How Do You Know?

Helping Students Build Justifications

NCTM 2016

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Introductions:

Why Justify?

Why Justify?

- Justification is an essential part of developing and communicating mathematical knowledge
  - (Hanna & Jahnke, 1996; Kitcher, 1984; Polya, 1981).
- Mathematicians use justification to validate claims and systematize knowledge
  - (Hanna, 2000).

Why Justify?

- Given its importance in doing and understanding mathematics, many have claimed that justification should be an important part of mathematics classrooms and that it ***should begin in early grades***
  - (Ball & Bass, 2003; Carpenter, Franke, & Levi, 2003; Hanna, 2000; Schoenfeld, 1992; Yackel & Hanna, 2003).

Common Core

- What does the common core say about justification?

Mathematical Practice #3

- Construct viable argument and critique the reasoning of others
  - ***Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions***. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

What does justification mean?

- For our purposes today, let's define it as:

a broad range of arguments used to convince another why a solution must be true

So if we agree it's important….

- How do we do it?
- What are some of the challenges?

The Challenges

- Justification has been found to be missing from many classrooms (Jacobs et al, 2006).
- When it is present, researchers found that students had a difficult time providing justifications (Chazan, 1993; Flores, 2002; Stylaniades, 2007)

How do you know?

- Elementary students often say:
  - "I just know it, I can't explain it"
  - "My teacher told me."
  - "That's how we learned it last year."
  - "That's the rule"
  - They give step by step procedural explanations

Why is it difficult?

- Lack of conceptual understanding
- Lack of exposure to sense-making tasks
- Lack of practice
- Lack of tools to develop justification

How do we help students justify?

- Negotiate norms for justification in mathematics
- Provide tools for them to use to build justifications
  - Manipulatives
  - Diagrams
  - Context
- Provide opportunities to practice and develop over time

Setting Norms

Setting Norms

- Sociomathematical norms (Yackel & Cobb, 1996)
  - Norms that specifically support mathematical thinking
  - Students relied on the textbook or teacher or described rules or procedures.
  - Justifications changed as students participated in tasks that ***made the mathematics personally meaningful***
  - In class discussions, the teacher and students negotiated what an acceptable justification was.

Videos

- The videos today come from the MARN project (Measurement Approach to Rational Number)
  - Fourth and fifth graders
  - Fraction and Ratio Concepts

Video

What did you notice?

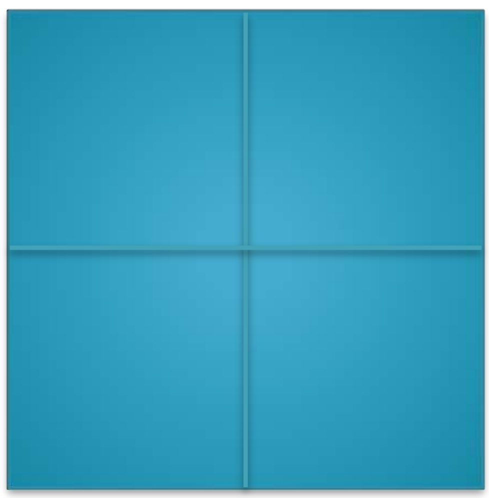
- About Kylie's understanding of what it means to justify?
- About how the instructor intervened to negotiate a justification?
- What were some challenges?

Setting Norms

- Justifications need to make sense of math
- They describe HOW we know something is true
- They are not procedural in nature. They are not list of steps that we follow that we don't understand.
- They need to negotiated throughout

The Skeptic

- Paper Folding Activity (youcubed.org)
  - Construct a square with exactly ¼ the area of the original square and convince your partner that it is a square and has ¼ of the area.
  - Students pair up and take turns being the convincer and the skeptic

