Geometry Tasks That Promote Habits of Mathematical Thinking

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(For information on how to obtain our tasks, please scroll to the bottom of this document.)

2 minutes of History

* Standards-based grading and synthesis (used principally as assessment)

Your first synthesis task

As you're completing this, think:

- What sets this apart from your typical assessment?
- What abilities do these problems require of a student?
- What content knowledge do these problems require of a student?

The three side lengths of a triangle are all integers. If two of the sides are 6 and 8, what is the ratio of the number of possible obtuse triangles to the total number of possible triangles?

Hint: The answer is a real number between 0 and 1.
The answer is NOT 3/11

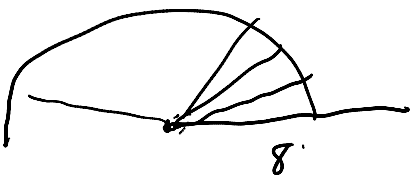
"Listed all the possibles with integer sides, it was 11 total [triangle inequality], found the obtuse ones, there were 3."

"Pythagorean Theorem. Triangle inequality. Get 6-8-10, therefore 6-8-11, 6-8-12, 6-8-13, makes 3 / 11."

But what if the third unknown side is *smaller* than 10? Smaller than 6?

Third cut-off side, use Pythagorean Theorem with 6 as a leg and 8 as a hypotenuse.

Yields 6-8-3, 6-8-4, and 6-8-5 as three more obtuse configurations.



What makes a problem a synthesis one?: Your Perspective
(Based on your thoughts on the above questions, and your experiences working through this problem)

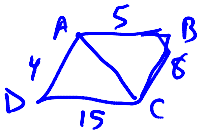
- Low prerequesite knowledge
- Combination of math concepts (critical thinking)
- Multiple entry points
- Multiple solution (how to proceed)
- Challenging (answer not immediately apparent)
- Not covered from direct instruction (student driven)
- Could lead to related problems
- "Quality control"

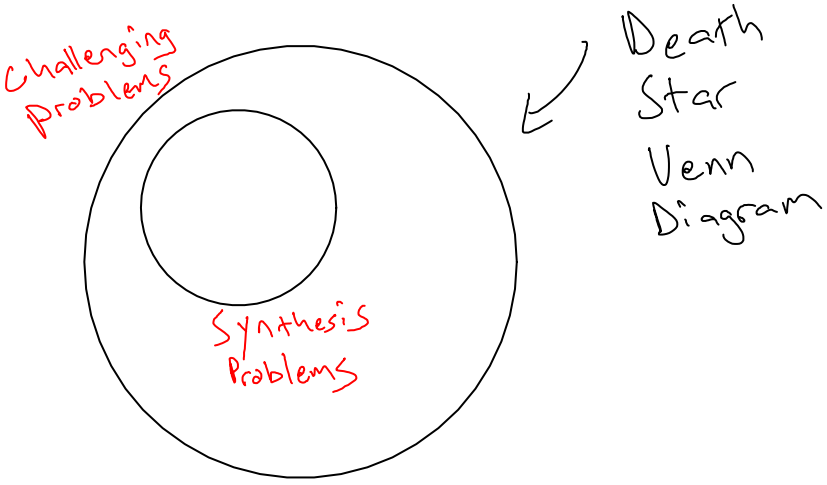
Compare: **What isn't synthesis?**:

- Here's a challenging, non-synthesis problem.

Quadrilateral ABCD has AB = 5, BC = 8, CD = 15, and DA = 4. If diagonal AC has integral length, find possible value(s) for AC; justify your answer.

AC = 12

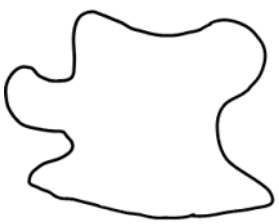




Death Star Venn Diagram

A gray-area (ha!) problem:

What is the area of the blob?



What is synthesis?: Our Perspective

- Used as assessment (but doesn't have to be)
- Problem-solving, on problems that aren't carbon-copies of ones you've seen in the textbook or done in class. NCTM calendar problems often a good source.
- Generally spans multiple concepts, but could just be a tougher question on a single concept
- Sometimes tell them what to study for
- Problems with multiple approaches and/or multiple entry points, optimally (this is based on what you've done in class)
- Open notes
- Historical note: Geometry, Algebra I, Algebra II
  - In all classes, not much consistency in what they looked like. This is a good thing.
- A work in progress! Tweak it, try it out, collect student feedback...

Let's look at a few more tasks - More time for awesome problems!

(All our tasks, as well as where you might find more, will be made available at the end of the presentation, and we've a bit more to talk about besides, so stick around…)

As you're completing these, keep an eye out for what we've discussed as distinguishes synthesis tasks. If you'd like, consider which (of multiple!) CC mathematical practices a given task embodies, using the CC handout which you've all surely memorized anyway, right?

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Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

A digital copy of this packet (only if you don't have ready access to the paper version):

http://bdwlf.com/0.pdf

We'll be walking around answering individual questions; the most common ones, we'll answer collectively in a bit.

Some tasks worth highlighting

Reflect and learn
Take home, complete ___ points worth of problems
Flagpole project
Semester exams

Common Questions

Places to find synthesis problems:
* NCTM calendar problems
* Art of Problem Solving
* Math team problems

Helping students grow at synthesis?
~~Corporal punishment~~

Our Tasks

Keep reading!

Greetings! Hopefully you enjoyed our 4/14/16 presentation. We're excited to share these tasks with you and to have you get started on implementing your own vision of synthesis. At heart, a synthesis task is nothing more than a problem (in our case, an assessment) designed to give you and your students a sense of, and appreciation for, their problem-solving abilities rather than their specific content-area skills.

This link will be up for a few weeks, but not indefinitely. To the extent we still use these as actual assessment tasks, we're reluctant to leave them floating around the Internet! If you find that the link is down, please email bjhyman@cps.edu.

https://drive.google.com/file/d/0B7NlQ5DXDUwkMVg1aWNOeVA3aU0/view

Good luck!

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