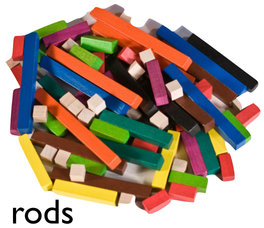
Exploring Cuisenaire Rods



- 3 attributes
  - distinctive features
  - relationships amongst the rods
  - relationship of the rods as a "whole set"
- if already familiar => build examples of your own identified feature

What is Number Sense?

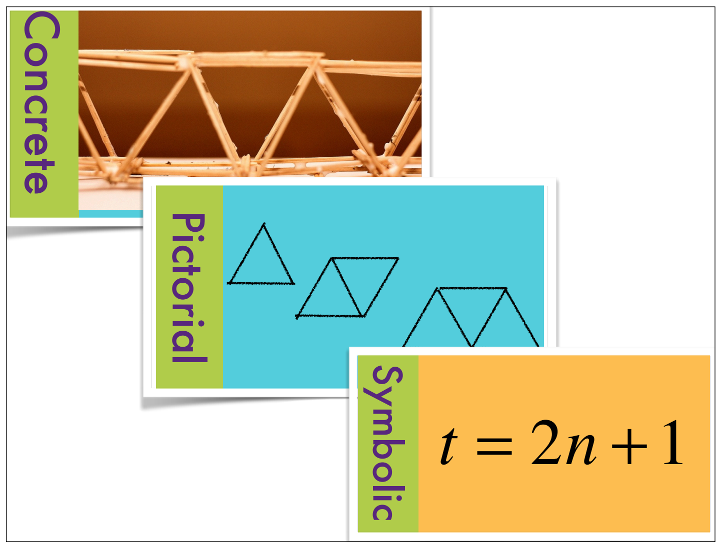
" Number Sense is an awareness and understanding about what numbers are, their relationships, their magnitude, the relative effect of operating on numbers, including the use of mental mathematics and estimation."

Fennel and Landis (1994)

What is Fluency?

Fluency is knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.

Parrish (2010)



- Direct Instruction
  Before - During - After
- Guided Practice
  Promote mastery & continued exploration
- Independent practice
  Practicing and mastering the strategy
  Thinking about meaning
  Communicating ideas

Students' role ...

wonder /struggle /connect with others / connect with concepts / understand

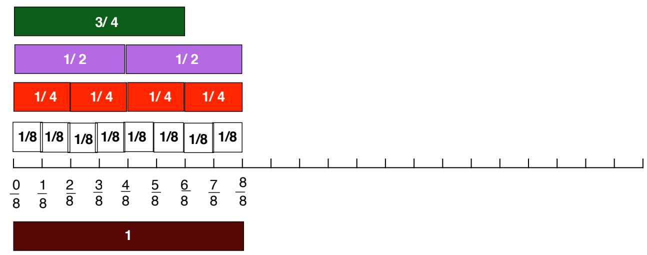
Your role ....

Ask questions to ...prompt / support / guide

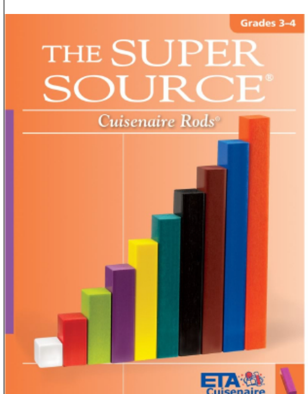
Fraction Big Ideas

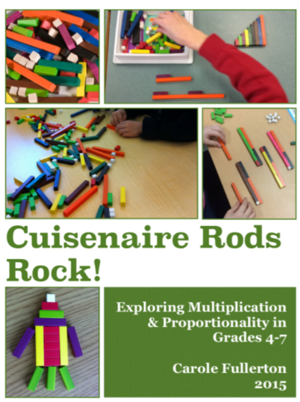
- Fractional parts are equal shares or equal-sized portions of a whole or unit. A unit can be an object or collection of things.
- Fractional parts (numerator & denominator) tell you relationship between parts to the whole.
- A fraction is not meaningful without knowing what the whole is.
- Renaming the fractions (equivalent fractions) enable comparison & computation.

Adapted from ***BIG IDEAS for Teaching Mathematics*** *(Marian Small)*



Here's What / So What / Now What?





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"Students with a strong number sense can think and reason flexibility with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among operations, figure mentally, and make reasonable estimates."