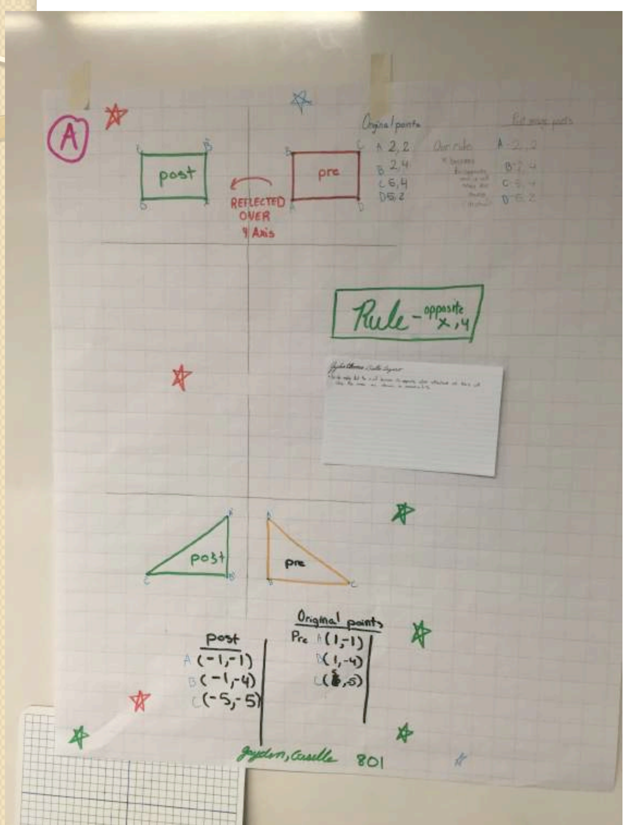


The Skeptic

- Sorting justifications
  - Create a chart of arguments the class comes up with
    - Sort into weak arguments and strong arguments
    - Create a shared understanding of what is convincing in our class

Extensions

- Plant skeptics in the room at random throughout activities
- Gallery Walks
  - Students create posters of justifications and skeptics circulate


A
post
pre
REFLECTED OVER y Axis
Original points
A 2,2
B 2,4
C 6,4
D 6,2
Rule - opposite x, y
post
pre
Original points
post
A (-1,-1)
B (-1,-4)
C (-5,-5)
Pre A (1,-1)
B (1,-4)
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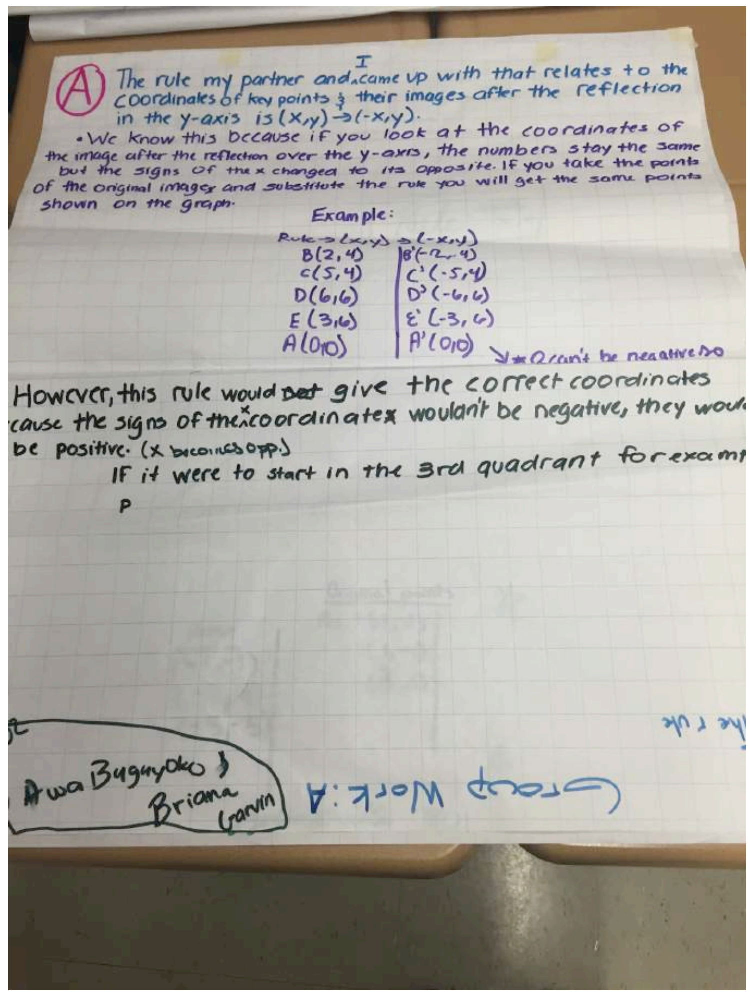

