Powerful Math Models to Develop Conceptual Understanding and Procedural Fluency

Arrays & Number Lines

Who am I?

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What are models?

Mathematical models are learners' representations of situations or problems. These models can develop into powerful tools for thinking and reasoning.

e.g., 10-frame, number line, array

Fosnot, ed. (2010). Models of Intervention in Mathematics.

What models do you see in your schools across different grades?

Introduce yourself to an elbow partner and share.

Why use models for math?

- ➢ Powerful mathematical tools for reasoning
- ➢ Support a variety of learning preferences
- ➢ Support the delivery of the Common Core State Standards

Students . . . “must learn to see, organize, and interpret the world through and with mathematical models.”

Fosnot (ed.). 2010. Models of Intervention in Mathematics. p. 21

When/how to use math models?

- ✓ To model specific situations in mathematics
- ✓ As a tool to represent students' strategies and mathematical thinking
- ✓ As a tool for thinking

Fosnot et al. (2005). Contexts for Learning Mathematics.

Learning goals for today

Why are the array and the number line powerful models of math?

How do they support students' mathematical development – conceptual understanding and procedural fluency?

Examples of Continuums of Learning Using a Math Model Across Grades

The Array; The Number Line

The Array

Why arrays?

"Array" appears in the Common Core State Standards for Mathematics 17 times

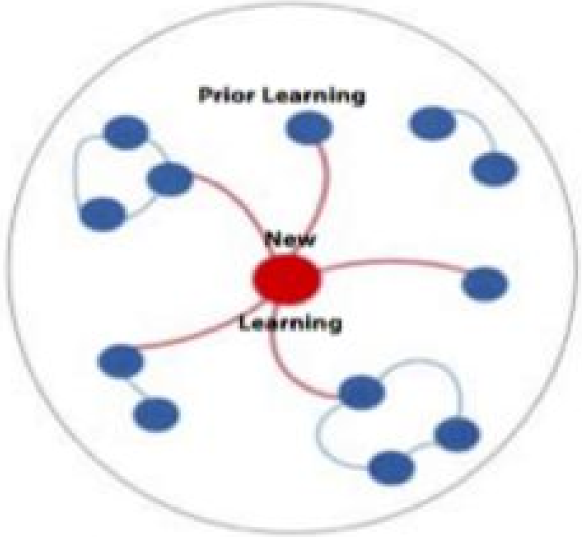
From kindergarten to grade 5

Interconnect with area models

More connections => increased understanding

“We understand something if we see how it is related or connected to other things we know.”
(Van de Walle, 2007, p. 4)

“The degree of understanding is determined by the number and strength of the connections.”
(Hiebert & Carpenter, 1992, p. 67)



Van de Walle

What is an **array**?

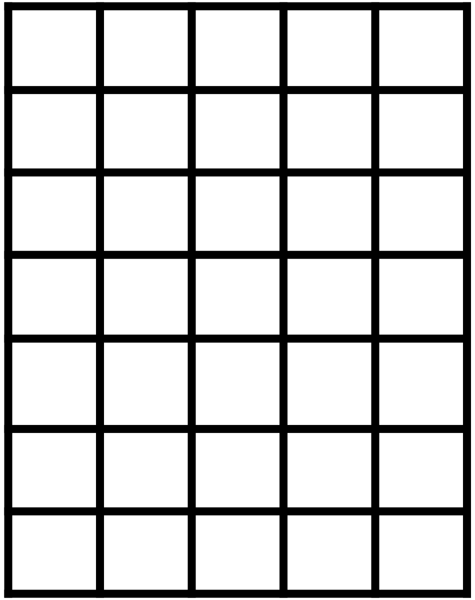
Array - A rectangular arrangement of objects into rows and columns, used to represent multiplication
(e.g., 5 x 3 can be represented by 15 objects arranged into 5 columns and 3 rows).

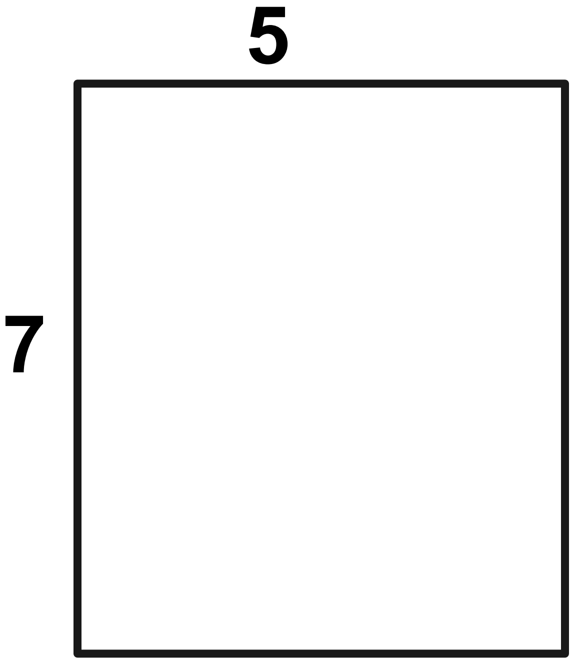
Source: Ontario Gr. 1-8 Math Curriculum

Use an array to model . . .

5 x 7

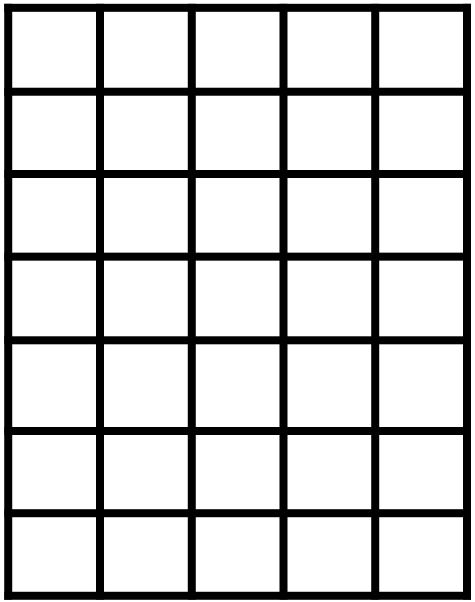
Does your **5 x 7** array look like any of these?

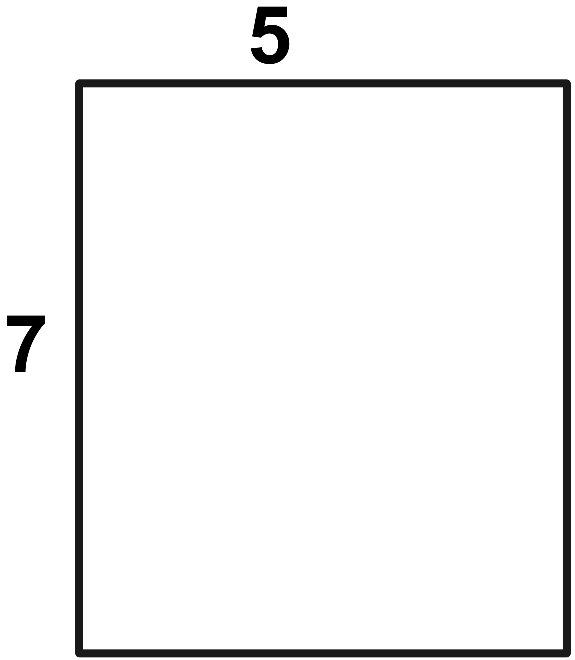




Continuum

"**closed**" to "**open**" arrays…"**area model**"





In which grades, have you seen “arrays” used?

