 \times 100) \times (3 \times 10) = ?$$

# Hot Spot #3: Losing Track of Partial Products

**b**

$22 \times 11 = 230$

200 ✓	20
10	

inking. Label your answers with the correct u

**b**

250

220

22

11	100	100	20
	10	10	10

32

30

clo



# Hot Spot #3: Losing Track of Partial Products

Fill in all of the blanks and boxes to solve  $25 \times 56$  with the area model, four partial products, and the standard multiplication algorithm.

Don't forget to label each region in the area model sketch with its correct area.

**a**

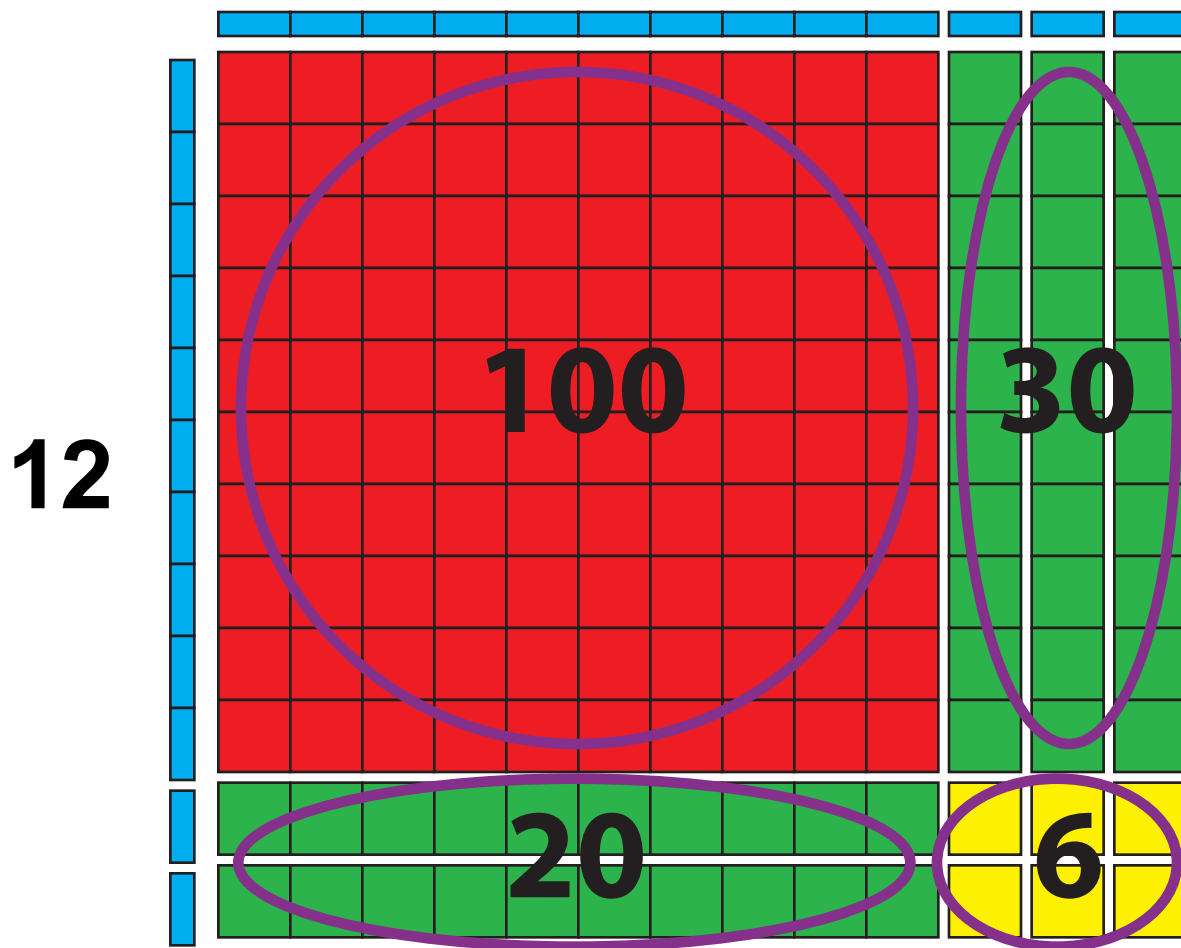
**b**

	56
	<u>× 25</u>
A	<u>6 × 5 = 30</u>
B	<u>20 × 5 = 100</u>
C	<u>50 × 6 = 300</u>
D	<u>20 × 50 = 1,000</u>
	Total = <u>1,430</u>

# Hot Spot #3: Possible Solutions

Build it with base 10 pieces.