A
The rule my partner and I came up with that relates to the coordinates of key points & their images after the reflection in the y-axis is (x,y) → (-x,y).
• We know this because if you look at the coordinates of the image after the reflection over the y-axis, the numbers stay the same but the signs of the x changed to its opposite. If you take the points of the original images and substitute the rule you will get the same points shown on the graph.
Example:
Rule → (x,y) → (-x,y)
B(2,4) | B'(-2,4)
C(5,4) | C'(-5,4)
D(6,6) | D'(-6,6)
E(3,6) | E'(-3,6)
A(0,0) | A'(0,0)
0 can't be negative so
However, this rule would give the correct coordinates
cause the signs of the x coordinate wouldn't be negative, they would be positive. (x becomes opp.)
If it were to start in the 3rd quadrant for example
P
Group Work: A
Awa Buguyoko &
Briana Garvin

Manipulatives: Fraction Bars

How do you know?

- Students can use manipulatives to show how they know and convince another why their solution must be correct

Fraction Bars

- Fraction Bars is an updated version of Java Bars, which came out of work at UGA. Fraction Bars was produced at UMass Dartmouth by James Burke and Chandra Orril
  - http://math.coe.uga.edu/olive/welcome.html

- file://localhost/Users/nicoraplaca/Documents/NCTM 2016/FB 20150214 w Make and Measure, Whole number corrected/index.html

Converting Mixed Number to Improper Fraction

- Task 1: If we made 1 ¾ all fourths, how many fourths would it be?

Video

Converting Mixed Number to Improper Fraction

- Task 2: What is the length of the bar created? 6 ¼ would be the same as what?

Video

- Converting Mixed Number to Improper Fraction
  - Task 3: If we have $2\ \frac{1}{4}$, what would be another way to say that as an improper fraction?

Video

What did you notice?

- About her justification?
- About the tasks?
- About the role of fraction bars?
- About the questions asked?

Task 5 Transcript

R: So here's a number, can you read it for me?

S: Mm. (pause) Two and one fourth.

R: Okay. So. If we have two and one fourth units. What would be another way to say that...as a... an improper fraction

S: Um. (pause) Nine fourths?

R: Nine fourths. Tell me how you got nine fourths.

S: Um. (pause) I put, I know that two times four is eight.

R: **Why'd you do two times four?**

S: Cause. There's. ... Um. Um. Each one of those

R**: One of what?**

S: Well two is. Two wholes.

R: **Two whole units?**

S: And each one of those wholes has four fourths in it. So that would be eight fourths. And then that would be, add the one fourth left and it's nine fourths

Using Manipulatives

- Gives students a way to unpack the operations even when not working directly with the bars
- Students need a chance to practice and build up to the justification

Video

- Find equivalent fractions to 7/4

What did you notice?

- About his justification?
- About the tasks?
- About the role of fraction bars?
- About the questions asked?

Questions to ask

- Questions about the quantities
  - What does the number refer to in the diagram?
- Questions about the process and operations
  - Why did you multiply?

Context

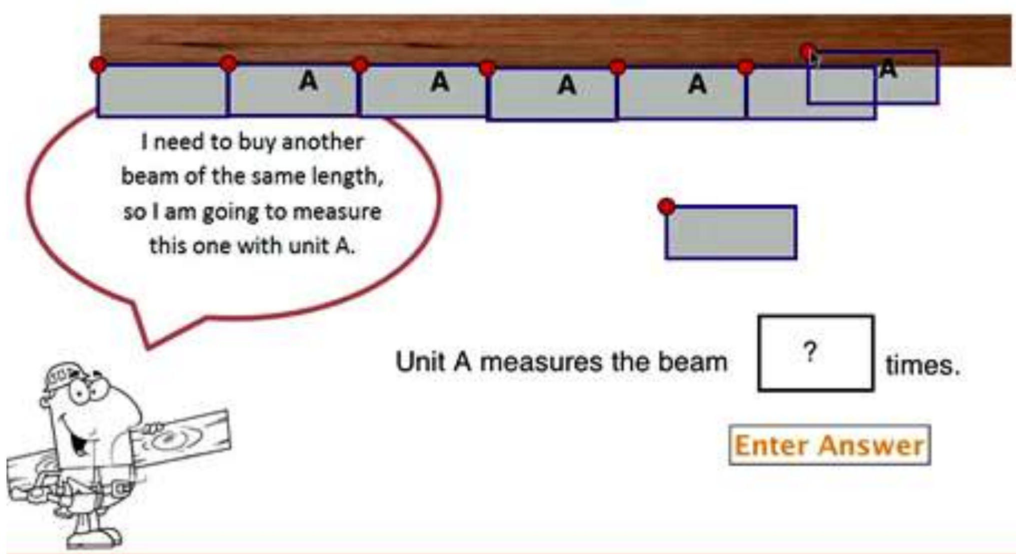
Context

- How do you know?
  - Giving students a context can help them make sense of the math and give them tools to justify
  - Measurement context for fractions