What did they look like?

Talk Time

Children who struggle to commit basic facts to memory often believe that there are "hundreds" to be memorized because they have little or no understanding of the relationships among them.

Source: Fosnot & Dolk. (2001). Young Mathematicians at Work: Constructing Multiplication and Division

Memorization OR Automaticity

- committing the results of operations to memory so that thinking is unnecessary

- relies on thinking about relationships

Source: Fosnot & Dolk. (2001). Young Mathematicians at Work: Constructing Multiplication and Division

Facts should eventually be memorized.

Question:
How should this memorization be achieved?

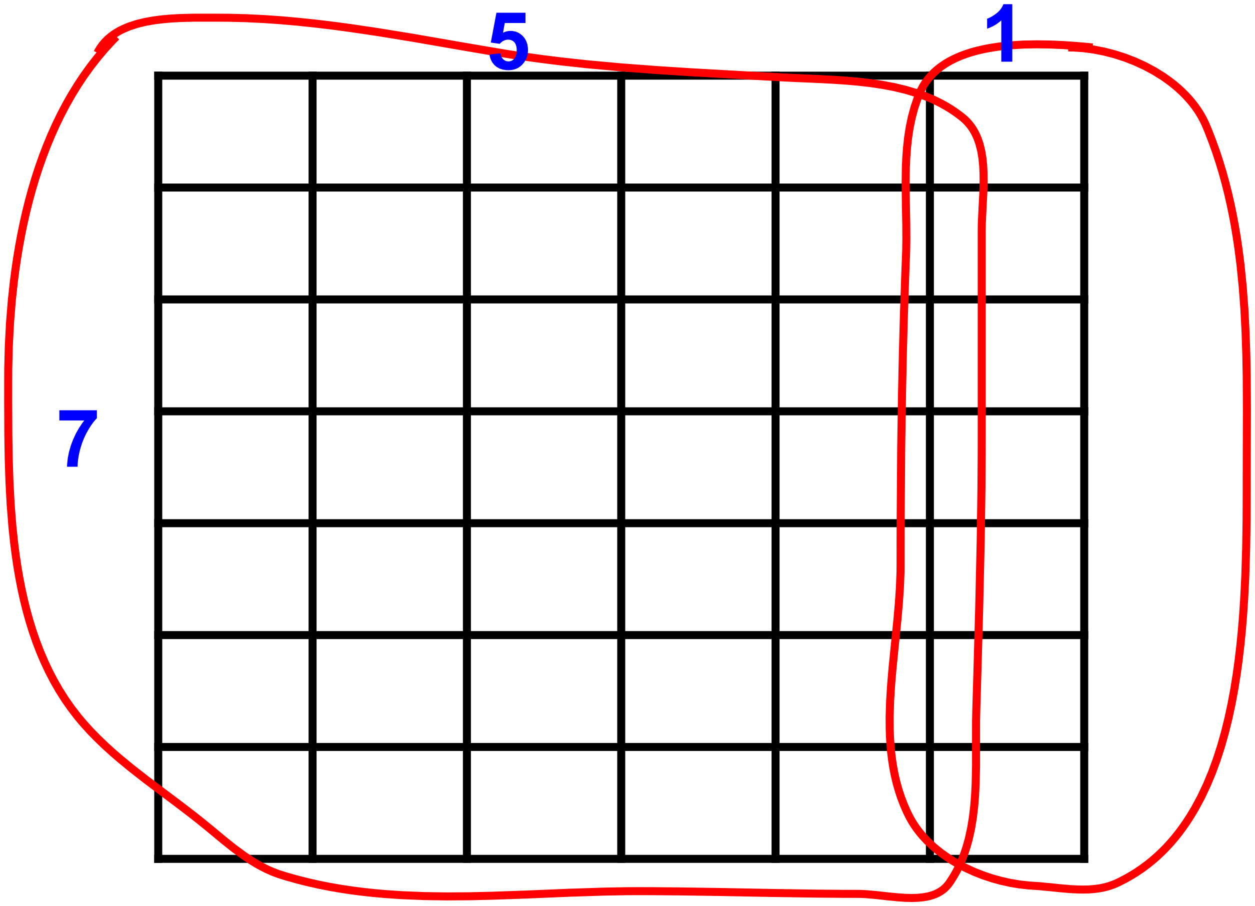
Rote drill and practise (memorization) **or** by focusing on relationships (automaticity)?

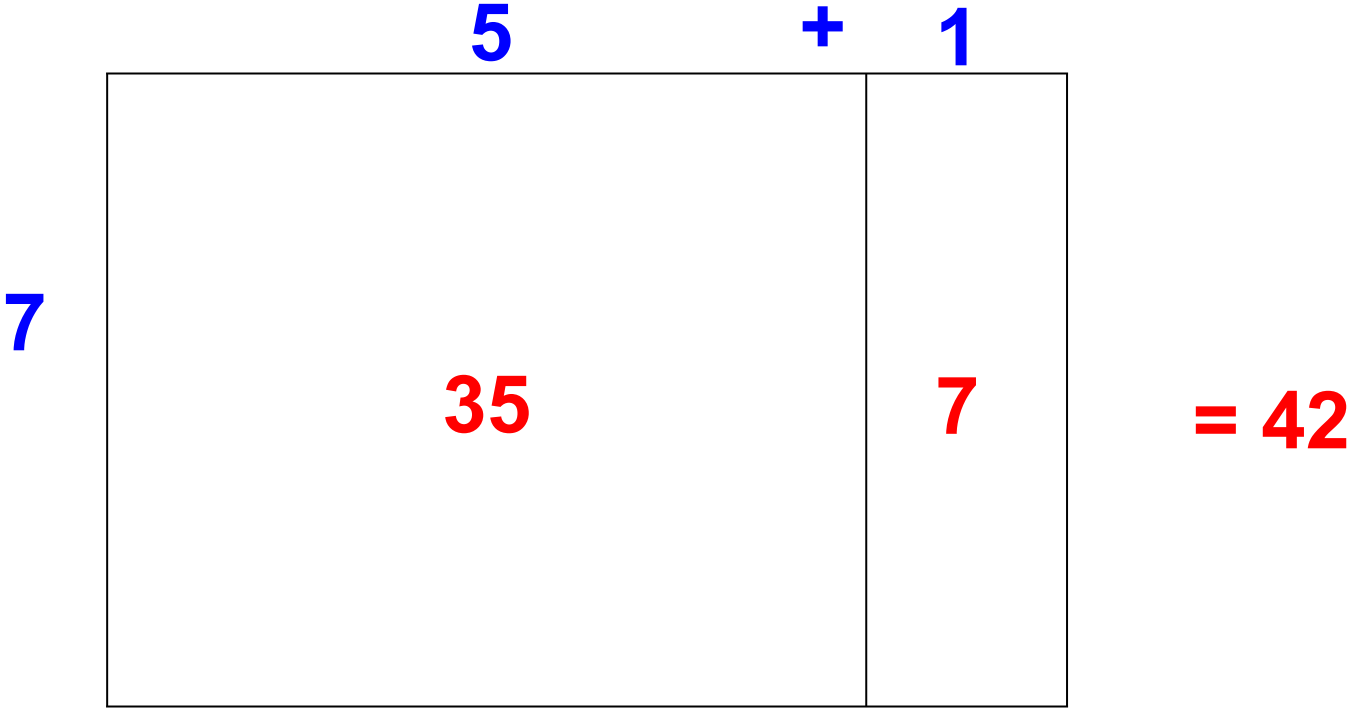
Source: Fosnot & Dolk. (2001). Young Mathematicians at Work: Constructing Multiplication and Division