Marilyn Burns

FRACTION PAIRS

NUMBER

- Fractions
- Spatial visualization
- Equivalence

Getting Ready

What You'll Need

Cuisenaire Rods, 1 set per pair

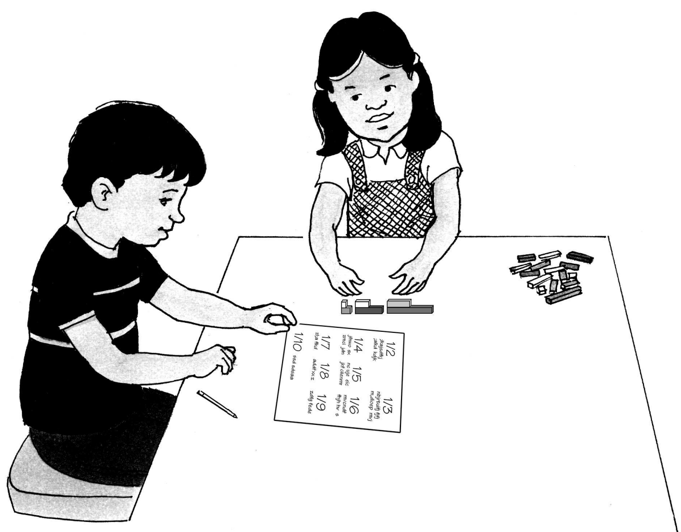
1-centimeter grid paper (optional), page 110

Overhead Cuisenaire Rods and or 1-centimeter grid paper transparency (optional)

Overview

Children find all pairs of Cuisenaire Rods that have a relationship that can be expressed in terms of a unit fraction. In this activity, children have the opportunity to:

- explore the meaning of fractions
- determine that the same fraction name can describe different rod pairs
- develop a mental picture of fractional parts of a whole



The Activity

Introducing

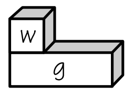
- Ask children to find a rod that is half the length of the orange rod. Have them explain their thinking.
- Verify that the yellow rod is half as long as the orange. Show children that this can be recorded either as $y = \frac{1}{2}o$ or $o = 2y$. Ask them why both recordings make sense.
- Ask children to find all other pairs of rods in which one rod is half the length of the other. Tell them to record their findings in the two ways you have described.
- Check to be sure that children understand the task and have recorded their findings correctly.

$p = \frac{1}{2}n$ or $n = 2p$	$r = \frac{1}{2}p$ or $p = 2r$
$g = \frac{1}{2}d$ or $d = 2g$	$w = \frac{1}{2}r$ or $r = 2w$

On Their Own

How many Cuisenaire Rod pairs can you find to show the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{1}{10}$?

- Work with a partner. Find a rod pair in which 1 rod is a third as long as another rod.
- Record your findings in 2 ways. Here is an example of how to record a white and light green rod pair:



$w = \frac{1}{3}g$ or $g = 3w$

- Find as many more rod pairs as you can that show $\frac{1}{3}$. Record each pair in 2 ways.
- Now, look for rod pairs that show $\frac{1}{4}$ and record each of those in 2 ways.
- Continue finding and recording rod pairs for all the fractions listed above until you think that you have found all the pairs possible.
- Be ready to explain why you think you have found all possible rod pairs for each of the fractions.

The Bigger Picture

Thinking and Sharing

Write the following fractions across the chalkboard: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$. Have children share their recordings by listing their sentences under the appropriate fraction. The list should begin to look like this.

1/2	1/3
$w = \frac{1}{2}r$ or $r = 2w$ $r = \frac{1}{2}p$ or $p = 2r$ $g = \frac{1}{2}d$ or $d = 2g$ $p = \frac{1}{2}n$ or $n = 2p$ $y = \frac{1}{2}o$ or $o = 2y$	$w = \frac{1}{3}g$ or $g = 3w$

Use prompts such as these to promote class discussion:

- What patterns do you notice for each fraction? for all the fractions?
- How do you know that the list for (name a fraction) is complete?
- Why isn't (name a rod) on this list?
- How can the same rod be used to represent two different fractions?
- Why are some fractions represented by fewer rod pairs than others?

Extending the Activity

Ask children to imagine that a red rod and an orange rod are combined in a train to form a new rod called "rorange." Have children repeat the activity and include the rorange rod when forming rod pairs.

Teacher Talk

Where's the Mathematics?

This activity can help deepen children's understanding of a fraction as a ratio of one whole number to another. As children place Cuisenaire Rods next to one another and decide how the length of the shorter rod may be expressed in terms of the length of the longer rod, they are modeling ratios that represent unit fractions.

The following table lists all possible answers. Individual pairs of children may not be able to find all these solutions on their own.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$w = \frac{1}{2}r$ or $r = 2w$	$w = \frac{1}{3}g$ or $g = 3w$	$w = \frac{1}{4}p$ or $p = 4w$
$r = \frac{1}{2}p$ or $p = 2r$	$r = \frac{1}{3}d$ or $d = 3r$	$r = \frac{1}{4}n$ or $n = 4r$
$g = \frac{1}{2}d$ or $d = 2g$	$g = \frac{1}{3}e$ or $e = 3g$	
$p = \frac{1}{2}n$ or $n = 2p$		
$y = \frac{1}{2}o$ or $o = 2y$		
$\frac{1}{5}$	**$\frac{1}{6}$**	**$\frac{1}{7}$**
$w = \frac{1}{5}y$ or $y = 5w$	$w = \frac{1}{6}d$ or $d = 6w$	$w = \frac{1}{7}k$ or $k = 7w$
$r = \frac{1}{5}o$ or $o = 5r$		
$\frac{1}{8}$	**$\frac{1}{9}$**	**$\frac{1}{10}$**
$w = \frac{1}{8}n$ or $n = 8w$	$w = \frac{1}{9}e$ or $e = 9w$	$w = \frac{1}{10}o$ or $o = 10w$

The question "How big is one half?" cannot be answered meaningfully until one knows the size of the whole unit; this is because a fraction indicates only a relationship between a part and the whole. As children look down the list for $\frac{1}{2}$, the idea of a fraction as a relationship, and not as an absolute quantity, is reinforced as they see the five different-sized pairs of rods used to represent one half. Additional reinforcement comes as children look across the lists and see that the same rod can have a variety of fractional names.