13



$$\begin{array}{r} 100 \\ 30 \\ 20 \\ + 6 \\ \hline 156 \end{array}$$

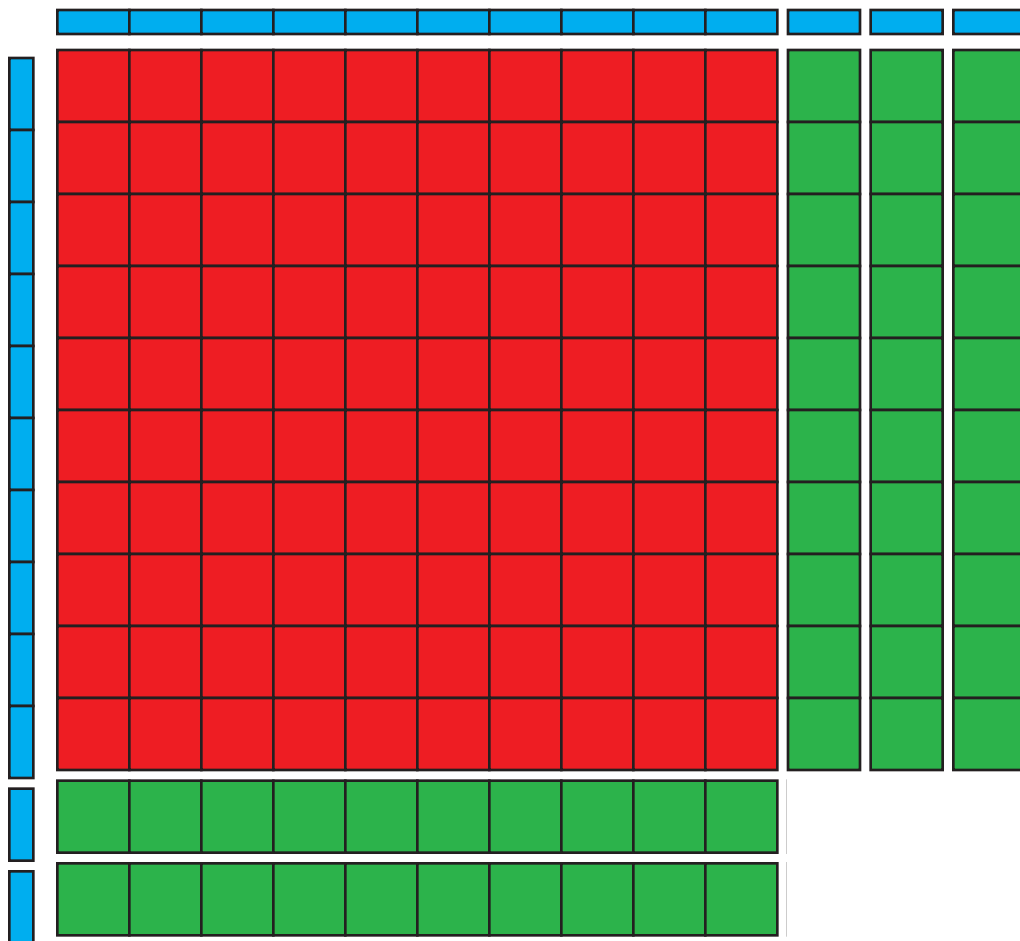


# Hot Spot #3: Possible Solutions, cont.

Reality bites...

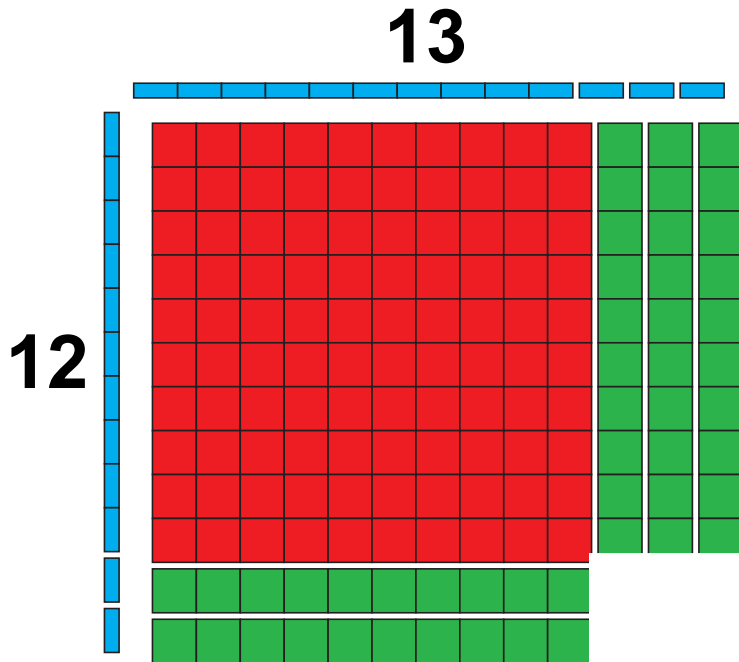
13

12



# Hot Spot #3: Possible Solutions, cont.

## Context helps



### Maggie's Chickens



#### Problem

- 1 Maggie's chickens live in a coop in her backyard. The coop is 12 feet by 13 feet. What is the area of the coop?