Build on your **5 x 7** array to model . . .

6 x 7

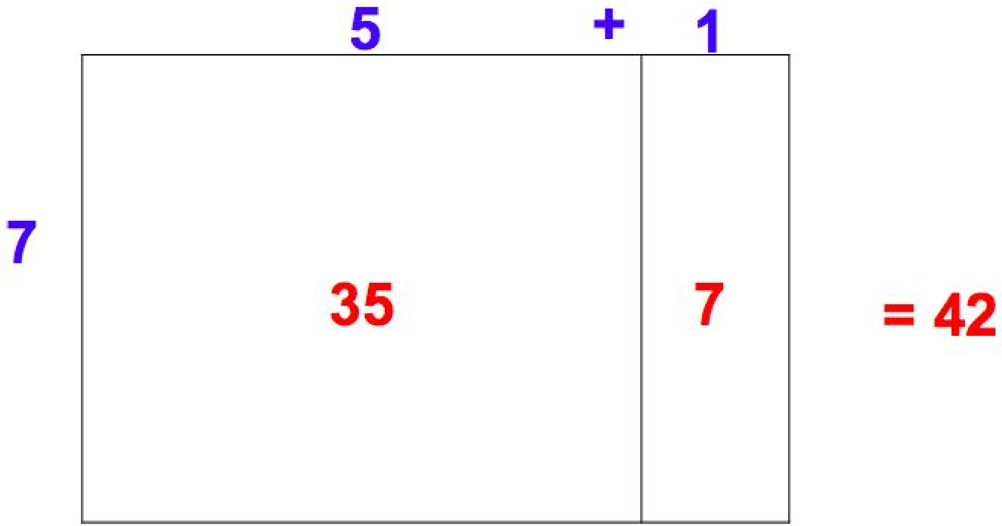
Circle and label the **5 x 7** part of the array.
What's left?
Circle and label it.


5
1
7


5
+
1
7
35
7
= 42

Open Array => Growth Mindset

Every open array represents a fact a student knows and can use



Open Array => Growth Mindset

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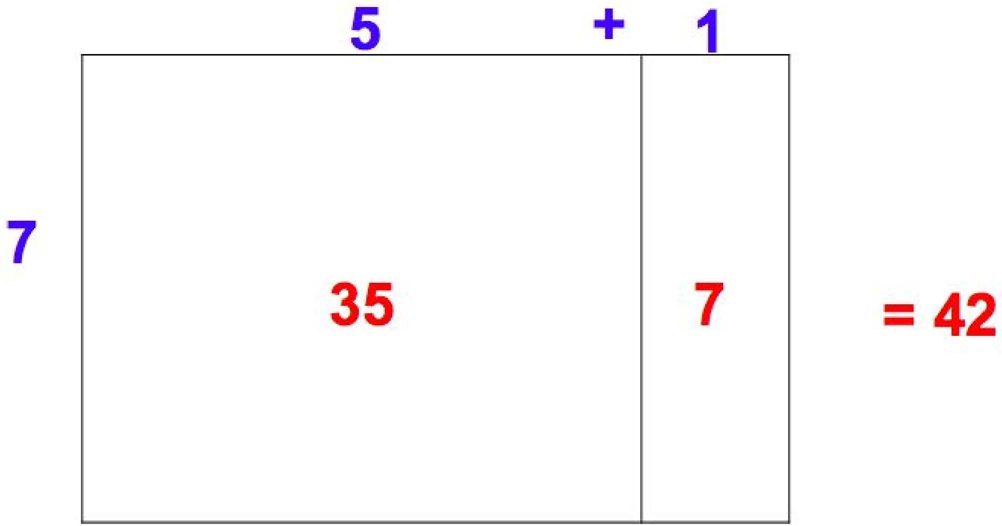


Partial Products & the Distributive Property

Use an open array (or area model) to model the distributive property and solve . . .

28 x 43

Example: (20 + 8) x (40 + 3)

Distributive Property → Partial Products

	20 +	8
40 +	800	320
3	60	24

28
X 43
24
60
320
800
1204

How does this connect with the standard/usual algorithm?

	20	+ 8
40	800	320
+		
3	60	24

$$
\begin{array}{r}
28 \\
\times\ \underline{43} \\
24 \\
60 \\
320 \\
\underline{800} \\
1204
\end{array}
$$

How does this connect with the standard/usual algorithm?

	20	+ 8
40	800	320
+		
3	60	24

28
X 43
24
60
320
800
1204

3
2
28
X 43
84
1120
1204

How does this connect with the standard/usual algorithm?

	20	+ 8	
40	800	320	= 1120
+			
3	60	24	= 84

28
X 43
24
60
320
800
1204

3
2
28
X 43
84
1120
1204

Remember – “Why arrays?”