Children may approach this task randomly, but once they have completed their lists for $^1/_2$ and $^1/_3$, they usually find an organized way of approaching the work. For example, when searching for rod pairs for $^1/_4$, many children start with the shortest rod (white), place four of them in a train, and find w = $^1/_4$ p or p = 4w; then they move on to the next shortest rod (red), place four of them in a train, and find r = $^1/_4$ n or n = 4 r. When they try to repeat the process for the next shortest rod (light green), they discover that the four-car train is longer than an orange rod, so they look no further.

Children who use this method of searching for rod pairs recognize that the denominator of the fraction indicates how many of the shorter rod to put down (or how many equal parts are required to make the whole). That is, if the denominator is 4, they place four rods in a train and look for a longer rod with a matching length.

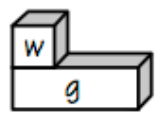
Children will notice that as they move along in the chart from $^1/_2$ to $^1/_{10}$, the lists get shorter. They will also notice that the white rod shows up on every list. When asked why certain rods are not on some lists, for example, "Why isn't purple on the $^1/_3$ list?" children may verbalize that $^1/_3$ means that 3 of one color equal the length of a longer rod. If they used 3 white rods, the length would be shorter than a purple and the next larger possibility, 3 reds, would be longer than a purple. Therefore, purple could not be used as the longer rod. If purple was used as the shorter rod, and three were placed in a train, the train would be longer than the orange rod. It is very important to emphasize that the denominator of a fraction indicates the number of equal parts that are required to make the whole.

Names: ______________________________

Fraction Pairs

How many Cuisenaire Rod pairs can you find to show the fraction $1/3$?

- Work with a partner. Find a rod pair in which 1 rod is a third as long as another rod.
- Record your findings in 2 ways. Here is an example of how to record a white and light green rod pair:



$w = \frac{1}{3} g$ or $g = 3w$

- Find as many more rod pairs as you can that show $\frac{1}{3}$. Record each pair in 2 ways.

- Now, look for rod pairs that show $\frac{1}{4}$ and record each of those in 2 ways.
- Continue finding and recording rod pairs for all the fractions listed above until you think that you have found all the pairs possible.
- Be ready to explain why you think you have found all possible rod pairs for each of the fractions.

Fractions of a Whole – How Many Ways?

Grade levels: 3/4
Math Concepts: Fractions of a whole, addition of fractions with like denominators
Task structure: Individual / Partners

Pose the following:

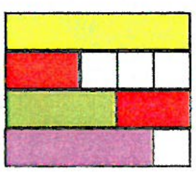
How many ways can you make the yellow rod?

Have students create trains of rods in any combination of colours that are the same length as a yellow. When students have exhausted all the ways, have them volunteer their favourites.

Explain that for this game, the yellow is the whole, or 1.
Ask:

- *If yellow is 1, what's the fractional name for a white?* (1/5)
- *If yellow is 1, what's the fractional name for a red?* (2/5)
- *A light green?* (3/5)
- *A pink/purple?* (4/5)
- *A yellow?* (5/5 or 1)

Once you've established names for each of the fractional pieces, have students pick their favourite train and record an equation to match it. In this example, a student wrote the equation to match the second train (red + white + white + white)



$$\frac{2}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5}$$

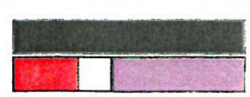
Explain that when mathematicians write a fraction, there is a top number (the **numerator**) and a bottom number (the **denominator**). The bottom number tells what we're counting, and the top number says how many we have. In the fraction 2/5, we are counting fifths, and we have 2 of them.

Now hold up a black rod (7) and tell students that this is now 1. Have students repeat the task above, re-defining each piece (white is now 1/7, red is now 2/7, etc) and creating as many different trains as they can to make one whole. This time, have students record all of their equations in a list.

Celebrate the many ways we can make one whole out of sevenths!

As an extension, have students read an equation aloud for a partner to build. Have both students confirm that the total is indeed 7 sevenths or 1 at the end.

$$\frac{2}{7} + \frac{1}{7} + \frac{4}{7} = \frac{7}{7}$$



Note: This task intentionally uses fifths and sevenths in order to avoid the use of equivalent fractions. Consider doing an introduction to equivalent fractions before extending this game to different rods with more factors.

1-CENTIMETER GRID PAPER

NAMING RODS

NUMBER

- Fractions
- Comparing
- Looking for patterns

Getting Ready

What You'll Need

Cuisenaire Rods, 1 set per pair

Overhead Cuisenaire Rods and/or 1-centimeter grid paper transparency (optional)

Overview

Children assign a value of one whole unit to a Cuisenaire Rod of their choice. They then identify each of the other rods as a number based on its relationship to the unit rod. In this activity, children have the opportunity to:

explore the meaning of rational numbers

see that the same fraction can name rods of different lengths

see that the same rod can be named with different fractions

The Activity

Introducing

p			
r		r	
w	w	w	w

Invite children to use their Cuisenaire Rods to build the one-color trains that are the same length as the purple.