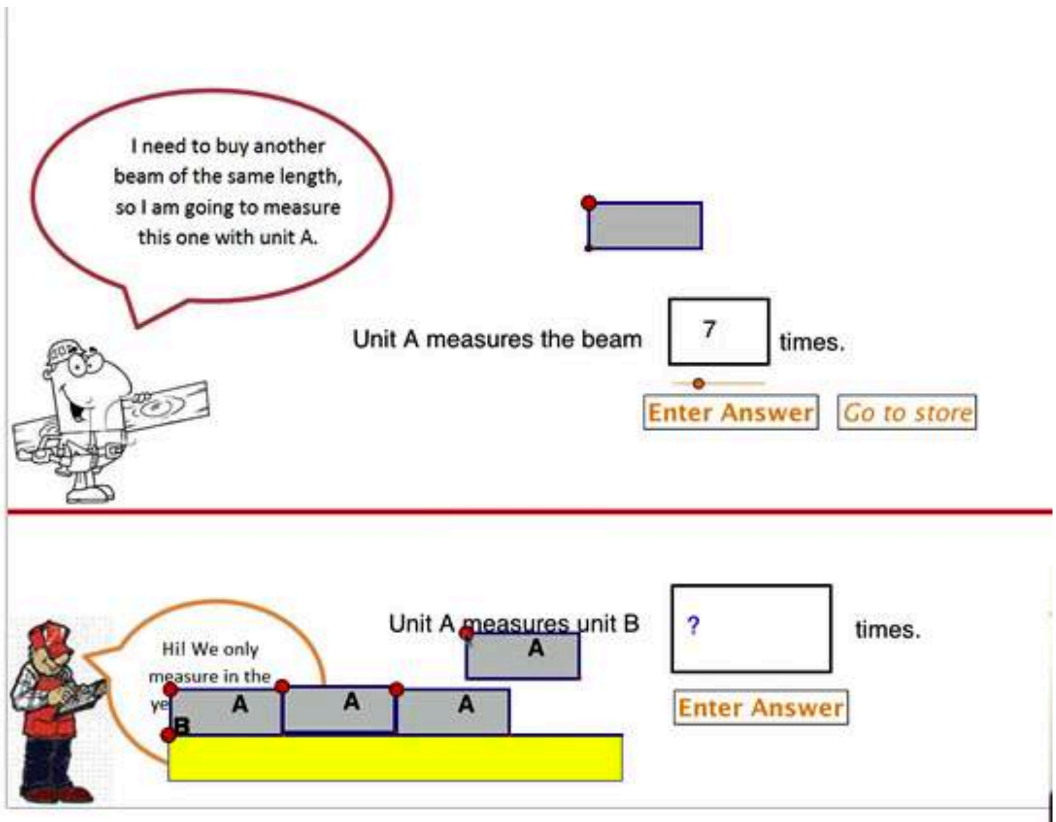
Task:

- Bob needs to buy a wooden beam that is the same size as the one he has at home. He measures it with Unit A at home. When he goes to the store, they only have Unit B.
- How can he tell the storekeeper the length he needs in Unit B?

