

# **The Scaling Factor**

## **NCTM San Francisco 2016**

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Learnzillion

# 5.NF.5

Interpret multiplication as scaling (resizing), by:

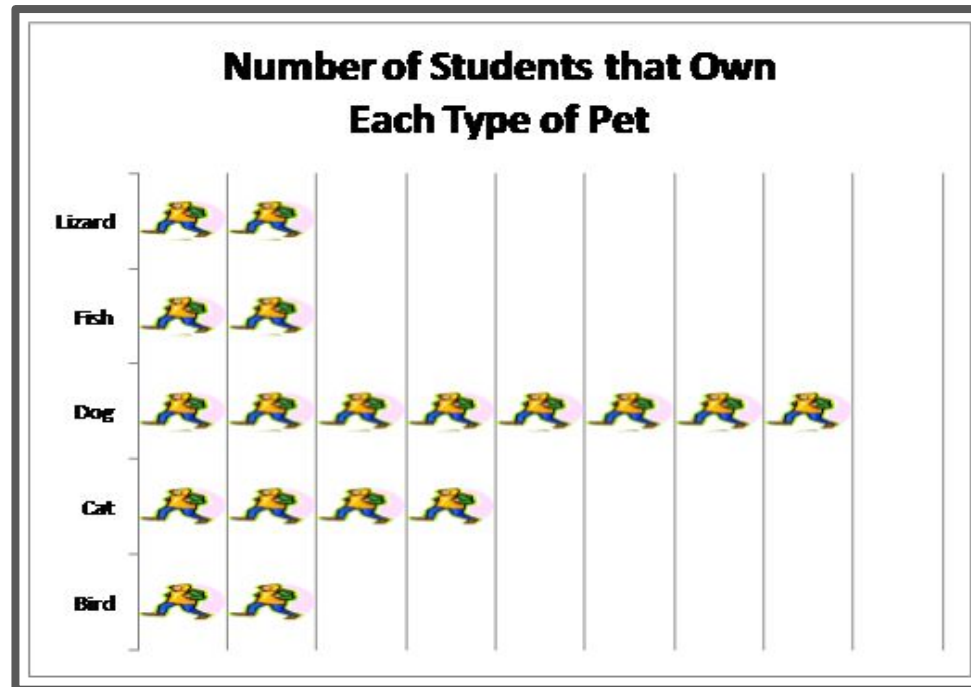
A. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

B. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1.

# 3.MD.3

Draw a **scaled** picture graph and scaled bar graph to represent data with several categories.

Solve “how many more” and “how many less one and two step problems



# 4.OA.1

Interpret a multiplication equation as a comparison

$$35 = 5 \times 7$$

“35 is five times as many as  
seven”  
and

“35 is seven times as many as five”

# 5.NF.5

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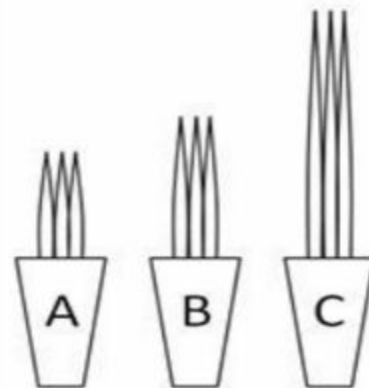
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# Multiplication of whole numbers

- Focus on the **relationship of the size** of the product to the size of the factors **not only repeated addition**
- The multiplier expresses the number of times the multiplicand increases or decreases; the multiplier is the **scaling factor**.

$$\text{multiplier} \times \text{multiplicand} = \text{product}$$

## 5.NF.5A Grass Seedlings



The students in Raul's class were growing seedlings in different conditions for a science project.

He noticed that Pablo's seedlings were one and a half times as tall as his own seedlings. He also saw that Celina's seedlings were three-fourths as tall as his own.

Which of the seedlings shown must belong to which student? Explain your reasoning.

## 5.NF.5A Grass Seedlings

### Solution

Since Pablo's seedlings are  $1\frac{1}{2}$  times as tall as Raul's, Pablo's seedlings must be taller than Raul's. Pablo's seedlings look like Raul's seedlings scaled by a factor larger than 1.

Since Celina's seedlings are  $\frac{3}{4}$  as tall as Raul's, Celina's seedlings must be shorter than Raul's. Celina's seedlings look like Raul's seedlings scaled by a factor smaller than 1.

Celina's seedlings are shorter than Raul's and Pablo's are taller than Raul's. Thus, the seedlings in pot A belong to Celina, the seedlings in pot B to Raul, and the seedlings in pot C belong to Pablo.



# Sticky Sort...



- Find the product of the two factors for each of your problems on the sticky notes.
- Place the sticky note under the appropriate heading:

greater than 3

equal to 3

less than 3

# Turn and Talk

- What did you notice about the factors with products that were less than three? Greater than three? Equal to three?
- Based on the patterns you see, what conjectures can you make?
- How could you use this activity with your students?

# A Number Talk Idea

$$1 \times 16$$

$$\frac{1}{2} \times 16$$

$$\frac{1}{4} \times 16$$

- What do you notice about the factors?
- Why are some of the products smaller than the whole number factor?
- Name a fractional factor that will increase the scale of the product.