# Hot Spot #4: Not Recognizing the Unreasonable

- 3 Fill in all of the blanks and boxes to solve  $25 \times 56$  with the area model, four partial products, and the standard multiplication algorithm.

*Don't forget to label each region in the area model sketch with its correct area.*

**a**

	50	6
20	D <u>1,000</u>	C <u>1,200</u>
5	B <u>2,500</u>	A <u>30</u>

**b**

	56
	× 25
A	<u>5 × 6 = 30</u>
B	<u>5 × 50 = 2,500</u>
C	<u>20 × 6 = 1,200</u>
D	<u>50 × 20 = 1,000</u>
Total	<u>5,000</u>

## Hot Spot #4: Not Recognizing the Unreasonable

- ✓ **Anticipate errors and use estimation to head them off.**

**Estimate the results of this multiplication combination. Explain your estimate.**

$$56 \times 25$$

## Hot Spot #4: Not Recognizing the Unreasonable

- ✓ Use ratio tables to keep students rooted in what they know.

$$56 \times 25$$

1	25
10	250
20	500
40	1,000
50	1,250
5	125
55	1,375
56	1,400



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# Division is the Inverse of Multiplication

Consistent use of the word **groups** helps students make the connection.

$$6 \times 7 = 42$$

*“6 **groups** of 7 is 42”*

$$42 \div 7 = ?$$

*“How many **groups** of 7 are there in 42?”*

$$42 \div 7 = 6$$

*“There are 6 **groups** of 7 in 42”*

# Use Contexts First, Not Last

*Mr. G. went to the office supply store to buy markers for his 5<sup>th</sup> graders. There are 15 markers in a pack. He got 390 markers. How many packs did he buy?*

<b>packs</b>	<b>1</b>	<b>10</b>	<b>20</b>	<b>5</b>	<b>25</b>	<b>26</b>
<b>markers</b>	<b>15</b>	<b>150</b>	<b>300</b>	<b>75</b>	<b>375</b>	<b>390</b>

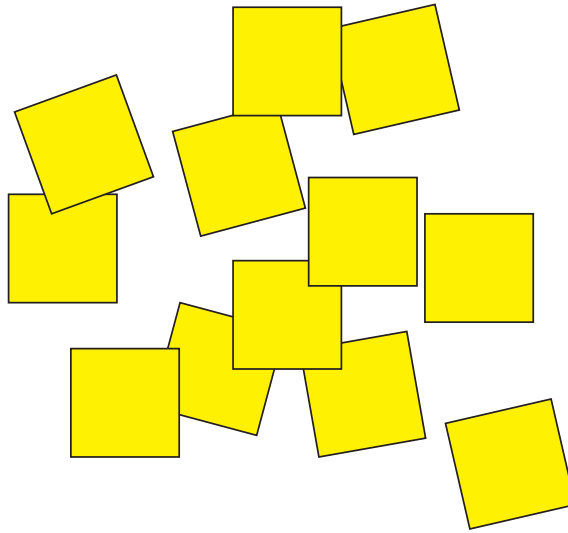
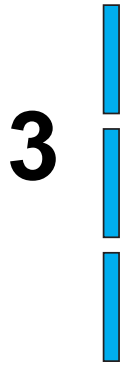
**We can use a ratio table to build up to the dividend.**

# Agenda

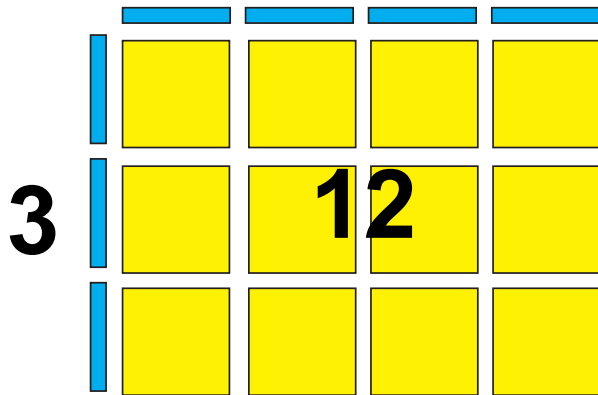
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# Arrays Also Work for Division