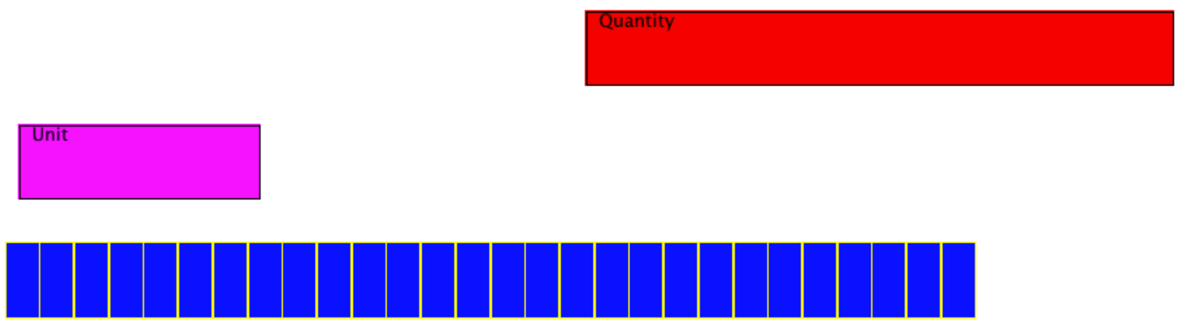
Measure the beam with Unit A

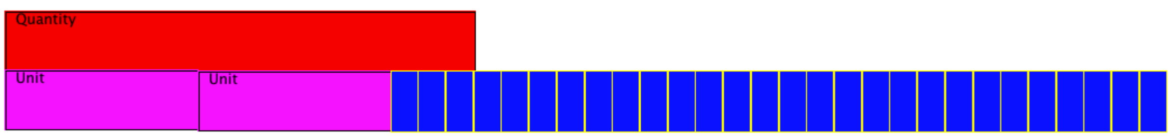


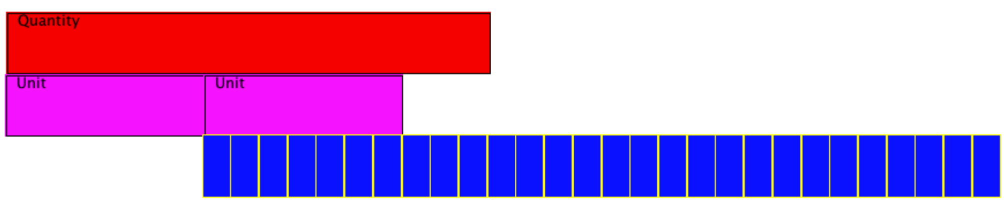
Measure Unit B with Unit A



<u>Task:</u> How many units long is the quantity?




Quantity
Unit
Unit


Quantity
Unit
Unit

Thinking about contexts

- Which contexts will allow students to make sense of the math?
- How will they use them to justify their solutions?
- Do they generalize?

Diagrams

How do you know?

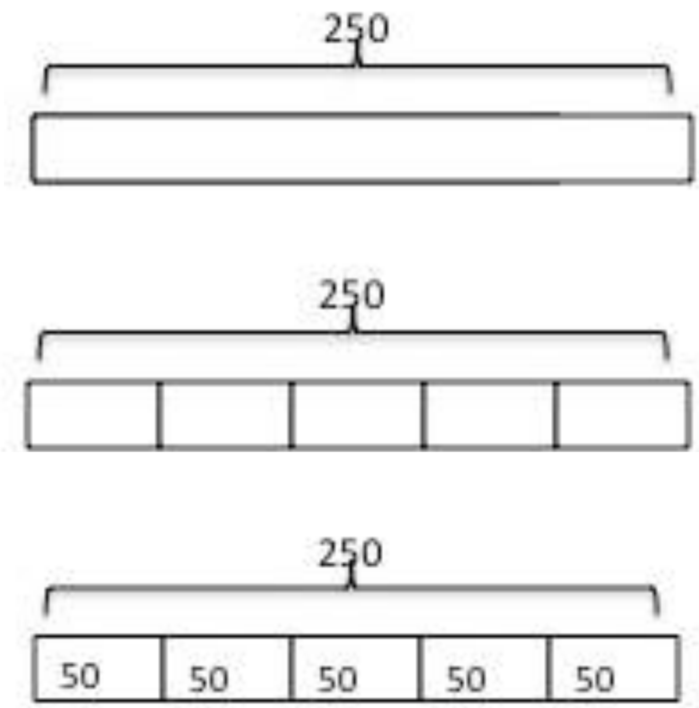
- Students can use a diagram to convince others of why a solution works

Tape Diagrams/Bar Models

- Can be used for initial whole number operations problems and then extend to fraction problems
- Students can make equal partitions to bars and count the iterations
- How is this similar/different to fraction bars?