# Problem:

Kaylea is putting bows on gifts.

Each bow uses three feet of ribbon.

She needs to make a bow that is four times larger than the other bows for a larger gift.

How many feet of ribbon will she need for the larger bow?

$$4 \text{ copies of } 3 \text{ feet} = \underline{\hspace{2cm}} \text{ feet}$$

$$4 \times 3 \text{ feet} = \underline{\hspace{2cm}} \text{ feet}$$

$$\text{multiplier} \times \text{multiplicand} = \text{product}$$

# Think about it as a compare problem:

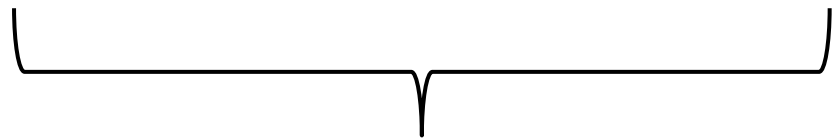
reference set:

3 feet of ribbon



multiplier:

4 times larger



Product

# Back to Kaylea



She is now going to make a much smaller bow. This bow will be one-third the size of the three foot bow.

How much ribbon will she need?

$$\frac{1}{3} \times 3 \text{ feet} = \underline{\hspace{1cm}} \text{ feet}$$

(multiplier  $\times$  multiplicand = product)

# Think about it as a compare problem:

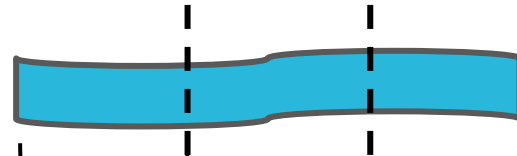
reference set:

3 feet of ribbon

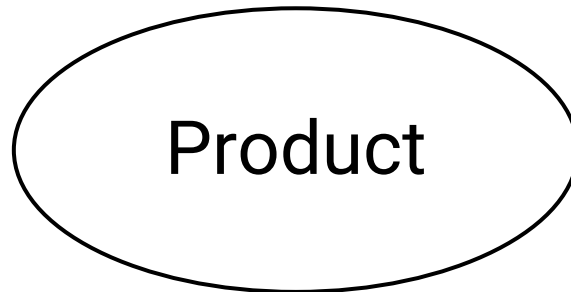


multiplier:

$\frac{1}{3}$  the size



Product



# Back to Kaylea...



She is now going to wrap a medium box.  
Again, the 3 foot bow will be too large!

This bow will need  $\frac{5}{12}$  the length of ribbon that  
she needs for the 3 foot bow.

How much ribbon will she need?

Avoid the algorithm and solve using a visual  
representation.



# Back to Kaylea...



She is now going to wrap a huge box.  
The 3 foot bow will be too small!

This bow will need to be  $2\frac{1}{2}$  times the size that she needs for the 3 foot bow.

How much ribbon will she need?

Avoid the algorithm and solve using a visual representation.

# Think of a copy machine...

- When copying the same size – you are not changing the size. You are multiplying by one (Identity property)
- When enlarging – the multiplier is greater than one – the product will be greater than the multiplicand.
- When shrinking – the multiplier is a fraction less than one – the product will be smaller than the multiplicand.

# Another Problem to consider:

Jane took  $\frac{2}{3}$  as much time to take a math test as Sandy.

Sandy took two hours to take the test, how long did it take Jane to take the test?

Express your answer in minutes.



What visual representations might your students use?



## Unit: Interpreting Multiplying Fractions as Scaling



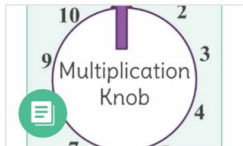
### 1. Understanding additive and multiplicative comparisons (C)

Lesson objective: Extend understanding that while additive comparisons show "n more than," multiplicative, or relative, comparisons show "n times as many" or "n times as large." Students bring prior knowledge...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP4, MP6

Tags: Conceptual, K-8 math curriculum



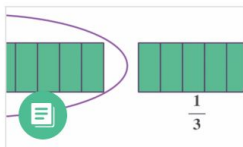
### 2. Understand that scaling does not always make numbers larger (C)

Lesson objective: Understand that scaling does not always make numbers larger and that if we multiply a number by a fraction less than one the number will scale down. Students bring prior knowledge of scaling by whole numbers and multiplying fractions...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP6

Tags: Conceptual, K-8 math curriculum



### 3. Practice scaling down (FP)

Lesson objective: Fluently predict products using the magnitude of the scaling factor. This lesson helps to build fluency with multiplication of whole numbers and fractions less than one. Tape diagrams and number lines are used here because...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, MP4

Tags: K-8 math curriculum, Procedural skills



### 4. Apply scaling down to a real-world situation (A)

Lesson objective: Apply understanding of multiplication of fractions less than one to solve real-world problems. This lesson provides an opportunity for students to apply their knowledge and understanding of multiplying fractions and whole numbers a ...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP4, MP6