Ask children to think of the purple rod as one whole unit and to name the fractions described by each of the red and white rods. Establish that if $p = 1$, $w = \frac{1}{4}$, and $r = \frac{1}{2}$ or $\frac{2}{4}$.

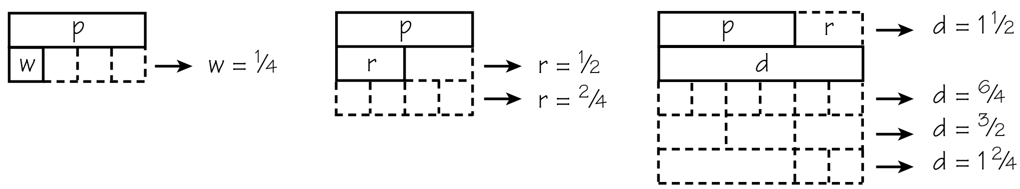
Have children find a fractional name for the light green rod based on the purple unit rod. Elicit the fact that since 1 light green = 3 whites and $w = \frac{1}{4}$, then $g = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, or $\frac{3}{4}$.

Have children find the names for the yellow rod. Establish that it has two fractional names: $1\frac{1}{4}$, since it is the same length as one purple and one white rod, and $\frac{5}{4}$, since it is as long as five white rods.

On Their Own

If you give a Cuisenaire Rod the value of 1, what are the numerical names for each of the other rods?

- Work with a partner. Choose a purple rod and assign it a value of 1.
- Find all the names of each of the other rods based on the fact that purple equals 1 whole. The names for the other rods may be fractions, mixed numbers, or whole numbers. Here are some solutions:



- Record all the solutions and look for patterns.
- Now choose a rod of another color and assign it the value of 1. Again, find and record all the numerical names for each of the other rods.
- Choose a different-colored rod and repeat the process.
- Be ready to explain how you know you have found all the possible solutions for the rods you chose to be 1.

The Bigger Picture

Thinking and Sharing

Have children share their solutions based on the purple rod and explain their methods. Then invite volunteers to share their solutions for other unit rods. Create a class chart with a column of solutions for each unit rod.

Use prompts such as these to promote class discussion:

- What is a good answer to this question: "How big is $\frac{1}{2}$?"
- How can you explain calling different rods by the same fractional names?
- How can you explain calling the same rod by so many different fractional names?
- What do you notice about names of rods shorter than 1? longer than 1?
- How can you explain that one fractional name can be represented by different rods?
- When you studied all your results, what did you notice?

Extending the Activity

1. Have children pick a rod and make trains in which that rod represents each of the following fractions: 1/2, 1/3, 1/4, 1/6, and 3/4.

Teacher Talk

Where's the Mathematics?

Naming Rods deepens children's understanding of the meaning of "fraction." As they place one Cuisenaire Rod next to another and decide how one may be expressed in terms of the other, they analyze part-to-whole relationships and form ratios to express the relationships. Children also begin to understand the reasoning behind fraction names: For example, they see that because it takes three light greens to make one blue, a light green rod can be named 1/3 when the blue rod is the unit.

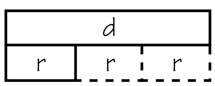
This table summarizes the fractional equivalents for each unit rod.

	w = 1	r = 1	g = 1	p = 1	y = 1	d = 1	k = 1	n = 1	e = 1	o = 1
w	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10
r	2	2/2 1	2/3	2/4 1/2	2/5	2/6 1/3	2/7	2/8 1/4	2/9	2/10 1/5
g	3	3/2	3/3	3/4	3/5	3/6 1/2	3/7	3/8	3/9 1/3	3/10
p	4	4/2 2	4/3 1 1/3	4/4 1	4/5	4/6 2/3	4/7	4/8 2/4 1/2	4/9	4/10 2/5
y	5	5/2 2 1/2	5/3 1 2/3	5/4 1 1/4	5/5 1	5/6	5/7	5/8	5/9	5/10 1/2
d	6	6/2 3	6/3 2	6/4 1 2/4 3/2 1 1/2	6/5 1 1/5	6/6 1	6/7	6/8 3/4	6/9 2/3	6/10 3/5
k	7	7/2 3 1/2	7/3 2 1/3	7/4 1 3/4	7/5 1 2/5	7/6 1 1/6	7/7 1	7/8	7/9	7/10
n	8	8/2 4	8/3 2 2/3	8/4 2	8/5 1 3/5	8/6 1 2/6 4/3 1 1/3	8/7 1 1/7	8/8 1	8/9	8/10 4/5
e	9	9/2 4 1/2	9/3 3	9/4 2 1/4	9/5 1 4/5	9/6 1 3/6 3/2 1 1/2	9/7 1 2/7	9/8 1 1/8	9/9 1	9/10
o	10	10/2 5	10/3 3 1/3	10/4 2 2/4 5/2 2 1/2	10/5 2	10/6 1 4/6 5/3 1 2/3	10/7 1 3/7	10/8 1 2/8 5/4 1 1/4	10/9 1 1/9	10/10 1

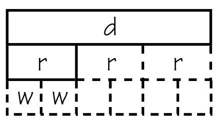
2. Have children repeat the original activity for rods longer than orange. For example, they might combine the red and orange rods to create a “rorange” rod and assign this rod a value of 1.