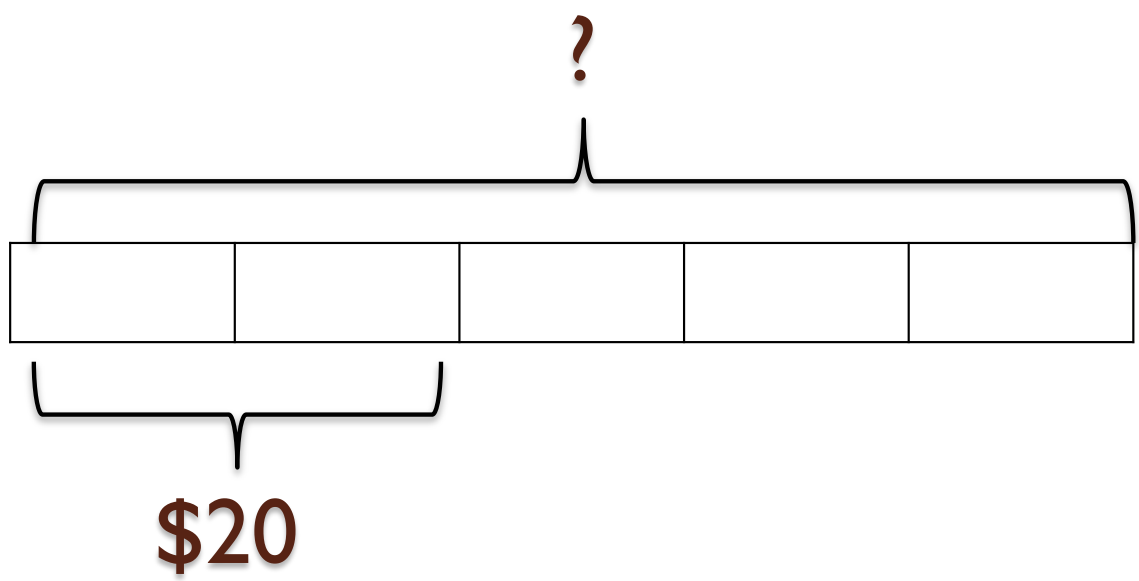
Fraction of a Set

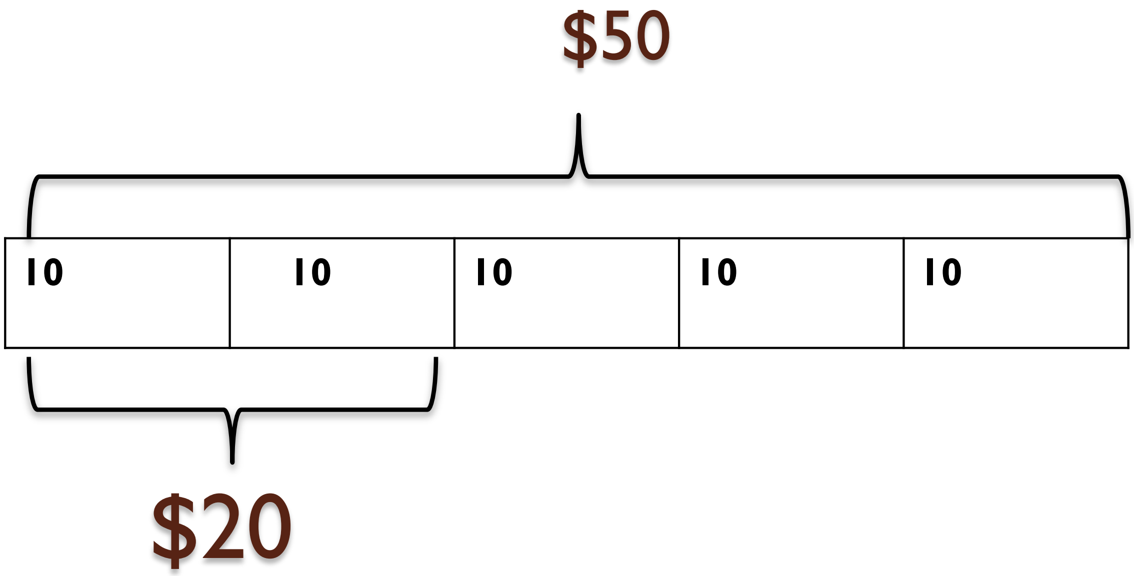
- There are 250 students in the fifth grade. Three-fifths of them speak two languages. How many speak two languages?