“Array” appears in the Common Core State Standards for Mathematics 17 times

From kindergarten to grade 5 **(and beyond!)**

Interconnect with area models **(not just 1 more thing)**

Distributive Property → Partial Products

	20	+ 8
40	800	320
+ 3	60	24

```
   28
 X 43
   24
   60
  320
  800
 1204
```

Use an open array to model the distributive property and expand . . .

$$(x + 3)\,(x + 2)$$

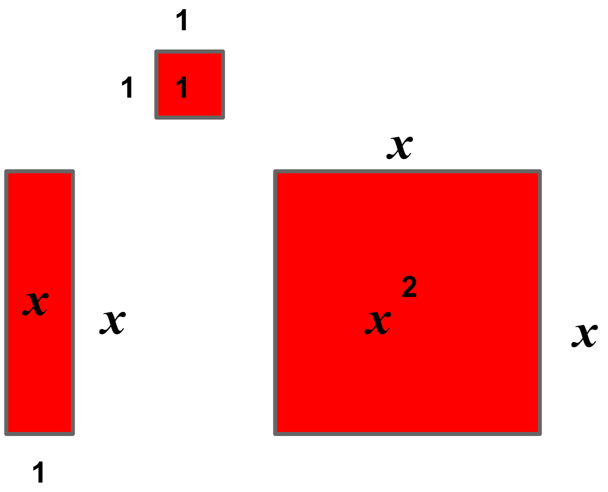
Partial Products ➔ Expanding

	x	$+ \; 3$
x	x^2	$3x$
$+ \; 2$	$2x$	6

$$(x + 3)\,(x + 2)$$

$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$



Algebra Tiles

Show that $2x \neq x^2$

Partial Products ➤ Expanding

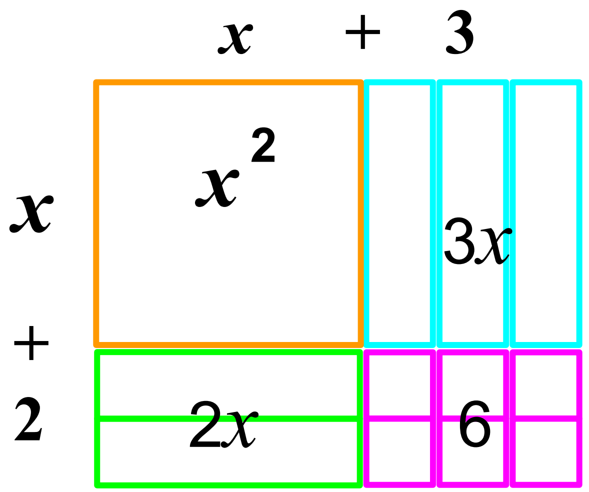
	x	$+ \; 3$
x	x^2	$3x$
$+ \; 2$	$2x$	6

$$(x + 3)\,(x + 2)$$

$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

Expanding - Look familiar?

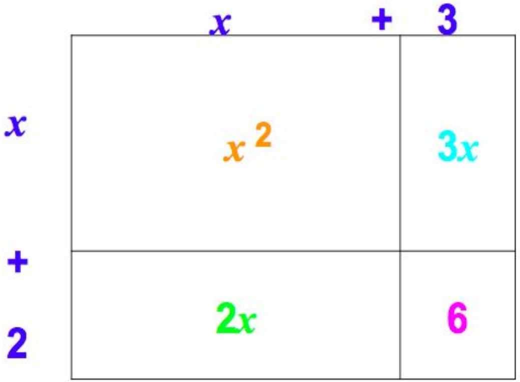


$$(x + 3)\ (x + 2)$$

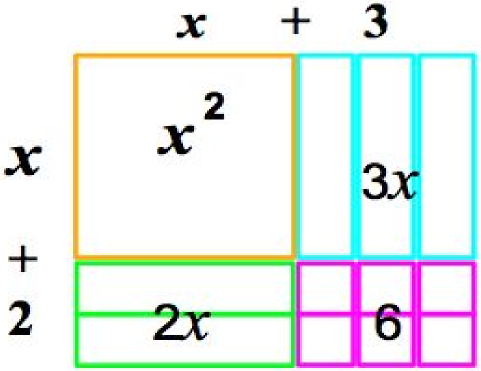
$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

Expanding - Look familiar?



Partial products with open arrays



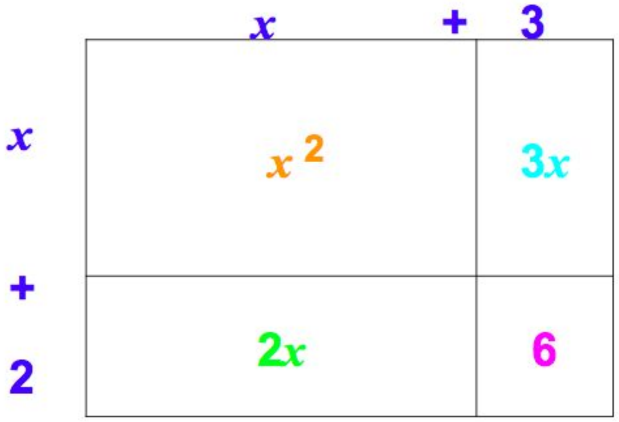
Using algebra tiles

$$(x + 3)(x + 2)$$

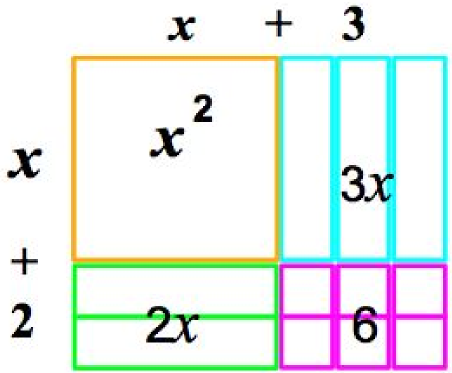
$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

Expanding - Look familiar?



Partial products with open arrays



Using algebra tiles

$$(x + 3)(x + 2)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Helping students make connections and build understanding

How can using **arrays** across grades support the development of student thinking in mathematics?
Conceptual understanding?
Procedural fluency?

Discussion

The Number Line

$$\begin{array}{r} 1005 \\ -\ 997 \\ \hline \end{array}$$

Your Answer?

Did you use...

$$
\begin{array}{r}
{}^{9}\,{}^{1}{}^{9}\,{}^{1} \\
\not{1}\,0\,\not{0}\,5 \\
-\quad 9\,9\,7 \\
\hline
8
\end{array}
$$

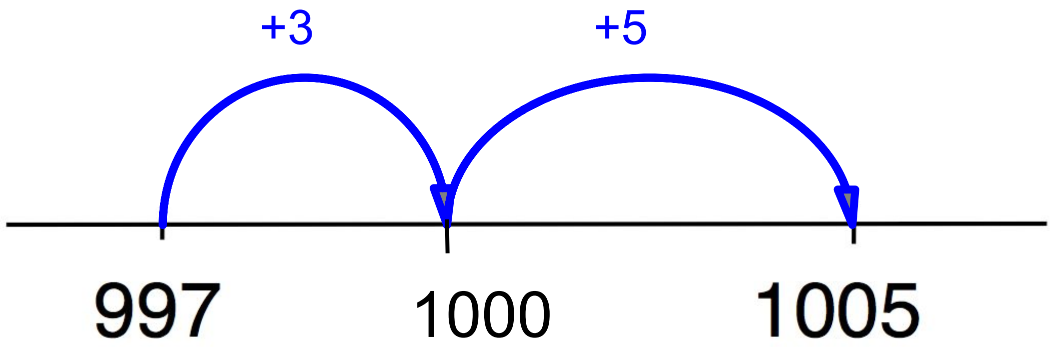
...the traditional algorithm?