Organizing children's data in a table similar to the one shown may lead them to notice certain patterns. One pattern is that rods that are shorter than the unit rod are named with proper fractions (numerator smaller than the denominator) and rods that are longer than the unit rod are named with improper fractions, mixed numbers, or whole numbers. Another pattern is that the numbers within the columns increase by the amount that represents the value of the white rod in that column. For example, when light green is the unit, the white rod is $\frac{1}{3}$ and the numbers in the column are $\frac{1}{3}$, $\frac{2}{3}$, 1 or $\frac{3}{3}$, $\frac{4}{3}$ or $1\frac{1}{3}$, and so on. Children who see such patterns may be able to find errors and fill in "holes" in their data.

Children will find different names based on the combination of rods they use. In doing so, children have an opportunity to explore the concept of equivalent fractions. In the examples below, dark green is one whole unit. Placing red rods next to the dark green would lead most children to conclude that r = $\frac{1}{3}$. If children think in terms of red and white rods, however, they might also correctly conclude that r = $\frac{2}{6}$.

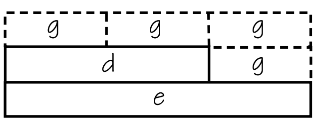


r = $\frac{1}{3}$

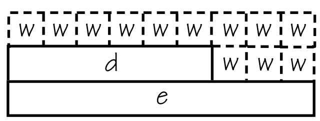


r = $\frac{2}{6}$

Similarly, children may conclude that the blue rod is $1\frac{1}{2}$ (or $\frac{3}{2}$) or $1\frac{3}{6}$ (or $\frac{9}{6}$).



e = $1\frac{1}{2}$ or $\frac{3}{2}$



e = $1\frac{3}{6}$ or $\frac{9}{6}$

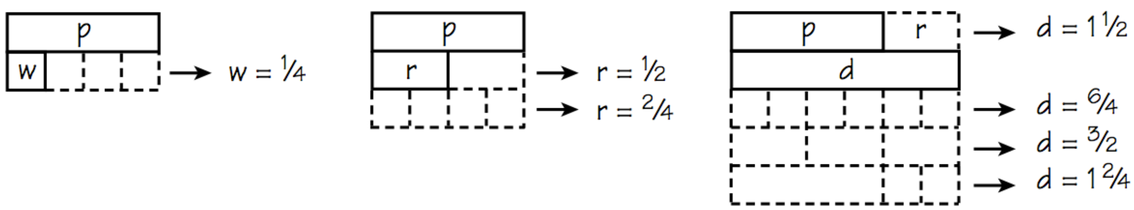
As children examine their results and see that $\frac{1}{2}$ can name all the rods from red through orange, they realize that the question "How big is $\frac{1}{2}$?" leads to another question: “$\frac{1}{2}$ of what?” That is, a fraction name expresses the relationship between the part and the whole, and the larger the whole, the larger the half. Children also see that one rod can have a variety of names. For example, the red rod can be called $\frac{1}{5}$ when the orange rod is the unit rod, whereas it can be called $\frac{2}{7}$ when the black rod is the unit. Children may then conclude that the red rod is $\frac{1}{5}$ of the orange rod, and that the red rod is *also* $\frac{2}{7}$ of the black rod.