$$12 \div 3$$



4



$$\begin{array}{r} 4 \\ 3 \overline{) 12} \end{array}$$

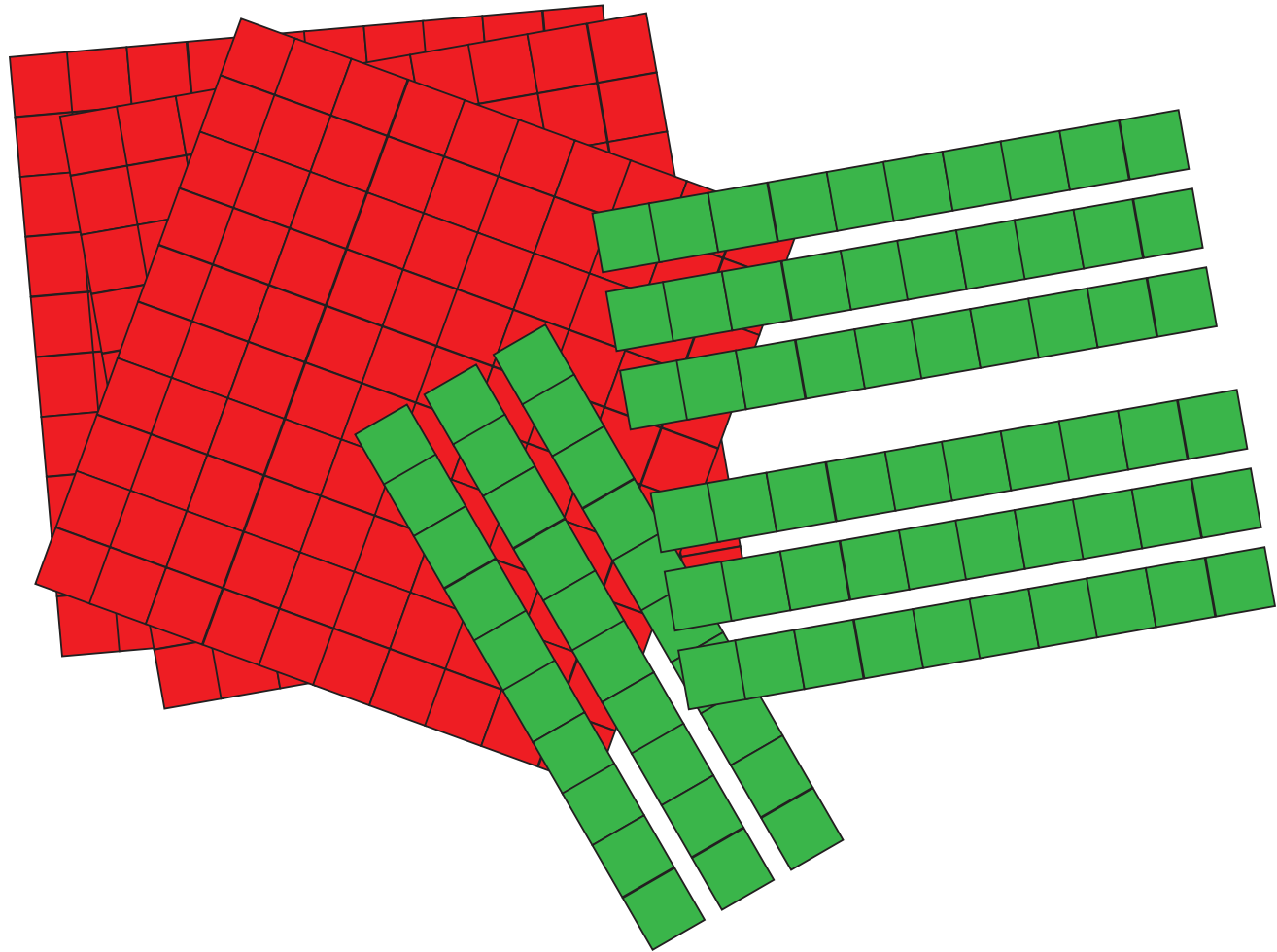
There are  
4 **groups**  
of 3 in 12.