Did you use your internal number line?

$$\begin{array}{r} 1005 \\ -\ \ 997 \\ \hline \end{array}$$

Did it look like this?

$$\begin{array}{r} 1005 \\ -\ \ 997 \\ \hline 8 \end{array}$$



In which grades, have you seen number lines used?
How were they used?

Talk Time

Why number lines?

- Much more than just a counting tool
- A powerful model for building mathematical thinking
- Applicable model for all grades
- Provides students with a visual representation of the linear nature of numbers
  - Implies direction
  - Fundamental to the understanding of integers
  - Precursor to algebraic reasoning

Frykholm, J. (2009). Inside math. Longmont, Colorado: Sopris West

Why number lines?

- Can be used to develop understanding of
  - Addition and subtraction
  - Multiplication and division
  - Decimals, fractions, and percents
- Fosters student engagement and participation
- Helps correct misconceptions immediately

Frykholm, J. (2009). Inside math. Longmont, Colorado: Sopris West

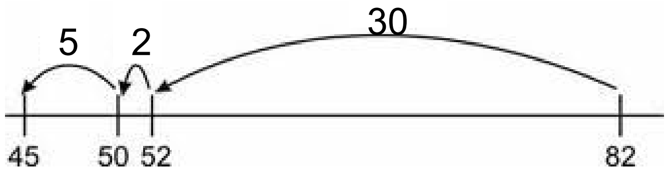
How do you tend to read this?

82 - 37

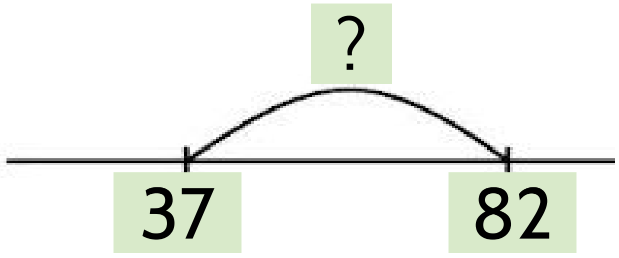
How would you calculate this?

82 - 37

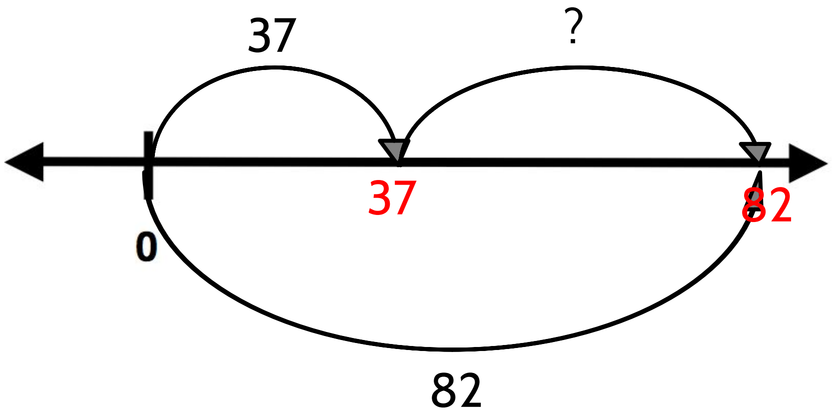
Did you use one of these subtraction methods?



Removal

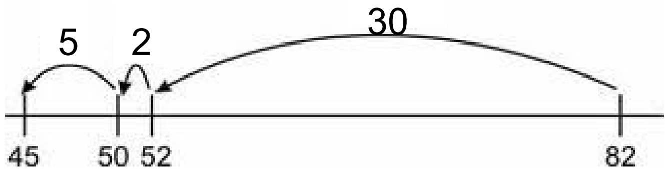


Difference

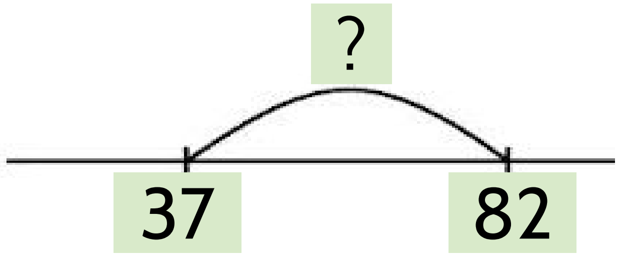


Comparison

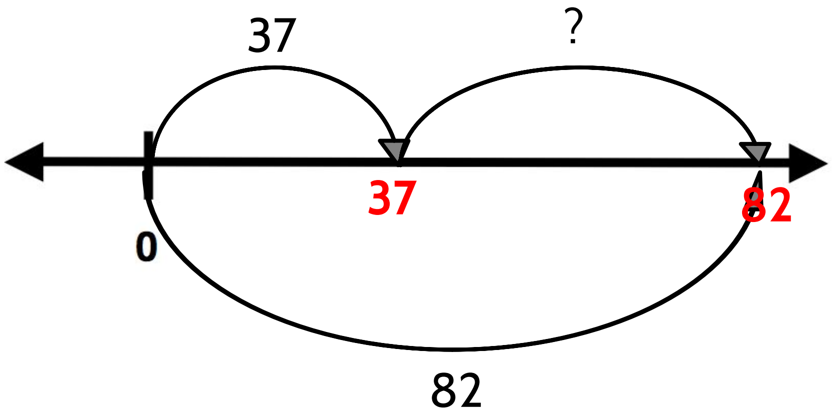
Did you use one of these **subtraction** methods?