Names: ______________________________________

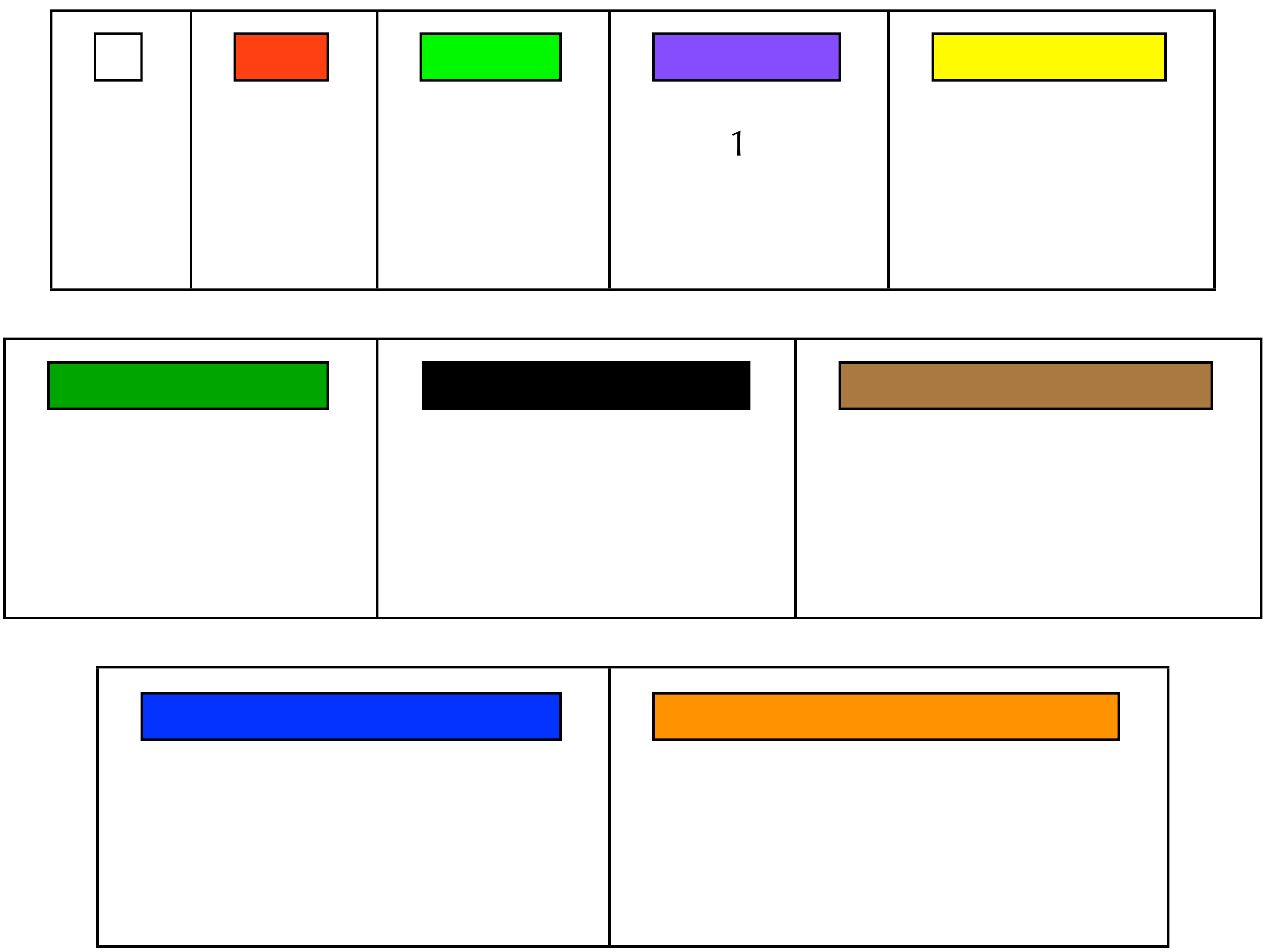
Naming Rods

If you give a Cuisenaire Rod the value of 1, what are the numerical names for each of the other rods?

- Work with a partner. Choose a purple rod and assign it a value of 1.
- Find all the names of each of the other rods based on the fact that purple equals 1 whole. The names for the other rods may be fractions, mixed numbers, or whole numbers. Here are some solutions:



If  = 1, then:



- Now try some more, and repeat the process.
- Be ready to explain how you know you have found all of the possible solutions for the rods you chose to be 1.

If ______________________________ = 1, then:

If ______________________________ = 1, then:

1-CENTIMETER GRID PAPER

Cuisenaire Comparison

How do each of these rods compare?

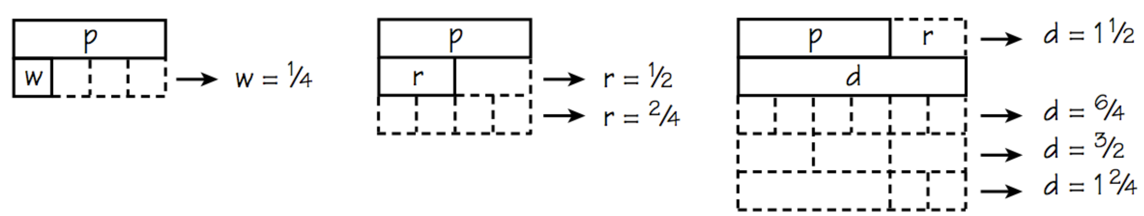
	White	Red	Light Green	Pink/ Purple	Yellow	Dark Green	Black	Brown	Blue	Orange
White										
Red										
Light Green										
Pink/ Purple										
Yellow										
Dark Green										
Black										
Brown										
Blue										
Orange										

Names: ______________________________

Naming Rods

If you give a Cuisenaire Rod the value of 1, what are the numerical names for each of the other rods?