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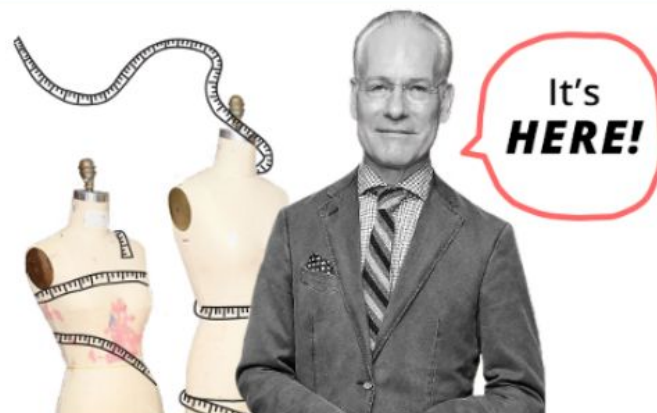


Lesson Plans



From the Creators of  
MATH 180

< See New Webisode



Premiere  
Webisode



Part 1: Meet Tim  
& Diane



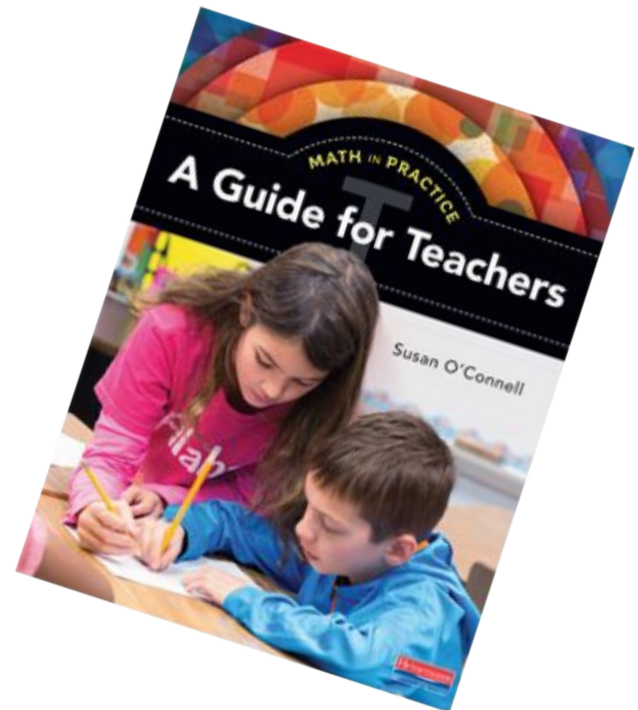
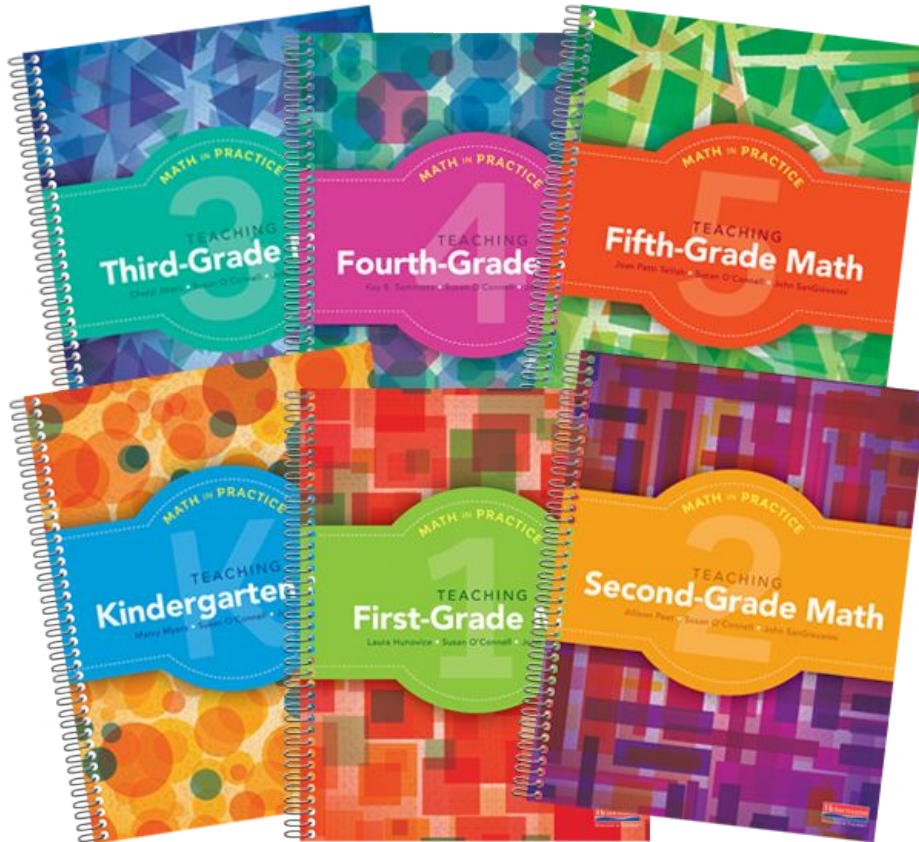
Part 2: Epic  
Scale





NCTM Booth# 316

## Math in Practice



Thank You!