Removal



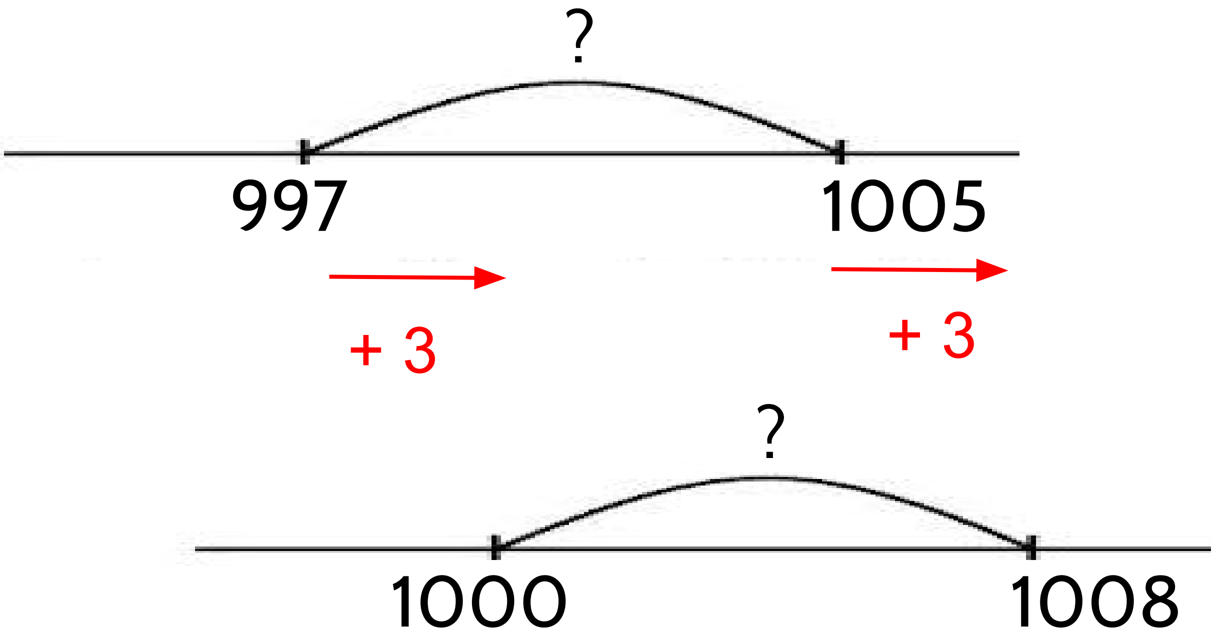
Difference



Comparison

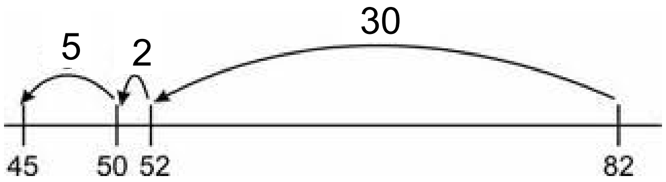
Are we using/recognizing these these different ways in our classes?

The power of the constant difference model

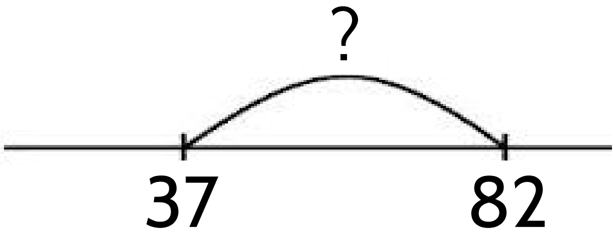


Use the number line to model 53 - 27 as removal and difference.

These samples might help guide your thinking:



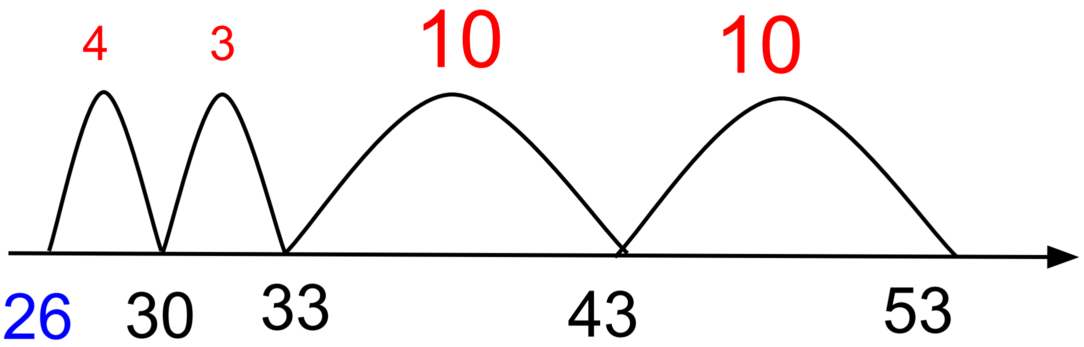
Removal



Difference

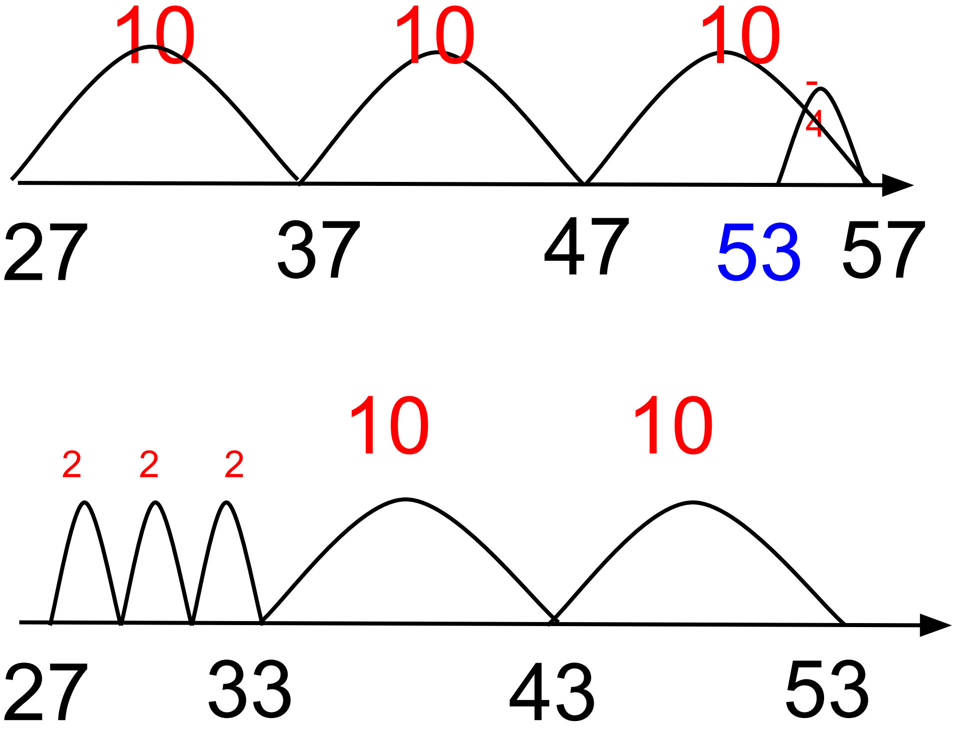
Removal or “take away”