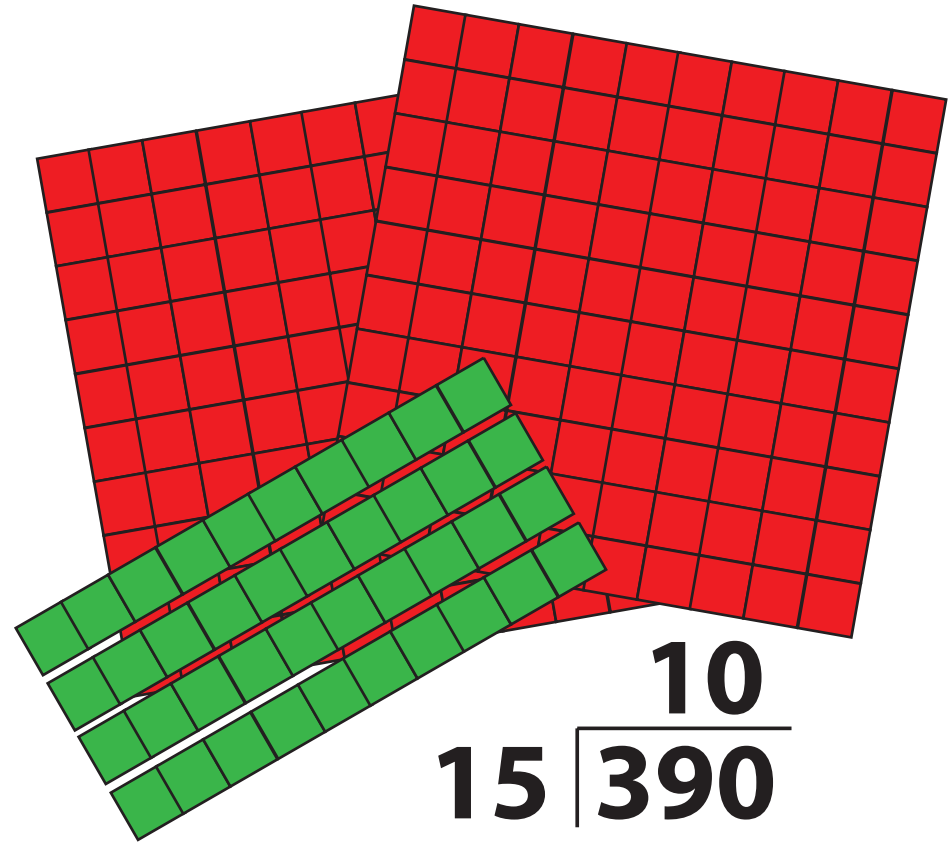
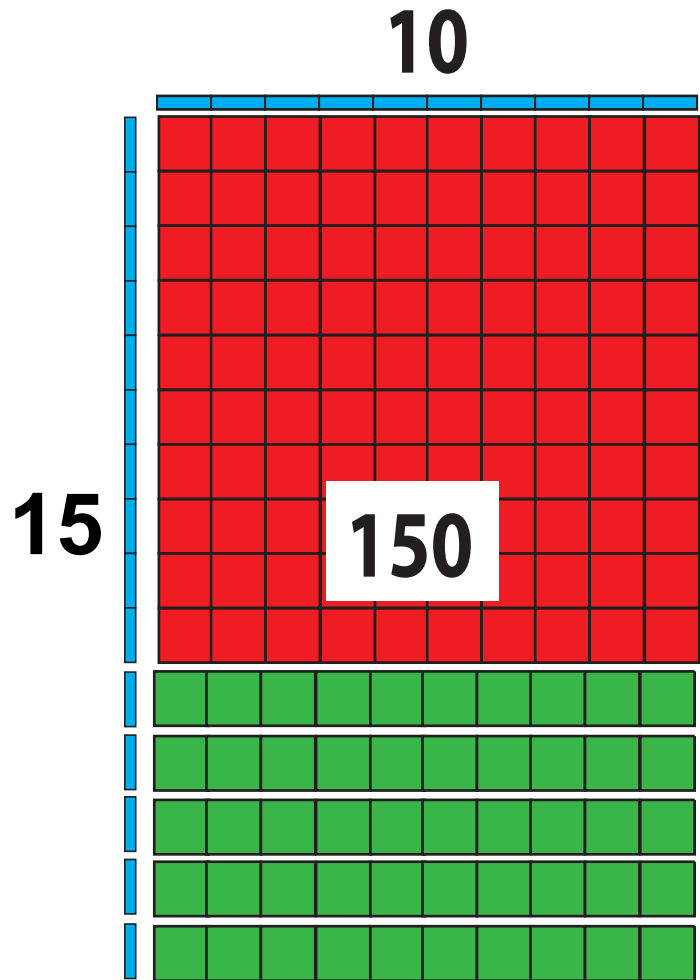
# Arrays Go Large!