250
250
250
50
50
50
50
50

Fraction of a Set

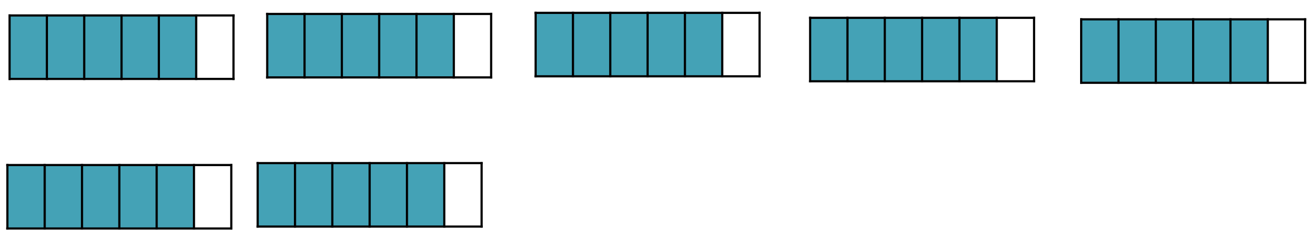
- Tom spent two-fifths of his money on a book. The book cost $20, how much money did he have at first?


?
$20


$50
10
10
10
10
10
$20

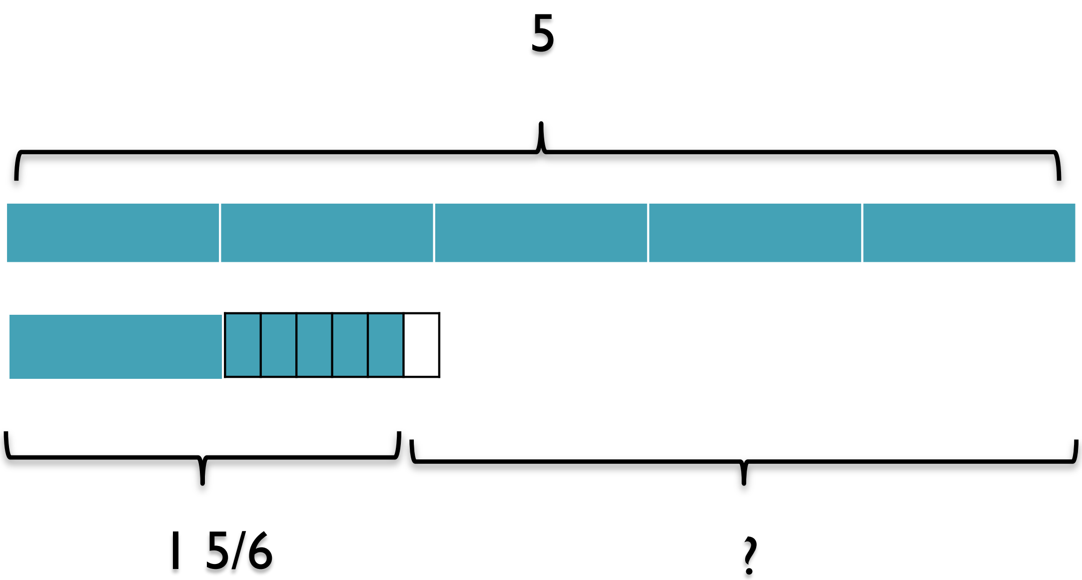
Fractions

- A recipe for a batch of cookies calls for 5/6 cup of flour. Tom wants to make 7 batches of cookies for a bake sale. How much flour does he need?



Fractions

- A builder began with 5 acres of land. He built a house on 1 5/6 acres of land. How much land was left unused?


5
1 5/6
?

Diagrams

- Students need to have time to develop meaning for the models
  - Danger is that it can become procedural
- Students can transition to using mental models
  - Imagine what you would draw for this problem
- Students can generalize from the model to the mathematical operations
  - How would you find x if you had a calculator?
  - How do you multiply a fraction by a whole number?

Other things to think about

- How do you decide what needs to be justified?
- How much practice do students need?
- How do you avoid making justification procedural?

In Summary

- Create learning opportunities for:
  - Practicing justification
  - Evaluating evidence used for justifications
  - Revising justifications
  - Making connections between one's activity and solution
  - Reasoning about mathematics (using manipulatives, diagrams, contexts)

Questions? Comments?

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