53 – 27 = 26

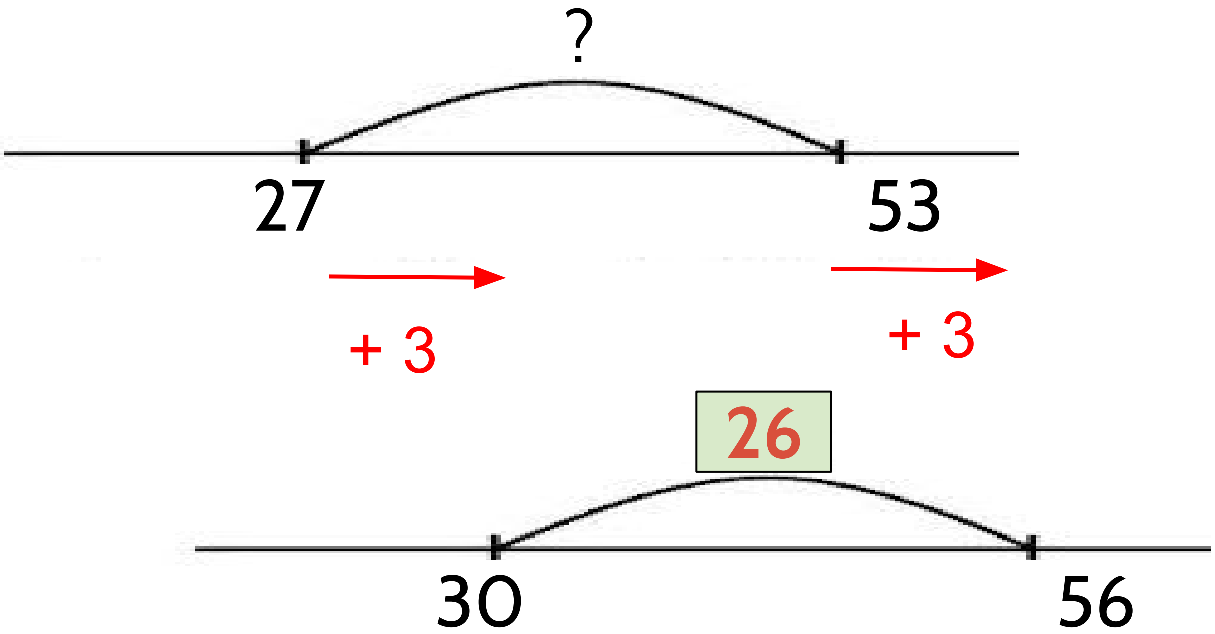


Difference

53 – 27 = 26



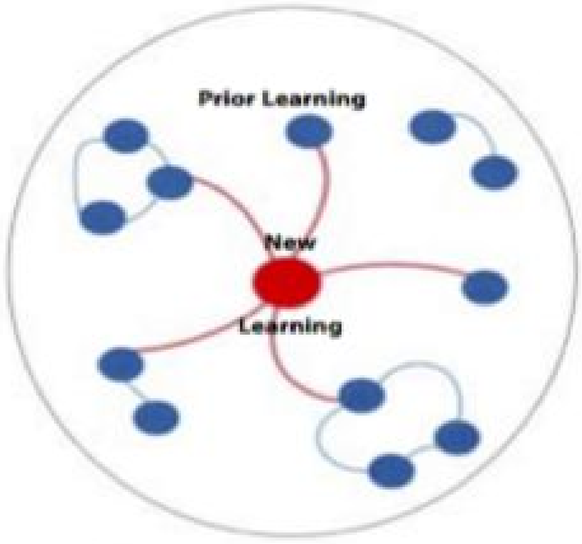
Constant Difference Model



But what about . . . BEYOND?

“We understand something if we see how it is related or connected to other things we know.”
(Van de Walle, 2007, p. 4)

“The degree of understanding is determined by the number and strength of the connections.”
(Hiebert & Carpenter, 1992, p. 67)



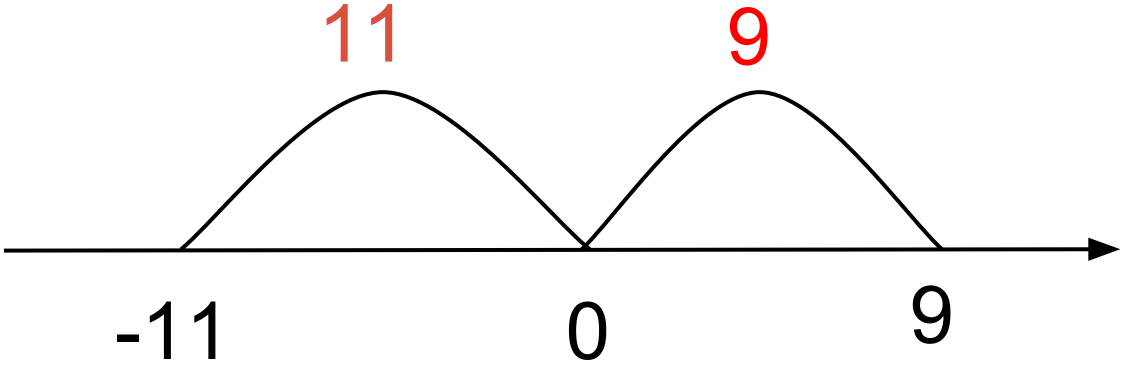
Van de Walle

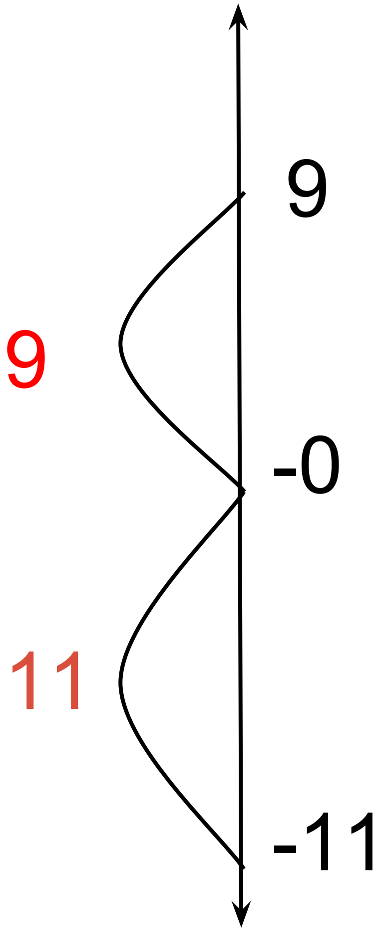
Use a number line to model

9 - (-11)