**15 | 390**

**15**



# 390 ÷ 15, cont.



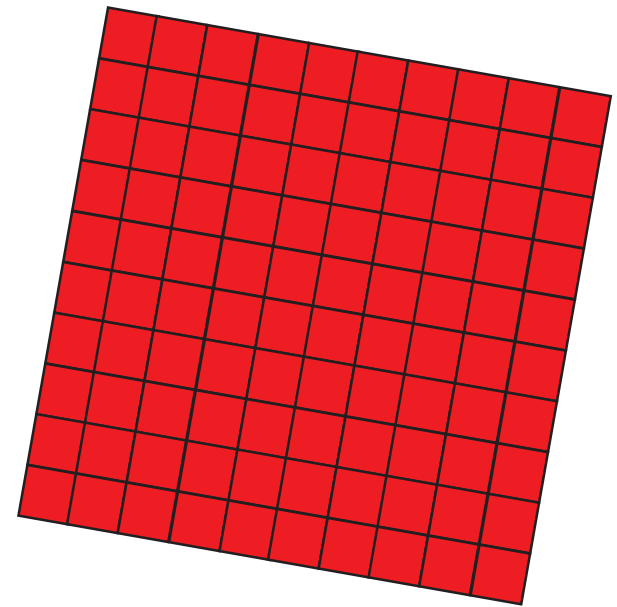
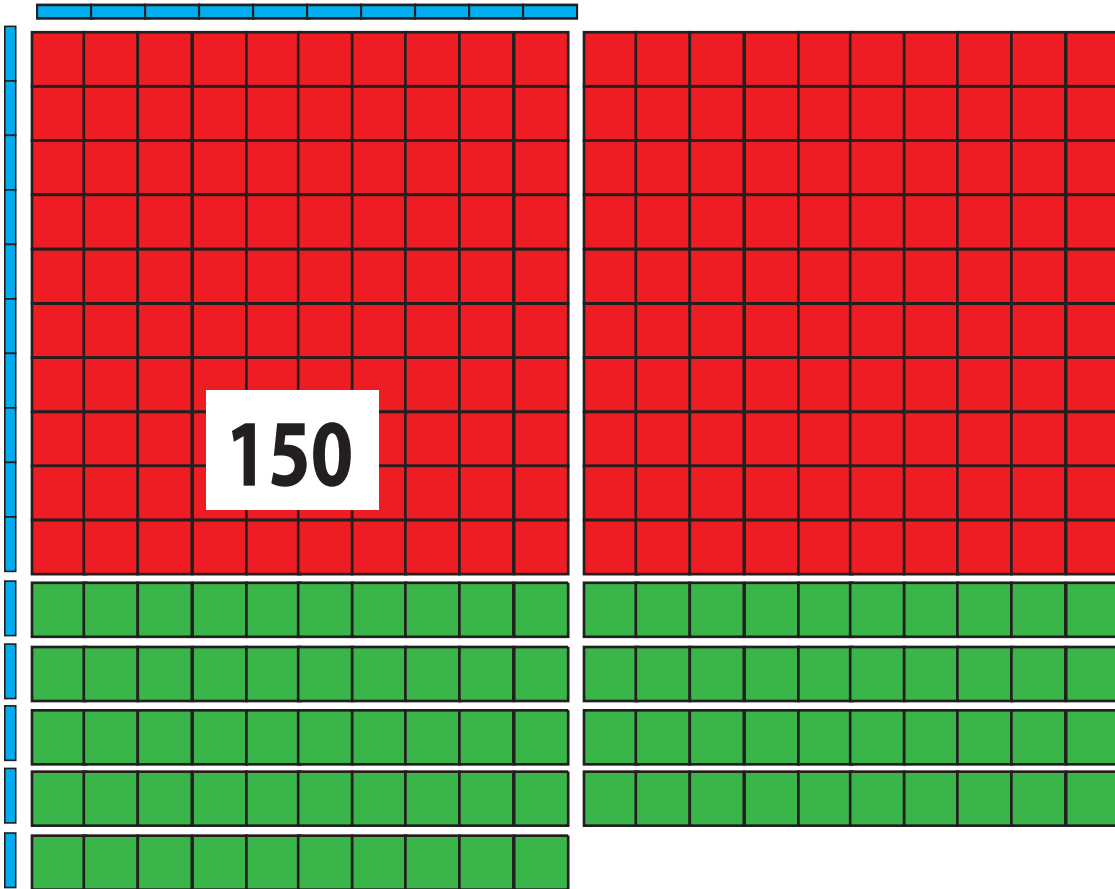
$$\begin{array}{r} 10 \\ 15 \overline{) 390} \\ - 150 \\ \hline 240 \end{array}$$