- Work with a partner. Choose a purple rod and assign it a value of 1.
- Find all the names of each of the other rods based on the fact that purple equals 1 whole. The names for the other rods may be fractions, mixed numbers, or whole numbers. Here are some solutions:

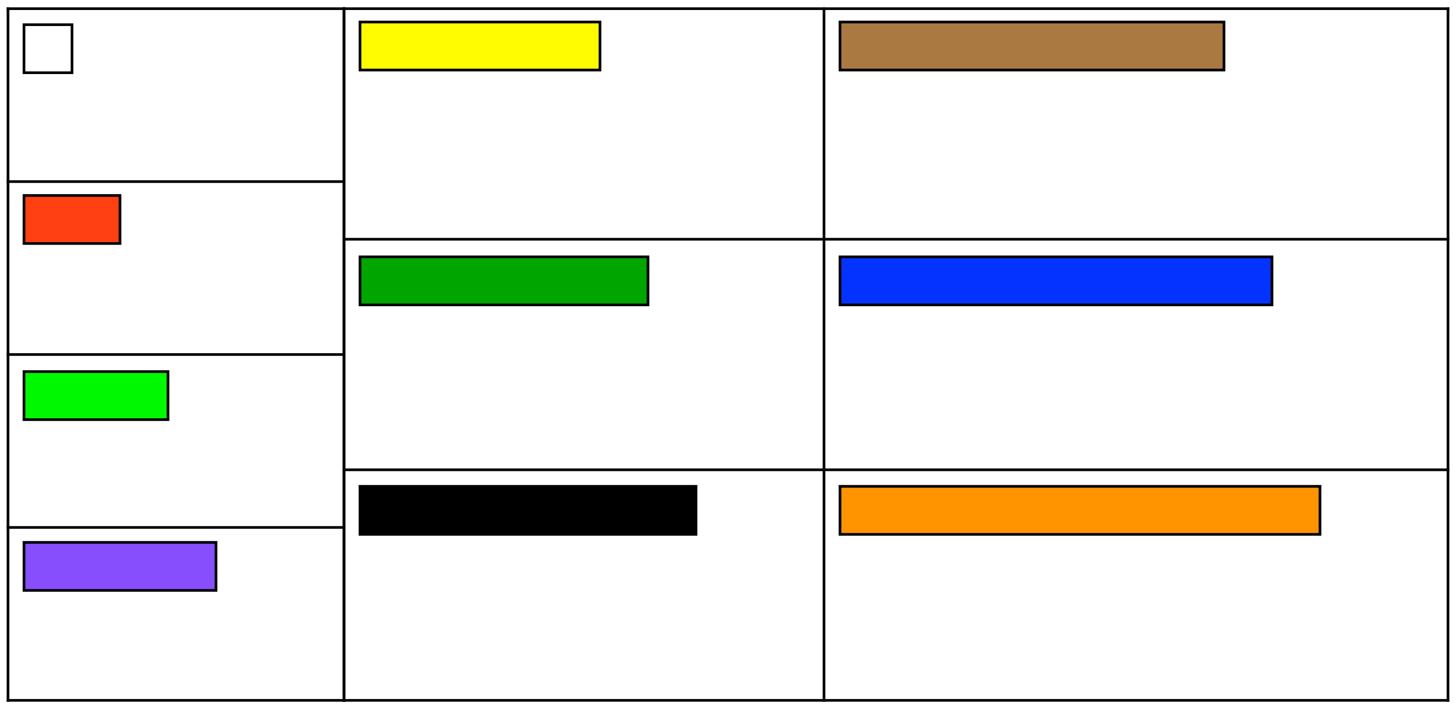


If = 1, then:

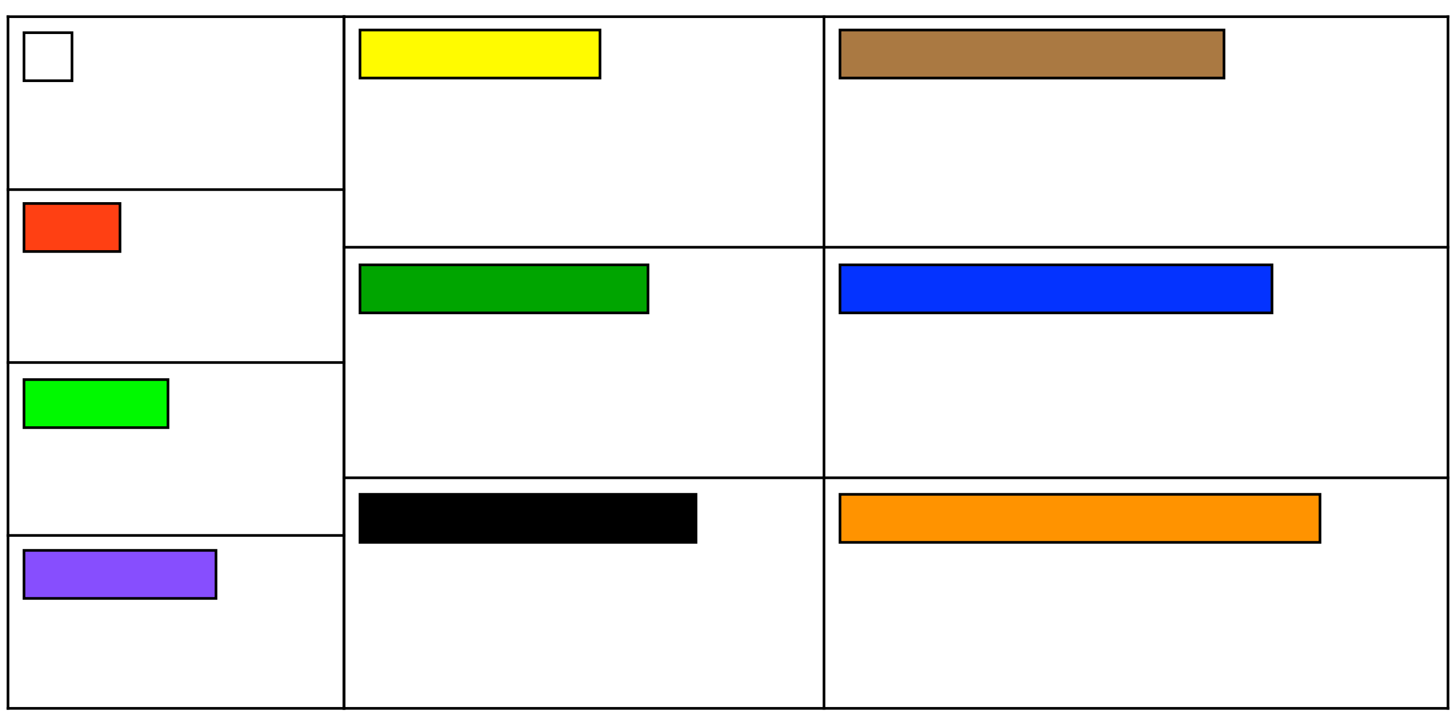
			1	

- Now try some more, and repeat the process.
- Be ready to explain how you know you have found all of the possible solutions for the rods you chose to be 1.

If ________________________________ = 1, then:



If ________________________________ = 1, then:



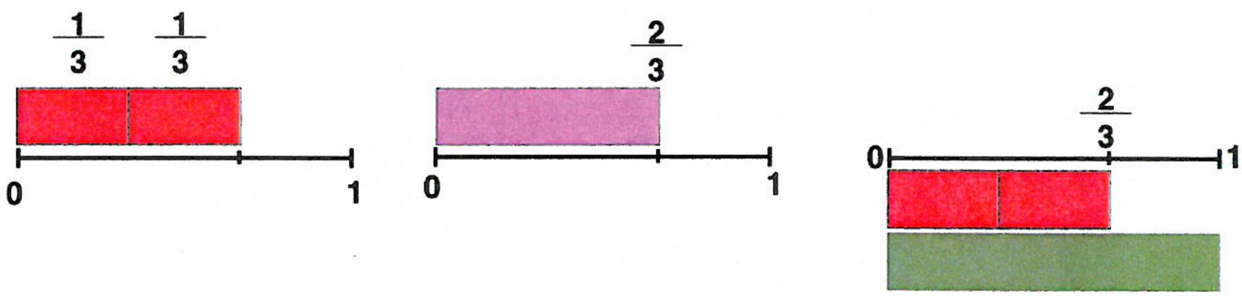
Plotting Fractions on a Number Line

Grade levels: 4/5/6
Math Concepts: Representing fractions and mixed numerals
Task structure: Whole class / Partners / Individual

Using a ruler, have students draw a line that's exactly 6 cm long. Tell them to them mark the left end with a 0 and the right end with a 1. Explain that this is a number line.

Next, ask:
Where would you put the fraction 2/3 on this line?
How can the rods help you?

Watch to see how students attack this problem; who uses the rods, the ruler or a combination of the two. Have students share their strategies for placing the mark on the line. For example, since 6 cm is the exact length of a dark green Cuisenaire rod, some may use rods to model:



Explain that the Cuisenaire rods can be used to mark referent points on a number line; that we can first build the fraction with the rods, use the denominator as the line length, and then build the corresponding fractional piece using another rod.

Have students use rods to model the following fractions and then plot them on a number line.

3/8

5/7

4/9

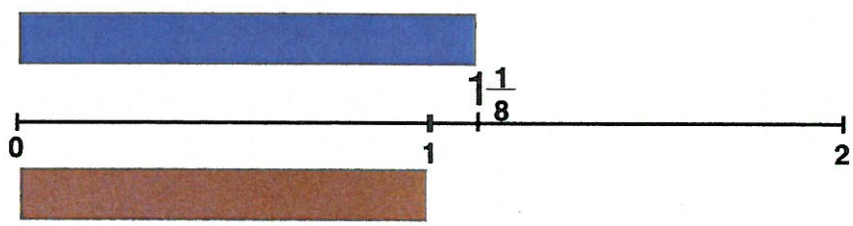
3/4 * (have students make this fraction in at least 2 ways)

When you're confident that students are comfortable with the process, give the next challenge.

Have students plot the fraction 9/8 on a number line. Give them time to struggle with this, and to sort out how the plotting of proper fractions is the connected to this more complex task.

Have students share their strategies before continuing. They will have lots of interesting methods to share!

Confirm that when we plot an improper fraction, it makes sense to create a longer number line, to make the new endpoints 0 and 2, (or even 3!), to record the fraction as a mixed numeral, etc.



Have students complete several other examples, using the rods for support. Consider:

7/6	8/5	5/3
9/7	10/6	9/4

For additional practice, have students draw 2 Cuisenaire rods from a pile and create an improper fraction. Students should then plot and label the corresponding number line.

FRACTION NUMBER LINES w Cuisneaire Rods