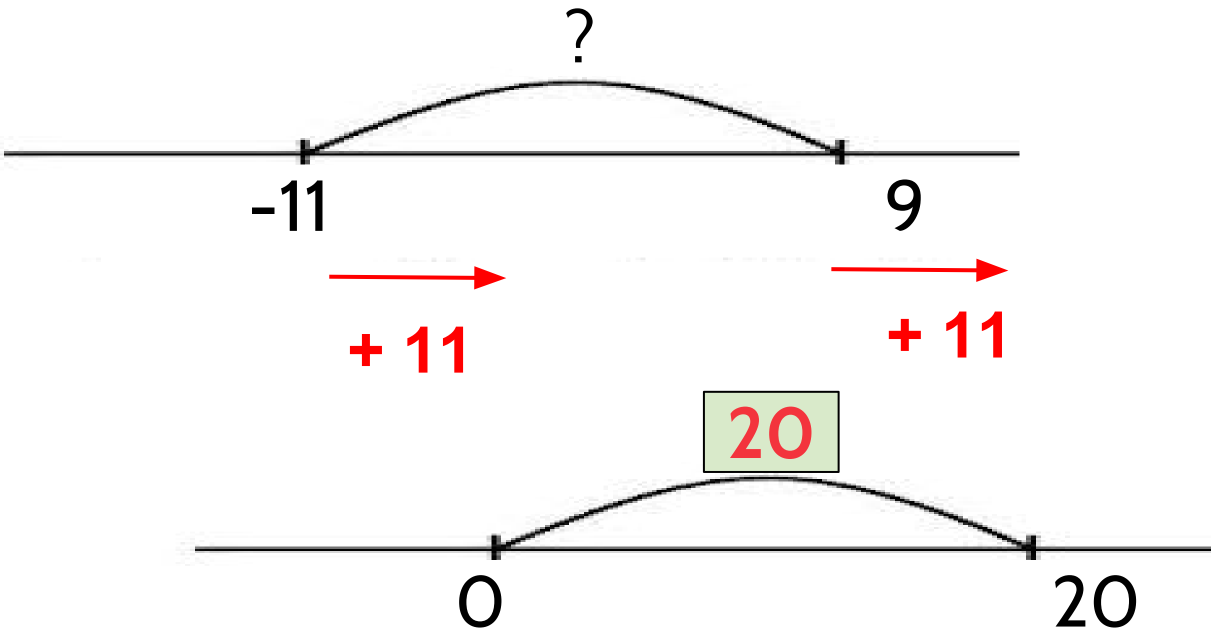
Difference

9 - (-11) = 20





Constant Difference Model



How can using **number lines** across grades support the development of student thinking in mathematics?

Conceptual understanding?

Procedural fluency?

Discussion

In my class?

Classroom look-fors

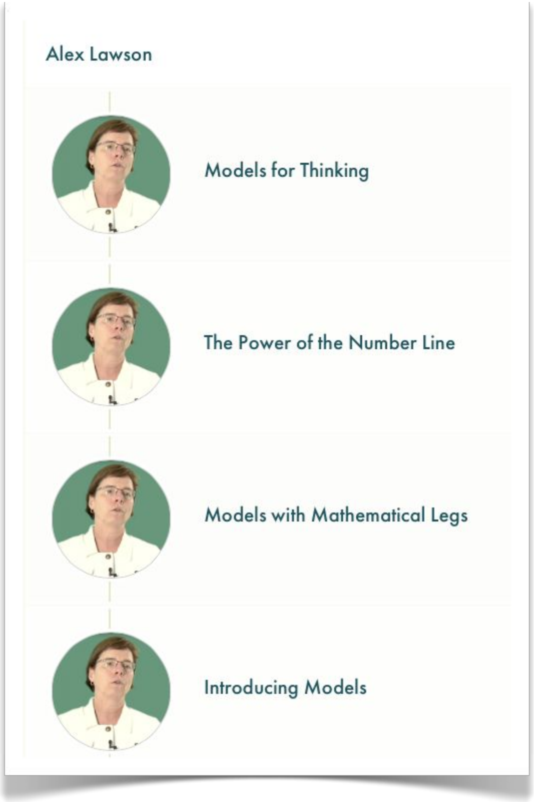
- A grade-appropriate, balanced approach to instruction including the use of concrete materials and abstract concepts
- Purposeful planning and use of models to meet specific needs of students and the standards

In my class?

Key questions you might ask yourself:

What model might I use to support this math concept?

How does this model help deepen or further students' thinking about this math concept?

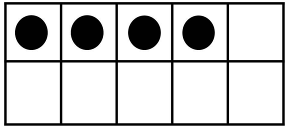
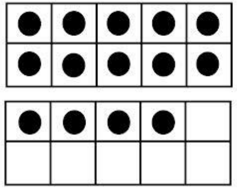


Worth Watching!

Dr. Alex Lawson
Lakehead University

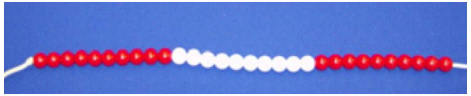
http://www.curriculum.org/k-12/en/projects/leaders-in-educational-thought-special-edition-on-mathematics

PROGRESSION OF MODELS - Number Lines

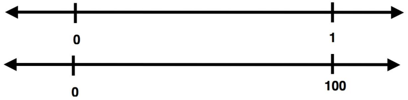
5 and 10 Frames

Rekenrek



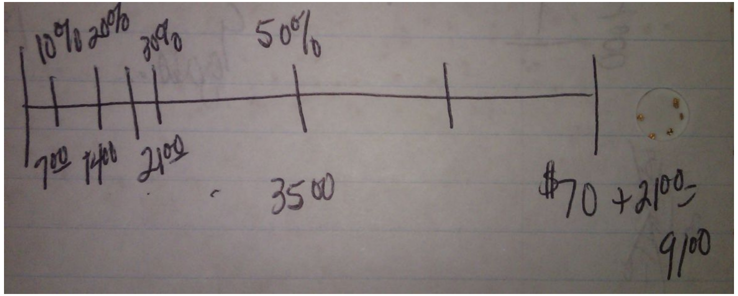
Bead Strings

Open and Closed Number Lines

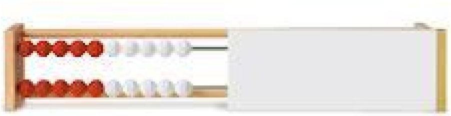
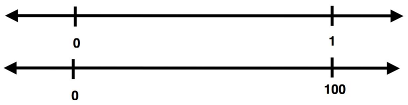
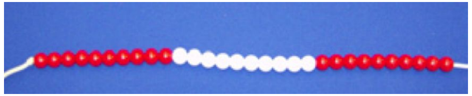


Number lines are appropriate in grade 2

Double Number Lines



FREE Resources

	Learning to Think Mathematically with the Rekenrek	http://cesa5mathscience.wikispaces.com/file/view/Learning+to+Think+Mathematically+with+the+Rekenrek.pdf
	Using the Rekenrek as a visual Model	http://bridges1.mathlearningcenter.org/media/Rekenrek_0308.pdf
	Learning to Think Mathematically with the Number Line	http://bridges1.mathlearningcenter.org/files/media/rekenrek/numberline_overview.pdf
	Getting to Grips with Bead Strings	http://maisdccssm.wikispaces.com/file/view/Ideas+for+Using+Bead+Strings.pdf
	Open Number Lines in a Problem Solving Lesson	http://www.contextsforlearning.com/samples/K3SampUnitOverFINAL.pdf

Models...the last word

The purpose of the models is not to fit the data, but to sharpen the questions.

Samuel Karlin

Learning goals for today?

Why are the array and the number line powerful models of math?

How do they support students' mathematical development – conceptual understanding and procedural fluency?

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https://goo.gl/sfLRnU