# 390 ÷ 15, cont.

10

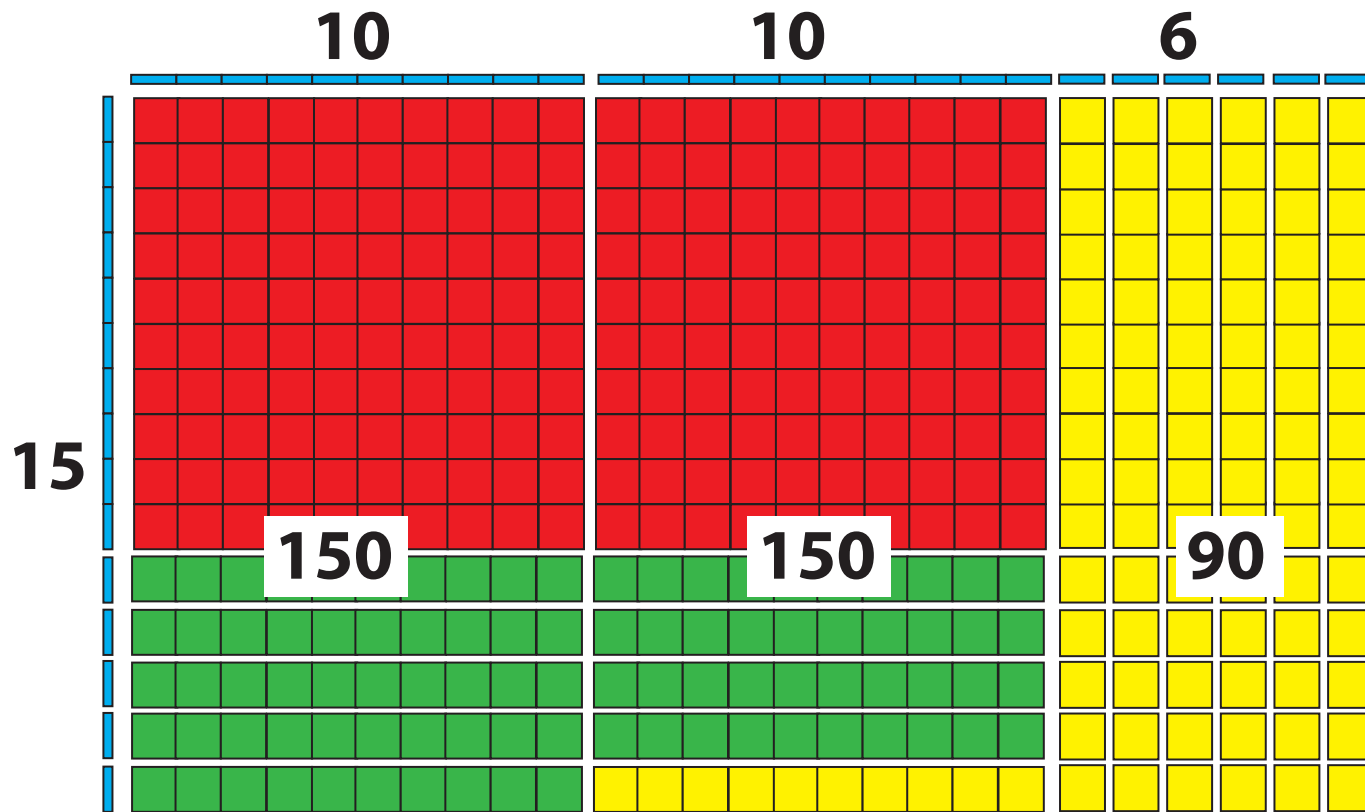
15

150



Hmmmm.....

# 390 ÷ 15, solved



$$\begin{array}{r} 6 \\ 10 \\ 10 \\ \hline 15 \overline{) 390} \\ - 150 \\ \hline 240 \\ - 150 \\ \hline 90 \\ - 90 \\ \hline 0 \end{array}$$

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# Putting it All Together

$$14 \overline{) 378}$$

Hmmm...okay, what do I know about 14 that might help?

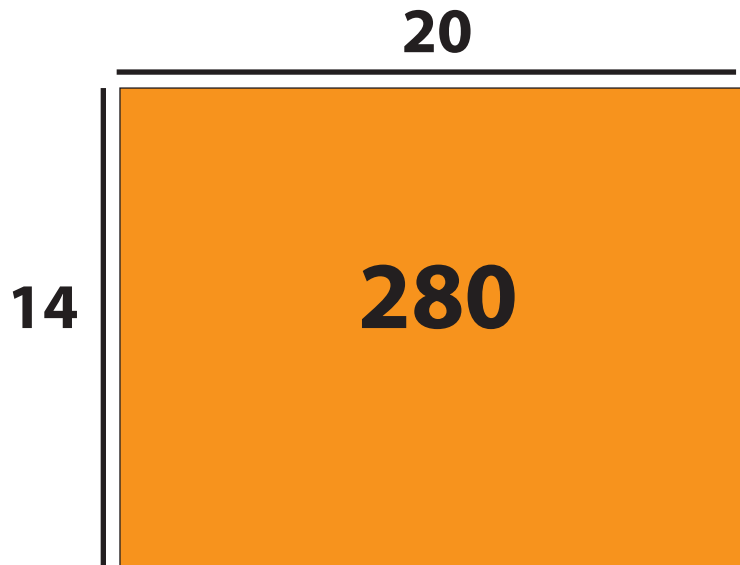
<b>1</b>	<b>10</b>	<b>20</b>	<b>5</b>		
<b>14</b>	<b>140</b>	<b>280</b>	<b>70</b>		

# Putting it All Together

<b>1</b>	<b>10</b>	<b>20</b>	<b>5</b>		
<b>14</b>	<b>140</b>	<b>280</b>	<b>70</b>		

I'm going to start with 20 groups of 14. That's 280.

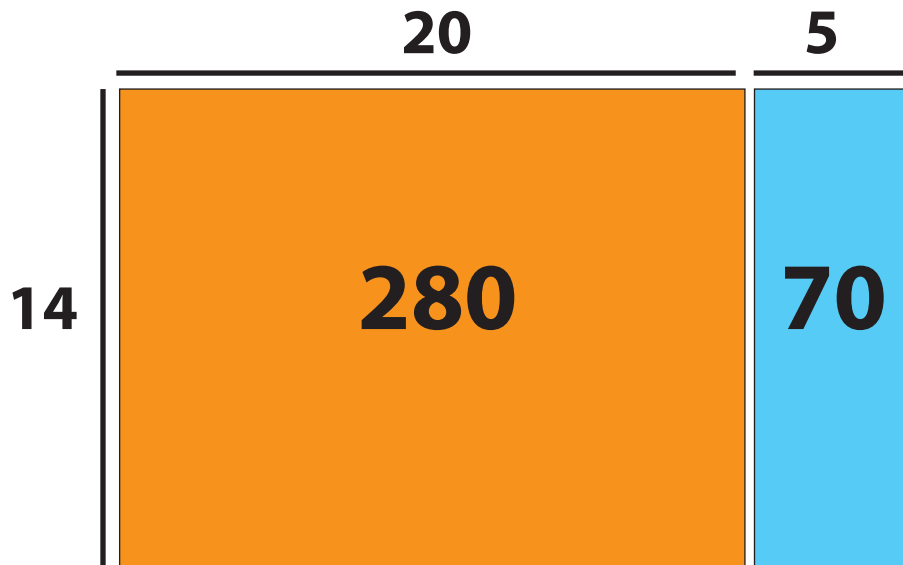
$$\begin{array}{r} 20 \\ 14 \overline{) 378} \\ \underline{- 280} \\ 98 \end{array}$$



# Putting it All Together

1	10	20	5	2	
14	140	280	70	28	

I have 98 left to go, so I can take another 5 groups of 14.

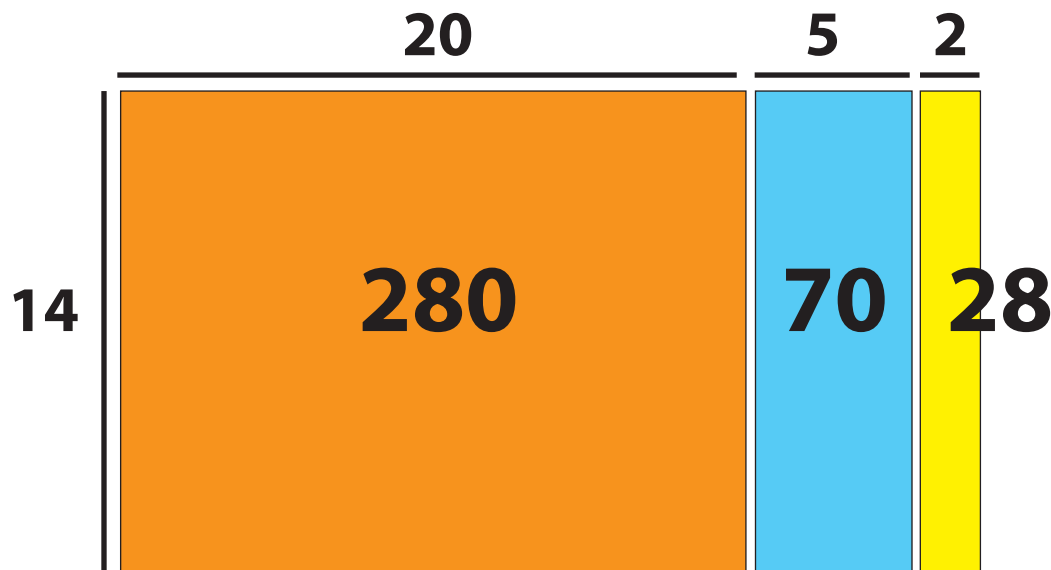


$$\begin{array}{r} 5 \\ 20 \\ \hline 14 \overline{) 378} \\ \underline{- 280} \\ 98 \\ \underline{- 70} \\ 28 \end{array}$$

Now what? Oh, wait! I see!

# Putting it All Together

1	10	20	5	2	
14	140	280	70	28	



$$\begin{array}{r} 2 \\ 5 \\ 20 \\ \hline 14 \overline{) 378} \\ \underline{- 280} \\ 98 \\ \underline{- 70} \\ 28 \\ \underline{- 28} \\ 0 \end{array} \quad \left. \begin{array}{l} 2 \\ 5 \\ 20 \end{array} \right\} 27$$

# Ratio Table + Scaffold Algorithm = Success!

1) Construct a ratio table for the divisor.

1	10	20	5	2	
14	140	280	70	28	

2) Use the table to identify reasonable groups to subtract from the dividend.

3) Continue to subtract groups until no longer possible.

$$\begin{array}{r} 2 \\ 5 \\ 20 \\ \hline 14 \overline{) 378} \\ \underline{- 280} \\ 98 \\ \underline{- 70} \\ 28 \\ \underline{- 28} \\ 0 \end{array}$$

} 27



# Questions & Comments

